Toulon, 7/11/2013

A diphasic low Mach model with phase change

The L(ow) M(ach) N(uclear) C(ore) Model

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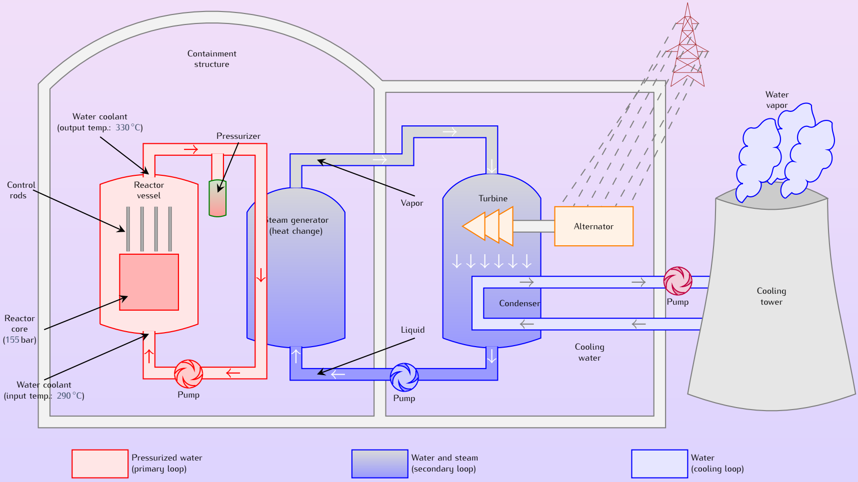
Outline

