Fourth Workshop on Compressible Multiphase Flows — Strasbourg, 30-31/5/2022

Thermal diffusion and phase change in a heat exchanger

A Low Mach Number model

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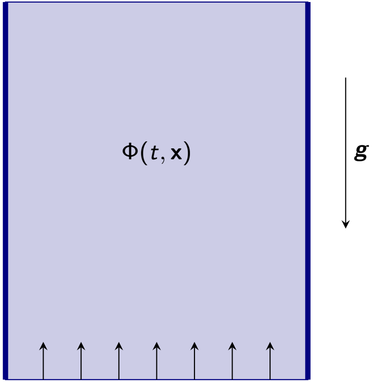
Outline

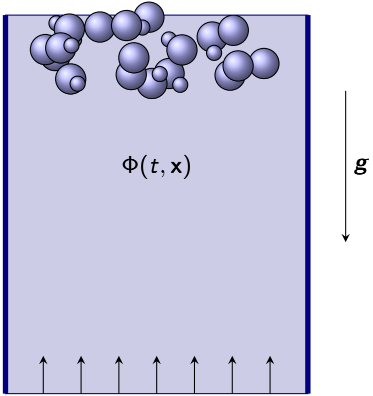
1. Context

A heat exchanger



$v_e(t, \mathbf{x})$ and $h_e(t, \mathbf{x})$

A heat exchanger



$v_e(t, \mathbf{x})$ and $h_e(t, \mathbf{x})$

1. Context

From compressible Navier-Stokes-Fourier to the LMNC model

Compressible Navier-Stokes-Fourier system ⇝ a low Mach number model

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \rho \boldsymbol{g} + \nabla \cdot \sigma(\mathbf{u}) \\ \partial_t (\rho h) + \nabla \cdot (\rho h \mathbf{u}) = \Phi + \nabla \cdot \left(\omega \nabla T\right) + \sigma(\mathbf{u}) : \nabla \mathbf{u} + \partial_t p + \mathbf{u} \cdot \nabla p \end{cases}$$

- Unknows
  - $(t, \mathbf{x}) \mapsto \mathbf{u}$ velocity field
  - $(t, \mathbf{x}) \mapsto h$ enthalpy
  - $(t, \mathbf{x}) \mapsto p$ pressure ⇝ $(t, \mathbf{x}) \mapsto \tilde{p}(t, \mathbf{x})$ perturbational pressure
  - ρ (specific density) and T (temperature) linked to h and p by an equation of state
- Given
  - $(t, \mathbf{x}) \mapsto \Phi \geq 0$ power density modelling the heating
  - $\boldsymbol{g}$ gravity field
  - ω heat conductivity
  - $\sigma(\mathbf{u})$ viscous effects
  - $p_* > 0$ thermodynamic pressure (constant)

Low Mach Number Regime

- Dimensionless System & $\mathcal{M} = \frac{\text{speed of fluid}}{\text{speed of sound}} \ll 1$
- Two pressure fields: $p(t, \mathbf{x}) = p_* + \mathcal{M}^2 \tilde{p}(t, \mathbf{x})$
  - $p_* > 0$ reference (or thermodynamic) pressure: an average pressure (constant in time and space)
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S. Dellacherie, *On A Low Mach Nuclear Core Model*, ESAIM: Proc., 35 (2012)

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A low Mach number model

Non-conservative formulation

$$\begin{cases} \nabla\cdot\mathbf{u} = \left[\Phi(t,\mathbf{y}) + \nabla\cdot\left(\omega(h,p_*)\nabla T(h,p_*)\right)\right] \left.\frac{\partial\tau}{\partial h}\right|_p (h,p_*) \\ \partial_t h + \mathbf{u}\cdot\nabla h = \left[\Phi(t,\mathbf{y}) + \nabla\cdot\left(\omega(h,p_*)\nabla T(h,p_*)\right)\right]\tau(h,p_*) \\ \partial_t\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \tau(h,p_*)\nabla\bar{p} = \mathbf{g} + \tau(h,p_*)\nabla\cdot\sigma(\mathbf{u}) \end{cases}$$

- Unknowns
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  - $(t,\mathbf{x}) \mapsto h$ enthalpy
  - $(t,\mathbf{x}) \mapsto \bar{p}(t,\mathbf{x})$ perturbational pressure
  - $\tau = 1/\rho$ (specific volume) and T (temperature) linked to h and p_* by an equation of state
- Given
  - $(t,\mathbf{x}) \mapsto \Phi \geq 0$ power density modelling the heating
  - $\boldsymbol{g}$ gravity field
  - ω heat conductivity (constant and isotropic for each phase)
  - $\sigma(\mathbf{u})$ viscous effects
  - $p_* > 0$ thermodynamic pressure (constant)

1. Context

Equation of state

- The fluid is in liquid phase (ℓ), vapour phase (g) or a mixture of them
- Pure phase κ:
  - compressible fluid governed by a given (complete) EoS $\implies$

$$(h,p)\mapsto \tau_\kappa(h,p) \text{ and } (h,p)\mapsto T_\kappa(h,p)$$

  - we can also define $(T,p)\mapsto h_\kappa(T,p)$
- Mixture: at saturation (same pressure p, temperature T, chemical potential g)
  - $g_\ell(T,p)=g_g(T,p) \implies p\mapsto T^{sat}(p)$ temperature at saturation
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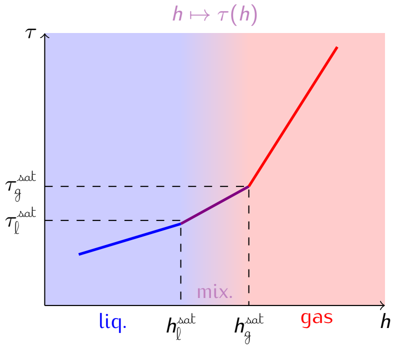
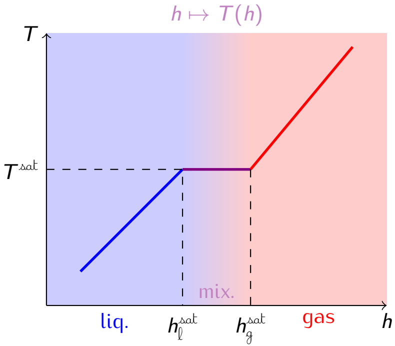
Equation of state

At constant pressure $p = p_*$





Diffusion term

Temperature in the LMNC model

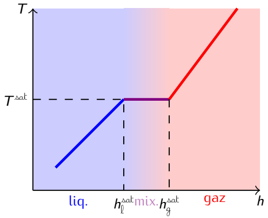
- LMNC model

$$\cdots = \left[\Phi(t, \mathbf{y}) + \nabla\cdot\left(\omega(h, p_*)\nabla T(h, p_*)\right)\right]\cdots$$

- Mixture at saturation and thermodynamic pressure p_* constant:

$$T = T^{sat}(p_*) \text{ when } h_\ell^{sat}(p_*) < h < h_g^{sat}(p_*)$$

$$\omega(h)\nabla T(h) = \begin{cases} \lambda_\ell \nabla h, & \text{if } h \le h_\ell^{sat}, \\ 0, & \text{if } h_\ell^{sat} < h < h_g^{sat}, \\ \lambda_g \nabla h, & \text{if } h \ge h_g^{sat}, \end{cases}$$



where $\lambda_\kappa \stackrel{\text{def}}{=} \frac{\omega_\kappa}{c_{p,\kappa}}$ and $c_{p,\kappa} \stackrel{\text{def}}{=} \left.\frac{\partial h}{\partial T}\right|_p$ is the isobar heat capacity of the phase $\kappa = \ell$ or g.