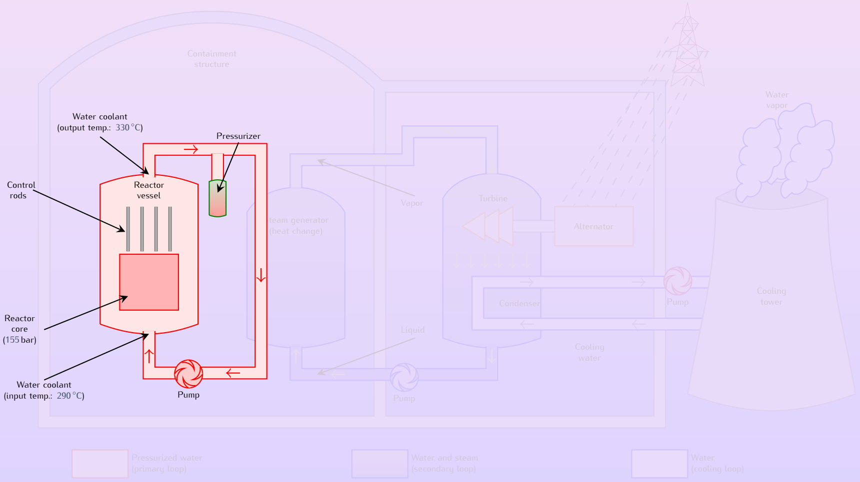
Section 1

Context

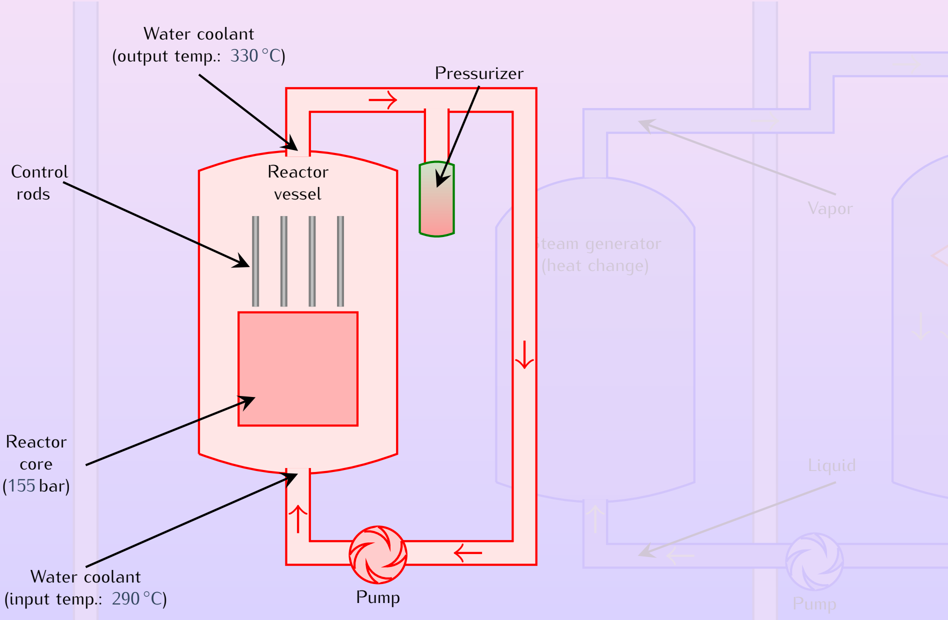
Pressurized Water Reactor



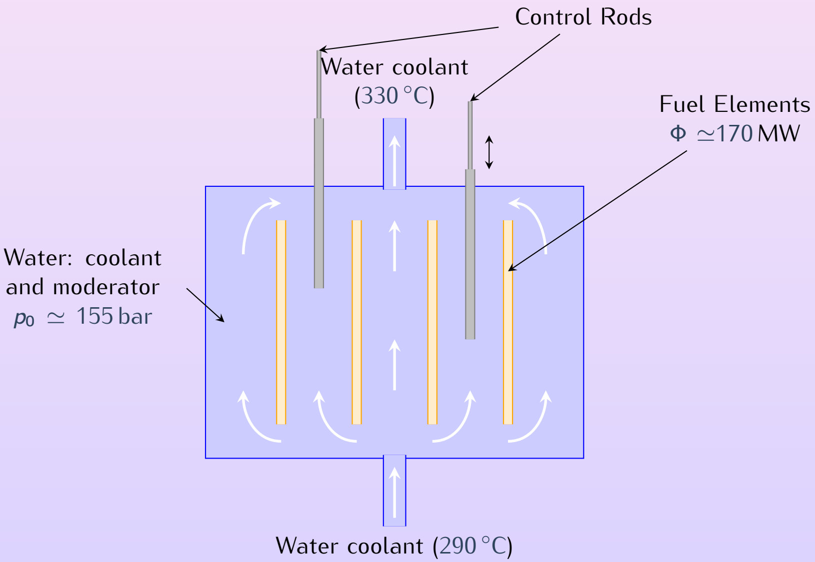
Pressurized Water Reactor



Pressurized Water Reactor



Core of a Pressurized Water Reactor



Section 2

The L(ow) M(ach) N(uclear) C(ore) model

Compressible Navier–Stokes system

$$\begin{cases} \partial_t \varrho + \operatorname{div}(\varrho \mathbf{u}) = 0 \\ \partial_t(\varrho \mathbf{u}) + \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \operatorname{div}(\sigma(\mathbf{u})) + \varrho \mathbf{g} \\ \partial_t(\varrho h) + \operatorname{div}(\varrho h \mathbf{u}) = \partial_t p + \mathbf{u} \cdot \nabla p + \sigma(\mathbf{u}) \colon \nabla \mathbf{u} + \Phi \end{cases}$$

where

$$\sigma(\mathbf{u}) \stackrel{\text{def}}{=} \nu\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) + \eta \nabla \mathbf{u}$$

- Unknowns
- Given quantities
- Equation Of State

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- **Unknowns**
  - $(t, \mathbf{x}) \mapsto \mathbf{u}$ velocity,
  - $(t, \mathbf{x}) \mapsto h$ enthalpy,
  - $(t, \mathbf{x}) \mapsto p$ pressure;
- **Given quantities**
- **Equation Of State**

Compressible Navier–Stokes system

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- Unknowns
- Given quantities
  - $(t, \mathbf{x}) \mapsto \Phi \geq 0$ power density,
  - $\mathbf{g}$ gravity;
- Equation Of State

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where

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- Unknowns
- Given quantities
- Equation Of State
  - $(h, p) \mapsto \nu, \eta$ such that $2\mu + 3\eta > 0$,
  - $(h, p) \mapsto \varrho$ density.

Compressible Navier–Stokes system → LMNC-model

Compressible Navier-Stokes system

$\downarrow$

Dimensionless compressible Navier-Stokes system

$\oplus$

$$M = \frac{\text{speed of fluid}}{\text{speed of sound}} \ll 1$$

$\downarrow$

LMNC system

p pressure

$$p \simeq p_0 + \bar{p}$$

p_0: thermodynamic pressure
$\bar{p}$: dynamic pressure

S. Dellacherie, *On A Low Mach Nuclear Core Model*, ESAIM: Proc., 35 (2012)

Compressible Navier-Stokes system → LMNC-model

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↓

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LMNC–MODEL

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \dfrac{\beta(h)}{p_0}\Phi, \\ \partial_t h + \mathbf{u}\cdot\nabla h = \dfrac{\Phi}{\varrho(h)}, \\ \varrho(h)\Big(\partial_t(\mathbf{u}) + (\mathbf{u}\cdot\nabla)\mathbf{u}\Big) + \nabla\bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h)\mathbf{g}, \end{cases}$$

where

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- Unknowns
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▼ Unknowns