The 1D model

In the following we neglect the viscous terms and the dependency on pressure p_*.

The LMNC model in a 1D nonconservative formulation

1. v and h solution of

$$\begin{cases} \partial_y v = \left[\Phi(t,\mathbf{y}) + \partial_y\left(\lambda(h)\partial_y h\right)\right]\tau'(h) \\ \partial_t h + v\partial_y h = \left[\Phi(t,\mathbf{y}) + \partial_y\left(\lambda(h)\partial_y h\right)\right]\tau(h) \end{cases}$$

2. $\bar{p}$ solution of

$$\partial_t v + v\partial_y v + \tau(h)\partial_y \bar{p} - \tau(h) = \mathbf{g}$$

$$\lambda(h) = \begin{cases} \lambda_\ell & \text{if } h \le h_\ell^{sat} \\ 0 & \text{if } h_\ell^{sat} < h < h_g^{sat} \\ \lambda_g & \text{if } h \ge h_g^{sat} \end{cases}$$

2. Toy Model for nonlinear diffusion

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The Toy model

- Enthalpy equation
- $\tau(h) \equiv 1$ for all h
- $v, \Phi = Cte > 0$

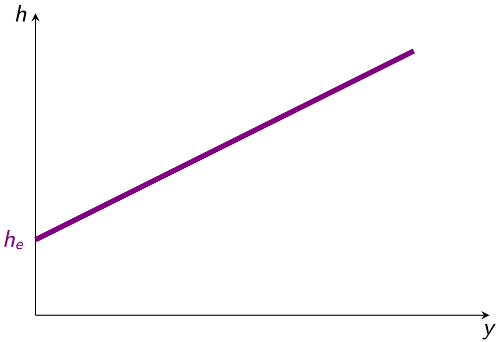
$$\partial_t h + v\partial_y h - \partial_y\left(\lambda(h)\partial_y h\right) = \Phi \quad \text{in } \mathbb{R}^+ \times \mathbb{R}^+$$

- Diffusion $\lambda(h) = \begin{cases} \lambda_\ell \geq 0 & \text{if } h \leq h_\ell^{sat} \\ 0 & \text{if } h_\ell^{sat} < h < h_g^{sat} \\ \lambda_g \geq 0 & \text{if } h \geq h_g^{sat} \end{cases}$
- Inlet condition $h(y=0,t) = h_e < h_\ell^{sat}$
- Initial condition $h(y, t=0) = h_{\text{init}}(y) = h_e$

2. Toy Model for nonlinear diffusion

Steady solution without diffusion

$$\cancel{\partial_t h} + v\partial_y h - \cancel{\partial_y\left(\lambda(h)\partial_y h\right)} = \Phi \implies h(y) = h_e + \frac{\Phi}{v} y$$



Steady solution with / without diffusion ⇝ Notebook Jupyter or ▶ Here

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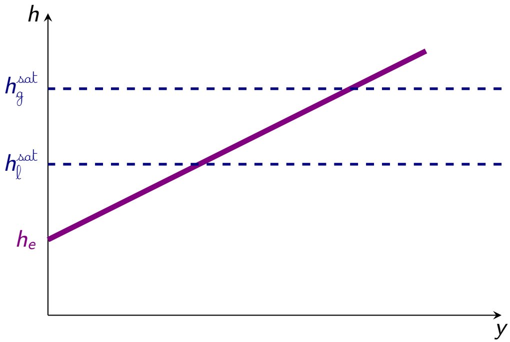

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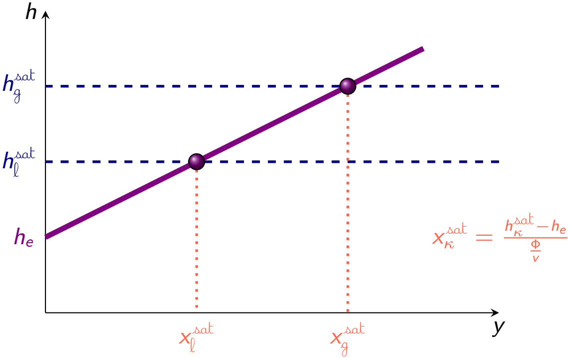

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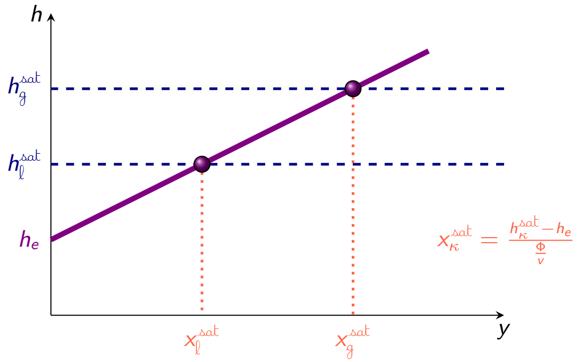

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Steady solution with diffusion – I: Liquid/mixture/gas

$$\text{Case } \frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$$

- The mixture is present
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \stackrel{\text{def}}{=} h_e + \frac{\Phi}{v} y + \frac{\Phi}{v}\frac{\lambda_\ell}{v}\left[1 - \exp\left(\frac{y}{\lambda_\ell/v}\right)\right]\exp\left(-\frac{-y_\ell^{sat}}{\lambda_\ell/v}\right) & \text{if } y \le y_\ell^{sat} \\ h_m^\infty(y) \stackrel{\text{def}}{=} h_\ell^{sat} + \frac{\Phi}{v}(y - y_\ell^{sat}) & \text{if } y_\ell^{sat} \le y < y_g^{sat} \\ h_g^\infty(y) \stackrel{\text{def}}{=} h_g^{sat} + \frac{\Phi}{v}(y - y_g^{sat}) & \text{if } y \ge y_g^{sat} \end{cases}$$

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- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \overset{\text{def}}{=} h_e + \frac{\Phi}{v} y + \frac{\Phi}{v}\frac{\lambda_\ell}{v}\left[1 - \exp\left(\frac{y}{\lambda_\ell/v}\right)\right]\exp\left(-\frac{-y_\ell^{sat}}{\lambda_\ell/v}\right) & \text{if } y \le y_\ell^{sat} \\ h_m^\infty(y) \overset{\text{def}}{=} h_\ell^{sat} + \frac{\Phi}{v}(y - y_\ell^{sat}) & \text{if } y_\ell^{sat} \le y < y_g^{sat} \\ h_g^\infty(y) \overset{\text{def}}{=} h_g^{sat} + \frac{\Phi}{v}(y - y_g^{sat}) & \text{if } y \ge y_g^{sat} \end{cases}$$

- The position y_ℓ^{sat} is implicitly defined by $h_\ell^\infty(y_\ell^{sat}) = h_\ell^{sat}$ and we have $\left(h_\ell^\infty\right)'(y_\ell^{sat}) = 0$
- The position y_g^{sat} is computed w.r.t. y_ℓ^{sat} by $y_g^{sat} = y_\ell^{sat} + \frac{v}{\Phi}(h_g^{sat} - h_\ell^{sat}) - \frac{\lambda_g}{v}$
- Gas diffusion reduces the mixture region for steady solution

$$(y_g^{sat} - y_\ell^{sat}) = (x_g^{sat} - x_\ell^{sat}) - \frac{\lambda_g}{v}$$

- The jump is in the mixture region and

Steady solution with diffusion – I: Liquid/mixture/gas

$$\text{Case } \frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$$

- The mixture is present
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \overset{\text{def}}{=} h_e + \frac{\Phi}{v} y + \frac{\Phi}{v}\frac{\lambda_\ell}{v}\left[1 - \exp\left(\frac{y}{\lambda_\ell/v}\right)\right]\exp\left(-\frac{-y_\ell^{sat}}{\lambda_\ell/v}\right) & \text{if } y \leq y_\ell^{sat} \\ h_m^\infty(y) \overset{\text{def}}{=} h_\ell^{sat} + \frac{\Phi}{v}(y - y_\ell^{sat}) & \text{if } y_\ell^{sat} \leq y < y_g^{sat} \\ h_g^\infty(y) \overset{\text{def}}{=} h_g^{sat} + \frac{\Phi}{v}(y - y_g^{sat}) & \text{if } y \geq y_g^{sat} \end{cases}$$

- The position y_ℓ^{sat} is implicitly defined by $h_\ell^\infty(y_\ell^{sat}) = h_\ell^{sat}$ and we have $\left(h_\ell^\infty\right)'(y_\ell^{sat}) = 0$
- The position y_g^{sat} is computed w.r.t. y_ℓ^{sat} by $y_g^{sat} = y_\ell^{sat} + \frac{v}{\Phi}(h_g^{sat} - h_\ell^{sat}) - \frac{\lambda_g}{v}$
- Gas diffusion reduces the mixture region for steady solution

$$(y_g^{sat} - y_\ell^{sat}) = (x_g^{sat} - x_\ell^{sat}) - \frac{\lambda_g}{v}$$

- The jump is in the mixture region and

Steady solution with diffusion – II: Liquid/gas (no mixture)

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$

- The mixture does not exist
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \overset{\text{def}}{=} h_e + \frac{\Phi}{v}y + \left[\left(h_g^{sat} - h_\ell^{sat}\right) - \left(\frac{\lambda_g}{v} - \frac{\lambda_\ell}{v}\right)\frac{\Phi}{v}\right]\left[1 - \exp\left(\frac{y}{\lambda_\ell/v}\right)\right]\exp\left(\frac{-y^{sat}}{\lambda_\ell/v}\right) & \text{if } y < y^{sat} \\ h_g^\infty(y) = h_g^{sat} + \frac{\Phi}{v}(y - y^{sat}) & \text{if } y > y^{sat} \end{cases}$$

- If $\lambda_\ell > 0$, the jump is constant (as in Stefan problems)

$$[\![h]\!](y^{sat}) = h_g^\infty(y^{sat}) - h_\ell^\infty(y^{sat}) = h_g^{sat} - h_\ell^{sat}$$

- The position $y^{sat} = y_\ell^{sat} = y_g^{sat}$ is implicitly defined by $h_\ell^\infty(y^{sat}) = h_\ell^{sat}$ and we have

$$\left(h_\ell^\infty\right)'(y^{sat}) = \frac{\frac{\lambda_g}{v}\frac{\Phi}{v} - \left(h_g^{sat} - h_\ell^{sat}\right)}{\frac{\lambda_\ell}{v}} \qquad \left[\geq 0 \text{ and } \xrightarrow[\lambda_\ell \to 0]{} +\infty\right]$$

- If $\lambda_\ell = 0$, the jump is in the liquid region and $h_\ell^\infty(y^{sat}) < h_\ell^{sat}$:

$$[\![h]\!](y^{sat}) = h_g^{sat} - h_\ell^\infty(y^{sat}) = \frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$$

Steady solution with diffusion – II: Liquid/gas (no mixture)

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$

- The mixture does not exist
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \stackrel{\text{def}}{=} h_e + \frac{\Phi}{v}y + \left[(h_g^{sat} - h_\ell^{sat}) - \left(\frac{\lambda_g}{v} - \frac{\lambda_\ell}{v}\right)\frac{\Phi}{v}\right]\left[1 - \exp\left(\frac{y}{\lambda_\ell/v}\right)\right]\exp\left(\frac{-y^{sat}}{\lambda_\ell/v}\right) & \text{if } y < y^{sat} \\ h_g^\infty(y) = h_g^{sat} + \frac{\Phi}{v}(y - y^{sat}) & \text{if } y > y^{sat} \end{cases}$$

- If $\lambda_\ell > 0$, the jump is constant (as in Stefan problems)

$$[\![h]\!](y^{sat}) = h_g^\infty(y^{sat}) - h_\ell^\infty(y^{sat}) = h_g^{sat} - h_\ell^{sat}$$

- The position $y^{sat} = y_\ell^{sat} = y_g^{sat}$ is implicitly defined by $h_\ell^\infty(y^{sat}) = h_\ell^{sat}$ and we have

$$\left(h_\ell^\infty\right)'(y^{sat}) = \frac{\frac{\lambda_g}{v}\frac{\Phi}{v} - (h_g^{sat} - h_\ell^{sat})}{\frac{\lambda_\ell}{v}} \qquad \left[\geq 0 \text{ and } \xrightarrow[\lambda_\ell \to 0]{} +\infty\right]$$

- If $\lambda_\ell = 0$, the jump is in the liquid region and $h_\ell^\infty(y^{sat}) < h_\ell^{sat}$:

$$[\![h]\!](y^{sat}) = h_g^{sat} - h_\ell^\infty(y^{sat}) = \frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$$

Steady solution with diffusion – II: Liquid/gas (no mixture)