- $(t,\mathbf{x}) \mapsto \mathbf{u}$ velocity,
- $(t,\mathbf{x}) \mapsto h$ enthalpy,
- $(t,\mathbf{x}) \mapsto \bar{p}$ dynamic pressure;

► Given quantities

► Equation Of State

LMNC-MODEL

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- Unknowns
- Given quantities
  - $(t,\mathbf{x}) \mapsto \Phi \geq 0$ power density,
  - $\mathbf{g}$ gravity,
  - $p_0 > 0$ thermodynamic pressure (constant),
- Equation Of State

LMNC–MODEL

$$
\begin{cases}
\operatorname{div}(\mathbf{u}) = \dfrac{\beta(h)}{p_0}\Phi, \\
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\end{cases}
$$

where

$$
\sigma(\mathbf{u}) \stackrel{\text{def}}{=} \nu(h)\big(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\big) + \eta(h)\nabla\mathbf{u}
$$

- Unknowns
- Given quantities
- Equation Of State
  - $h \mapsto \nu, \eta$ such that $2\mu + 3\eta > 0$,
  - $h \mapsto \varrho$ density,
  - $h \mapsto \beta \stackrel{\text{def}}{=} -\frac{p_0}{\varrho^2(h)}\varrho'(h)$ compressibility coefficient.

Section 3

Equation of State

- Thermodynamic of the Phase Change
- Mixture
- Pure Phase EoS
  - Stiffened Gas EoS
  - Tabulated EoS

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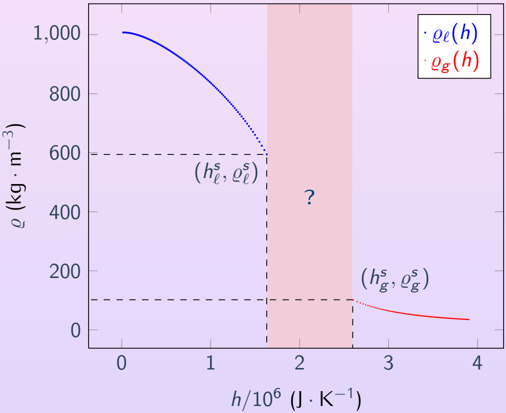
$h \mapsto \varrho$ AT $p_0 = 155\,\text{bar}$ FOR WATER FROM NIST[1]

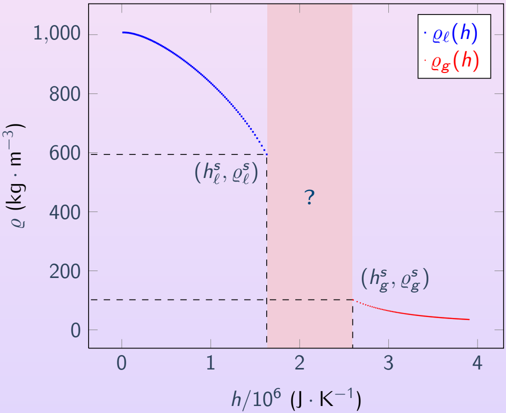


Goals:

1. define $h \mapsto \varrho_m$ for $h \in [h^s_\ell; h^s_g]$: mixture EoS
2. define $h \mapsto \varrho_\kappa$ for $h \le h^s_\ell$ if $\kappa = \ell$ and for $h \ge h^s_g$ if $\kappa = g$: pure phase EoS

[1] Source: `http://webbook.nist.gov/chemistry/fluid/`

$h \mapsto \varrho$ AT $p_0 = 155\,\text{bar}$ FOR WATER FROM NIST[1]



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Section 3

Equation of State

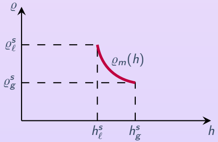
- Thermodynamic of the Phase Change
- Mixture
- Pure Phase EoS