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$

- The mixture does not exist
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \stackrel{\text{def}}{=} h_e + \frac{\Phi}{v} y + \left[(h_g^{sat} - h_\ell^{sat}) - \left(\frac{\lambda_g}{v} - \frac{\lambda_\ell}{v}\right)\frac{\Phi}{v}\right] \left[1 - \exp\left(\frac{y}{\lambda_\ell/v}\right)\right] \exp\left(\frac{-y^{sat}}{\lambda_\ell/v}\right) & \text{if } y < y^{sat} \\ h_g^\infty(y) = h_g^{sat} + \frac{\Phi}{v}(y - y^{sat}) & \text{if } y > y^{sat} \end{cases}$$

- If $\lambda_\ell > 0$, the jump is constant (as in Stefan problems)

$$[\![h]\!](y^{sat}) = h_g^\infty(y^{sat}) - h_\ell^\infty(y^{sat}) = h_g^{sat} - h_\ell^{sat}$$

- The position $y^{sat} = y_\ell^{sat} = y_g^{sat}$ is implicitly defined by $h_\ell^\infty(y^{sat}) = h_\ell^{sat}$ and we have

$$\left(h_\ell^\infty\right)'(y^{sat}) = \frac{\frac{\lambda_g}{v}\frac{\Phi}{v} - (h_g^{sat} - h_\ell^{sat})}{\frac{\lambda_\ell}{v}} \qquad \left[\geq 0 \text{ and } \xrightarrow[\lambda_\ell \to 0]{} +\infty\right]$$

- If $\lambda_\ell = 0$, the jump is in the liquid region and $h_\ell^\infty(y^{sat}) < h_\ell^{sat}$:

$$[\![h]\!](y^{sat}) = h_g^{sat} - h_\ell^\infty(y^{sat}) = \frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$$

Steady solution with diffusion – II: Liquid/gas (no mixture)

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$

- The mixture does not exist
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \stackrel{\text{def}}{=} h_e + \frac{\Phi}{v} y + \left[\left(h_g^{sat} - h_\ell^{sat} \right) - \left(\frac{\lambda_g}{v} - \frac{\lambda_\ell}{v} \right) \frac{\Phi}{v} \right] \left[1 - \exp\left(\frac{y}{\lambda_\ell / v} \right) \right] \exp\left(\frac{-y^{sat}}{\lambda_\ell / v} \right) & \text{if } y < y^{sat} \\ h_g^\infty(y) = h_g^{sat} + \frac{\Phi}{v}(y - y^{sat}) & \text{if } y > y^{sat} \end{cases}$$

- If $\lambda_\ell > 0$, the jump is constant (as in Stefan problems)

$$[\![h]\!](y^{sat}) = h_g^\infty(y^{sat}) - h_\ell^\infty(y^{sat}) = h_g^{sat} - h_\ell^{sat}$$

- The position $y^{sat} = y_\ell^{sat} = y_g^{sat}$ is implicitly defined by $h_\ell^\infty(y^{sat}) = h_\ell^{sat}$ and we have

$$\left(h_\ell^\infty\right)'(y^{sat}) = \frac{\frac{\lambda_g}{v}\frac{\Phi}{v} - \left(h_g^{sat} - h_\ell^{sat}\right)}{\frac{\lambda_\ell}{v}} \qquad \left[\geq 0 \text{ and } \xrightarrow[\lambda_\ell \to 0]{} +\infty \right]$$

- If $\lambda_\ell = 0$, the jump is in the liquid region and $h_\ell^\infty(y^{sat}) < h_\ell^{sat}$:

$$[\![h]\!](y^{sat}) = h_g^{sat} - h_\ell^\infty(y^{sat}) = \frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$$

Steady solution with diffusion – II: Liquid/gas (no mixture)

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$

- The mixture does not exist
- The solution satisfies

$$h^\infty(y) = \begin{cases} h_\ell^\infty(y) \stackrel{\text{def}}{=} h_e + \frac{\Phi}{v} y + \left[(h_g^{sat} - h_\ell^{sat}) - \left(\frac{\lambda_g}{v} - \frac{\lambda_\ell}{v} \right) \frac{\Phi}{v} \right] \left[1 - \exp\left(\frac{y}{\lambda_\ell / v} \right) \right] \exp\left(\frac{-y^{sat}}{\lambda_\ell / v} \right) & \text{if } y < y^{sat} \\ h_g^\infty(y) = h_g^{sat} + \frac{\Phi}{v}(y - y^{sat}) & \text{if } y > y^{sat} \end{cases}$$

- If $\lambda_\ell > 0$, the jump is constant (as in Stefan problems)

$$[\![h]\!](y^{sat}) = h_g^\infty(y^{sat}) - h_\ell^\infty(y^{sat}) = h_g^{sat} - h_\ell^{sat}$$

- The position $y^{sat} = y_\ell^{sat} = y_g^{sat}$ is implicitly defined by $h_\ell^\infty(y^{sat}) = h_\ell^{sat}$ and we have

$$\left(h_\ell^\infty\right)'(y^{sat}) = \frac{\frac{\lambda_g}{v}\frac{\Phi}{v} - (h_g^{sat} - h_\ell^{sat})}{\frac{\lambda_\ell}{v}} \qquad \left[\geq 0 \text{ and } \xrightarrow[\lambda_\ell \to 0]{} +\infty \right]$$

- If $\lambda_\ell = 0$, the jump is in the liquid region and $h_\ell^\infty(y^{sat}) < h_\ell^{sat}$:

$$[\![h]\!](y^{sat}) = h_g^{sat} - h_\ell^\infty(y^{sat}) = \frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$$

2. Toy Model for nonlinear diffusion

Numerical Time-Dependent solution

The unified model

$$\partial_t h + v\partial_y h - \partial^2_{yy}(L(h)) = \Phi, \qquad L(h) \overset{\text{def}}{=} \begin{cases} \lambda_\ell(h - h_\ell^{sat}) & \text{if } h \leq h_\ell^{sat} \\ 0 & \text{if } h_\ell^{sat} < h < h_g^{sat} \\ \lambda_g(h - h_g^{sat}) & \text{if } h \geq h_g^{sat} \end{cases}$$

(so that $L'(h) = \lambda(h)$)

is solved by a

Fully implicit scheme

$$\frac{h^{n+1} - h^n}{\delta t} + v\partial_y h^{n+1} - \partial^2_{yy}(L(h^{n+1})) = \Phi \quad \text{in } \mathbb{R}^+$$

associated to a gradient scheme [Eymard et al. 2013]

1. No diffusion; influence of Φ on Δx_κ^{sat} the width of the mixture zone
   - $\Phi = 2, \lambda_\ell = 0, \lambda_g = 0$ | $\Phi = 1, \lambda_\ell = 0, \lambda_g = 0$
2. Diffusion in the liquid phase: influence on y_ℓ^{sat} and the slope at y_ℓ^{sat}
   - $\Phi = 1, \lambda_\ell = 0 \rightarrow 2, \lambda_g = 0$
3. Diffusion in the gas phase: influence on Δy_κ^{sat} the width of the mixture zone and jump at y_g^{sat}
   - $\Phi = 1, \lambda_\ell = 0, \lambda_g = 0 \rightarrow 1$

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4. Diffusion in liquid and vapour phases, existence of the mixture zone
   - $\Phi = 1, \lambda_\ell = 0 \rightarrow 1, \lambda_g = 1$
5. Diffusion in liquid and vapour phases, disappearence of the mixture zone
   - $\Phi = 1, \lambda_\ell = 1, \lambda_g = 1 \rightarrow 2$

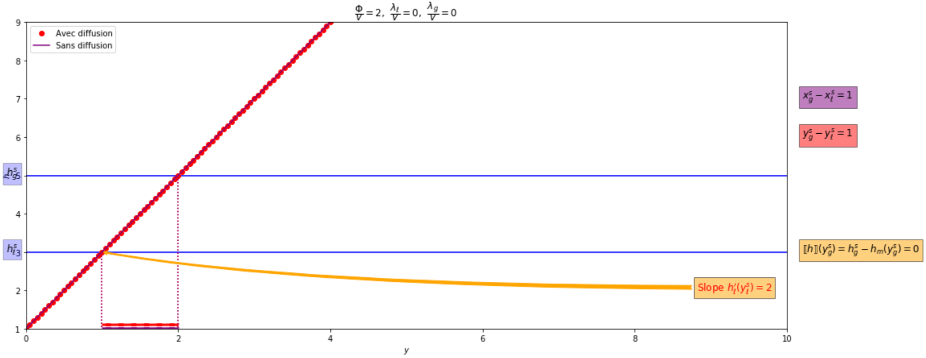
---

6. Diffusion in liquid and vapour phases, no mixture zone, focus on slope at y_ℓ^{sat}
   - $\Phi = 1, \lambda_\ell = 1, \lambda_g = 2 \rightarrow 3$ | $\Phi = 1, \lambda_\ell = 1, \lambda_g = 3 \rightarrow 4$
7. Diffusion in liquid and vapour phases, no mixture zone, jump in liquid phase
   - $\Phi = \frac{1}{2}, \lambda_\ell = 1, \lambda_g = 5$ | $\Phi = \frac{1}{2}, \lambda_\ell = 0.1, \lambda_g = 5$ | $\Phi = \frac{1}{2}, \lambda_\ell = 0, \lambda_g = 5$

Return

$\frac{\Phi}{v} = 2, \frac{\lambda_\ell}{v} = 0, \frac{\lambda_g}{v} = 0$

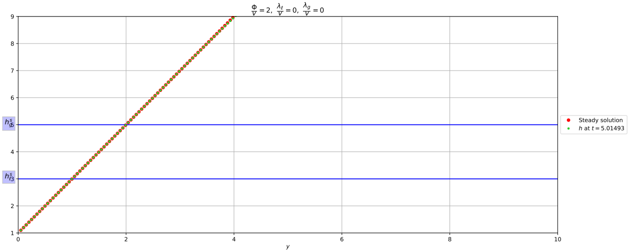
Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state



No diffusion; influence of Φ on Δx_κ^{sat} the width of the mixture zone

$\frac{\Phi}{v} = 2, \frac{\lambda_\ell}{v} = 0, \frac{\lambda_g}{v} = 0$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

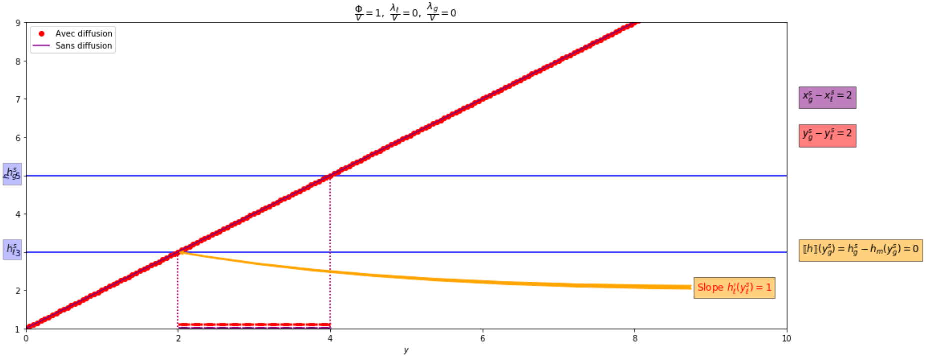


No diffusion; influence of Φ on Δx_κ^{sat} the width of the mixture zone

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 0, \frac{\lambda_g}{v} = 0$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

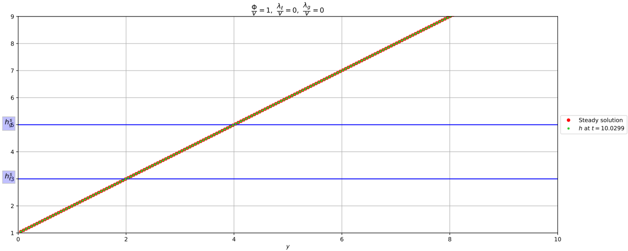


No diffusion; influence of Φ on Δx_κ^{sat} the width of the mixture zone

Return

$\frac{\Phi}{v} = 1, \ \frac{\lambda_\ell}{v} = 0, \ \frac{\lambda_g}{v} = 0$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

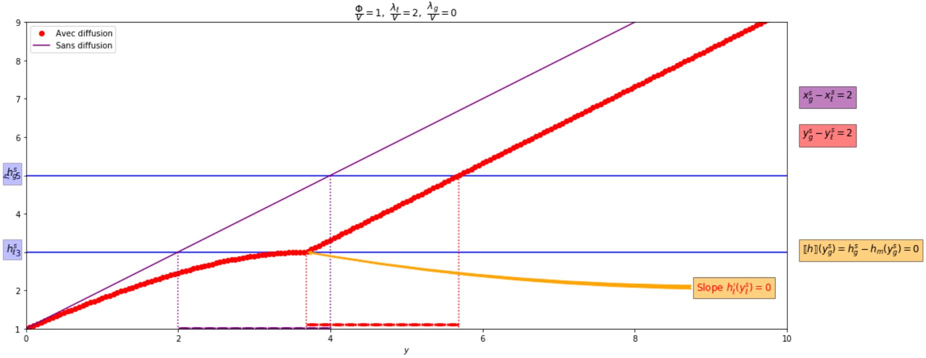


No diffusion; influence of Φ on Δx_κ^{sat} the width of the mixture zone

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 2, \frac{\lambda_g}{v} = 0$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

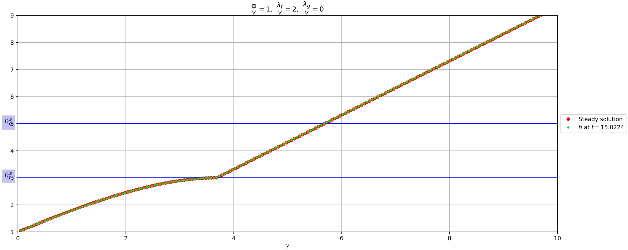


Diffusion in the liquid phase: influence on y_ℓ^{sat} and the slope at y_ℓ^{sat}

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 2, \frac{\lambda_g}{v} = 0$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

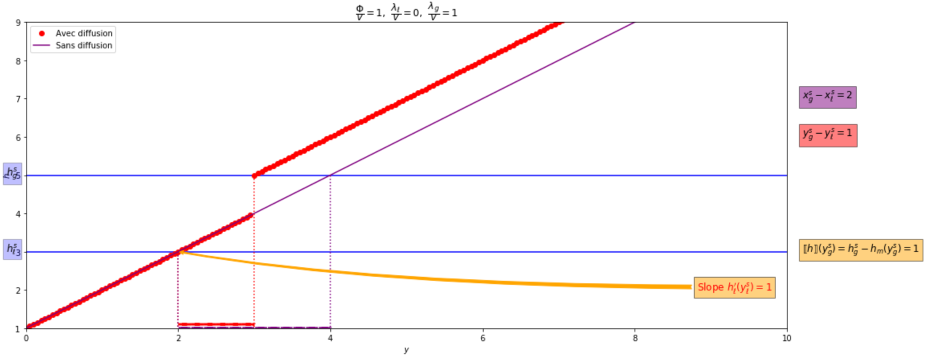


Diffusion in the liquid phase: influence on y_ℓ^{sat} and the slope at y_ℓ^{sat}

Return

$\frac{\Phi}{v} = 1,\ \frac{\lambda_\ell}{v} = 0,\ \frac{\lambda_g}{v} = 1$

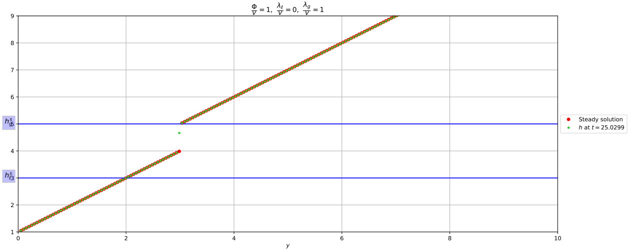
Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state



Diffusion in the gas phase: influence on Δy_κ^{sat} the width of the mixture zone and jump at y_g^{sat}

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 0, \frac{\lambda_g}{v} = 1$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

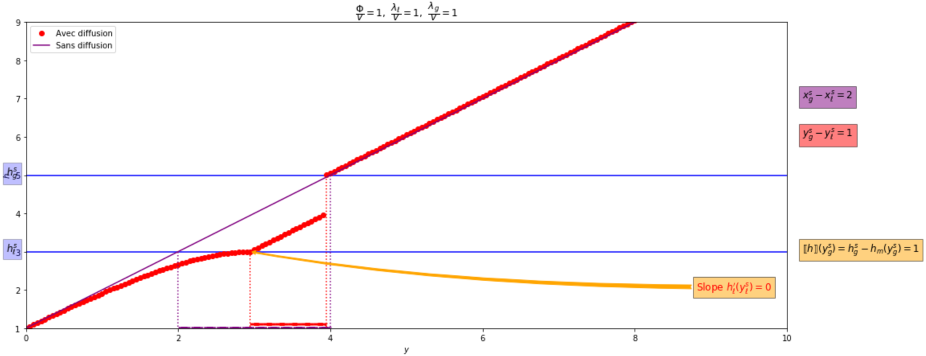


Diffusion in the gas phase: influence on Δy_κ^{sat} the width of the mixture zone and jump at y_g^{sat}

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 1$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

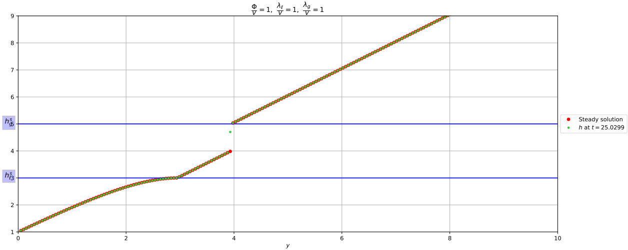


Diffusion in liquid and vapour phases, existence of the mixture zone at steady state

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 1$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} < h_g^{sat} - h_\ell^{sat}$: mixture zone at steady state

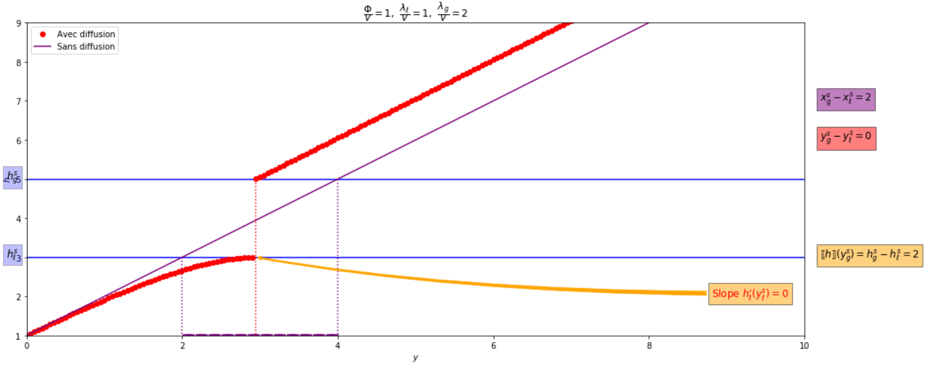


Diffusion in liquid and vapour phases, existence of the mixture zone at steady state

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 2$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

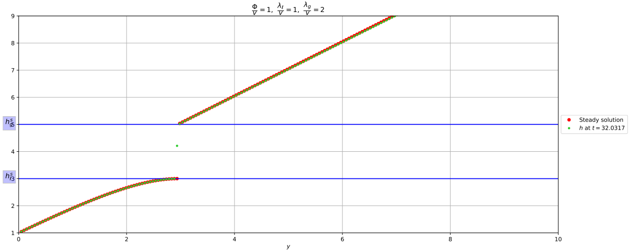


Diffusion in liquid and vapour phases, disappear of the mixture zone at steady state

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 2$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

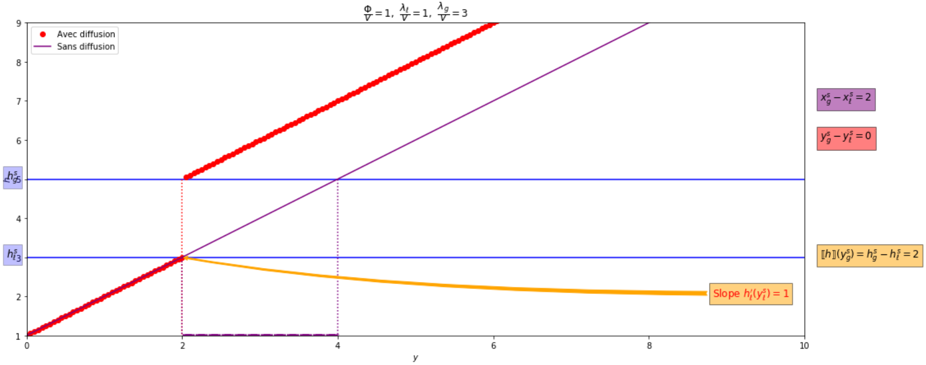


Diffusion in liquid and vapour phases, disappear of the mixture zone at steady state

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 3$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

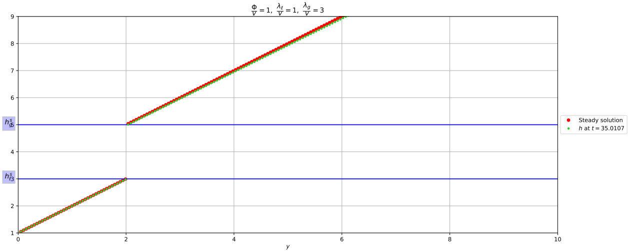


Diffusion in liquid and vapour phases, no mixture zone, focus on the slope at $y^{sat,-}$

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 3$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

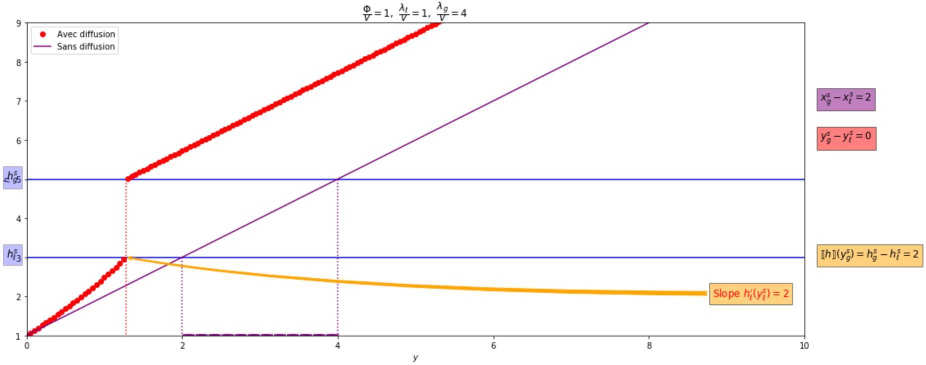


Diffusion in liquid and vapour phases, no mixture zone, focus on the slope at $y^{sat,-}$

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 4$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

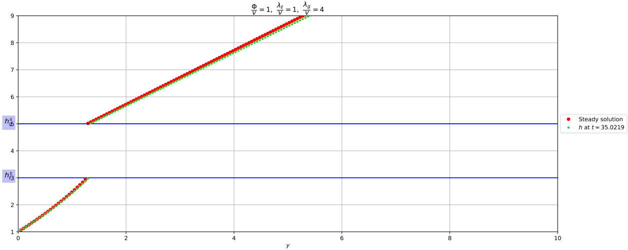


Diffusion in liquid and vapour phases, no mixture zone, the slope at $y^{sat,-}$ can be $> \frac{\Phi}{v}$

Return

$\frac{\Phi}{v} = 1, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 4$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

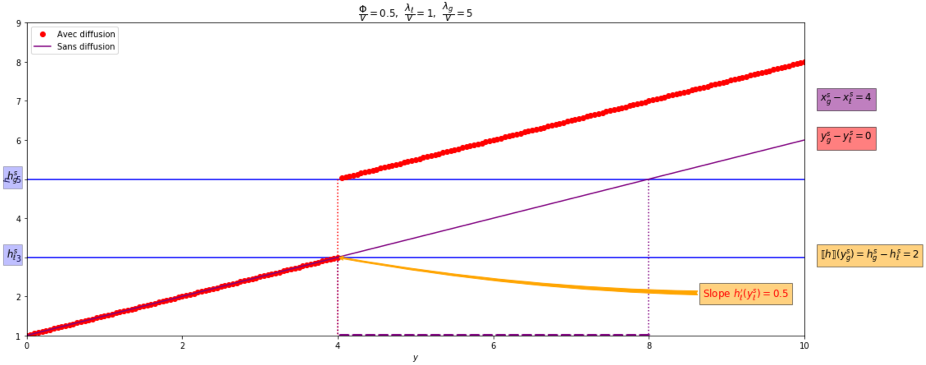


Diffusion in liquid and vapour phases, no mixture zone, the slope at $y^{sat,-}$ can be $> \frac{\Phi}{v}$

Return

$\frac{\Phi}{v} = \frac{1}{2}, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 5$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

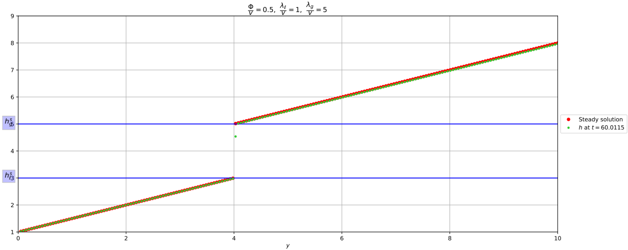


Diffusion in liquid and vapour phases, no mixture zone, focus on the slope at $y^{sat,-}$

Return

$\frac{\Phi}{v} = \frac{1}{2}, \frac{\lambda_\ell}{v} = 1, \frac{\lambda_g}{v} = 5$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state

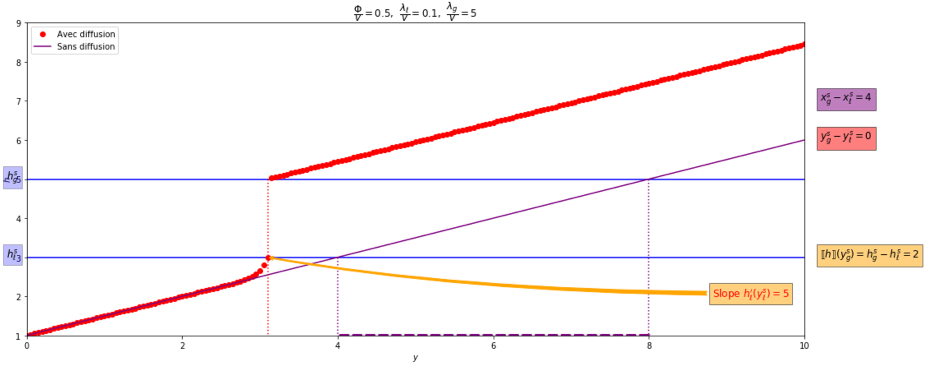


Diffusion in liquid and vapour phases, no mixture zone, focus on the slope at $y^{sat,-}$

Return

$\frac{\Phi}{v} = \frac{1}{2}, \ \frac{\lambda_\ell}{v} = 0.1, \ \frac{\lambda_g}{v} = 5$

Case $\frac{\lambda_g}{v}\frac{\Phi}{v} \geq h_g^{sat} - h_\ell^{sat}$: no mixture zone at steady state



Diffusion in liquid and vapour phases, no mixture zone, focus on the slope at $y^{sat,-}$ when $\lambda_\ell \to 0$

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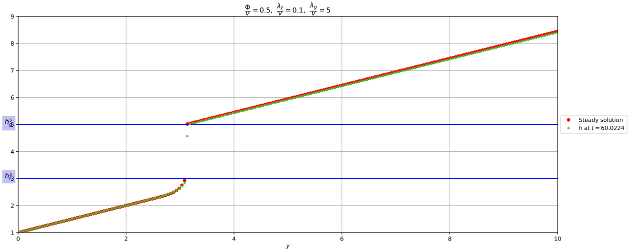

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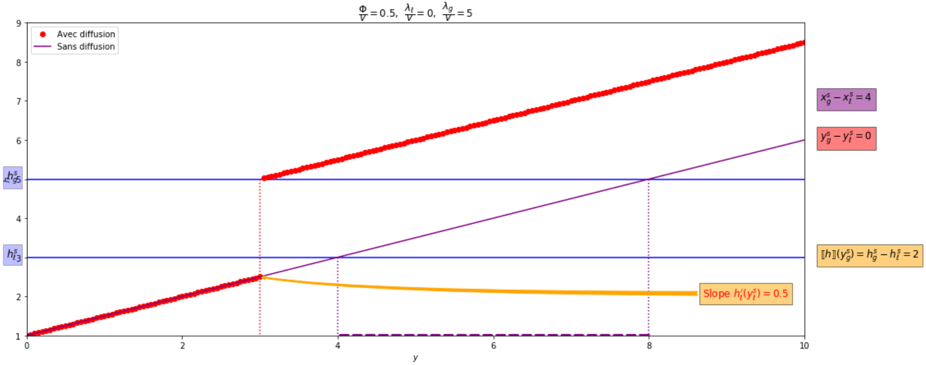

Diffusion in vapour phase, no mixture zone, jump in liquid phase at y^{sat}

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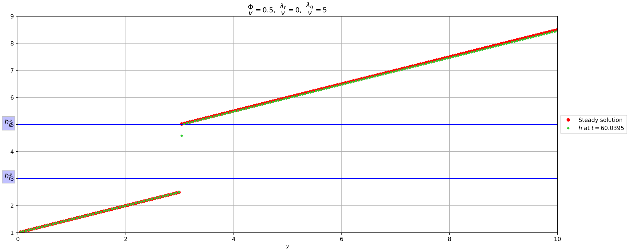

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3. Conclusion

Conclusion & Perspectives

1. Without diffusion, the mixture zone is always present
2. Diffusion in gas reduces the mixture zone and can disappear if sufficiently high:
   - diffusion in gas can be neglected if
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3. Robust numerical scheme with appearance/disappearances of phases and jumps

Next steps

- Add variable specific volume $v(h)$
- Add Diffusion for full LMNC model (*i.e.* divergence equation)

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4. Appendix

4.1 Jump relations in evolution problem

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Jump relations for liquid/mixture

Assuming a liquid/mixture transition at $y_\ell^{sat}(t)$

$$\partial_t h + v\partial_y h - \partial_y\left(\lambda(h)\partial_y h\right) = \Phi, \qquad \lambda(h) = \begin{cases}\lambda_\ell, & \text{if } h \leq h_\ell^{sat}\\ 0, & \text{otherwise}\end{cases}$$

$$\left(v - (y_\ell^{sat})'(t)\right)\left(h(y_\ell^{sat,+}, t) - h(y_\ell^{sat,-}, t)\right) = 0\partial_y h(y_\ell^{sat,+}, t) - \lambda_\ell \partial_y h(y_\ell^{sat,-}, t)$$

If $v > (y_\ell^{sat})'(t)$ and $h(\cdot, t)$ increasing this implies that

$$\begin{cases} h(y_\ell^{sat,-}(t), t) = h(y_\ell^{sat,+}(t), t) = h_\ell^{sat}\\ \partial_y h(t, y_\ell^{sat,-}(t)) = 0\end{cases}$$

Jump relations for mixture/gas

Assuming a mixture/gas transition at $y_g^{sat}(t)$

$$\partial_t h + v\partial_y h - \partial_y\left(\lambda(h)\partial_y h\right) = \Phi, \qquad \lambda(h) = \begin{cases} 0, & \text{if } h < h_g^{sat} \\ \lambda_g, & \text{otherwise} \end{cases}$$

$$\left(v - (y_g^{sat})'(t)\right)\left(h(y_g^{sat,+}, t) - h(y_g^{sat,-}, t)\right) = \lambda_g \partial_y h(y_g^{sat,+}, t) - 0\partial_y h(y_g^{sat,-}, t)$$

If $v > (y_g^{sat})'(t)$ and $h(\cdot, t)$ increasing this implies that

$$\begin{cases} h(y_g^{sat,+}(t), t) = h_g^{sat} \\ (v - (y_g^{sat})'(t))(h_g^{sat} - h(y_g^{sat,-}, t)) = \lambda_g \partial_y h(y_g^{sat,+}, t) \end{cases}$$

Jump relation for liquid/gas (Stefan-like model)

Assuming a liquid/gas transition at $y^{sat}(t) \stackrel{\text{def}}{=} y_\ell^{sat}(t) = y_g^{sat}(t)$

$$\partial_t h + v\partial_y h - \partial_y\left(\lambda(h)\partial_y h\right) = \Phi, \qquad \lambda(h) = \begin{cases} \lambda_\ell, & \text{if } h \leq h_\ell^{sat} \\ \lambda_g, & \text{if } h \geq h_g^{sat} \end{cases}$$

$$\left(v - (y^{sat})'(t)\right)\left(h(y^{sat,+}, t) - h(y^{sat,-}, t)\right) = \lambda_g \partial_y h(y^{sat,+}, t) - \lambda_\ell \partial_y h(y^{sat,-}, t)$$

If $v > (y^{sat})'(t)$ and $h(\cdot, t)$ increasing this implies that

$$\begin{cases} h(t, y^{sat,-}(t)) = h_\ell^{sat} \\ h(t, y^{sat,+}(t)) = h_g^{sat} \\ \left(v - (y^{sat})'(t)\right)\left(h_g^{sat} - h_\ell^{sat}\right) = \lambda_g \partial_y h(t, y^{sat,+}(t)) - \lambda_\ell \partial_y h(t, y^{sat,-}(t)) \end{cases}$$