Mixture

$$\begin{cases} \varrho = \alpha \varrho_g^s + (1-\alpha)\varrho_\ell^s \\ \varrho h = \alpha \varrho_g^s h_g^s + (1-\alpha)\varrho_\ell^s h_\ell^s \end{cases} \qquad \text{for } h \in [h_\ell^s; h_g^s]$$

$$\Downarrow$$

$$\boxed{\varrho_m(h) = \frac{p_0/\beta_m}{h - q_m}}$$



where

$$\beta_m \stackrel{\text{def}}{=} -p_0 \frac{\varrho_g^s - \varrho_\ell^s}{\varrho_g^s \varrho_\ell^s (h_g^s - h_\ell^s)} = -\frac{p_0}{\varrho_m(h)} \varrho_m'(h) \qquad q_m \stackrel{\text{def}}{=} \frac{\varrho_g^s h_g^s - \varrho_\ell^s h_\ell^s}{\varrho_g^s - \varrho_\ell^s}$$

Section 3

Equation of State

- Thermodynamic of the Phase Change
- Mixture
- Pure Phase EoS
  - Stiffened Gas EoS
  - Tabulated EoS

Pure phase EoS: Stiffened Gas law

$$\varrho_\kappa(h) = \frac{\gamma_\kappa}{\gamma_\kappa - 1}\frac{p_0 + \pi_\kappa}{h - q_\kappa}$$

where

- $\gamma_\kappa > 1$ adiabatic coefficient,
- π_κ reference pressure,
- q_κ binding energy.

$$\Downarrow$$

$$\beta_\kappa = -\frac{p_0}{\varrho_\kappa^2(h)}\varrho'_\kappa(h) = \frac{\gamma_\kappa - 1}{\gamma_\kappa}\frac{p_0}{p_0 + \pi_\kappa} \qquad \text{constant}$$

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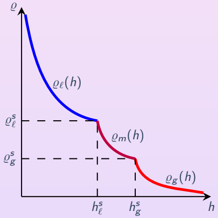
Diphasic Stiffened Gas EOS with Phase Transition

$$\varrho(h) = \frac{p_0/\beta(h)}{h - q(h)}$$



where

$$\beta(h) = \begin{cases} \beta_\ell, & \text{if } h \le h_\ell^s, \\ \beta_m & \text{if } h_\ell^s < h < h_g^s, \\ \beta_g, & \text{if } h \ge h_g^s, \end{cases}$$

$$q(h) = \begin{cases} q_\ell, & \text{if } h \le h_\ell^s, \\ q_m & \text{if } h_\ell^s < h < h_g^s, \\ q_g, & \text{if } h \ge h_g^s, \end{cases}$$

Section 3

Equation of State

- Thermodynamic of the Phase Change
- Mixture
- Pure Phase EoS
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Saturation values

- Liquid $\kappa = \ell$ and vapor $\kappa = g$ are characterized by their EoS

$$(h,p) \mapsto \varrho_\kappa = \frac{\gamma_\kappa}{\gamma_\kappa - 1}\frac{p+\pi_\kappa}{h-q_\kappa}$$

(see Le Metayer and Saurel for parameters of liquid water and steam)

- Second principle of thermodynamics: when phases coexist, they have the same pressures, the same temperatures and their chemical potentials are equal:

$$g_\ell(p,T) = g_g(p,T) \qquad \Longrightarrow \qquad T = T^s(p).$$

- We define saturation values at $p = p_0$:

$$h^s_\kappa \stackrel{\text{def}}{=} h_\kappa\big(p_0, T^s(p_0)\big), \qquad \varrho^s_\kappa \stackrel{\text{def}}{=} \varrho_\kappa\big(h^s_\kappa, p_0\big).$$

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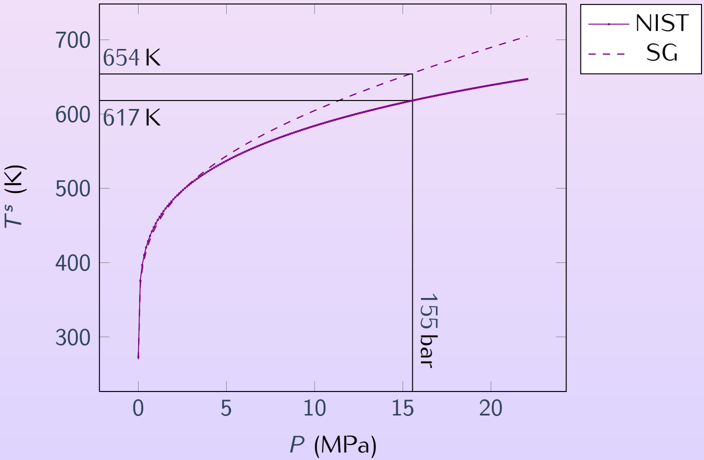
$p \mapsto T^s$

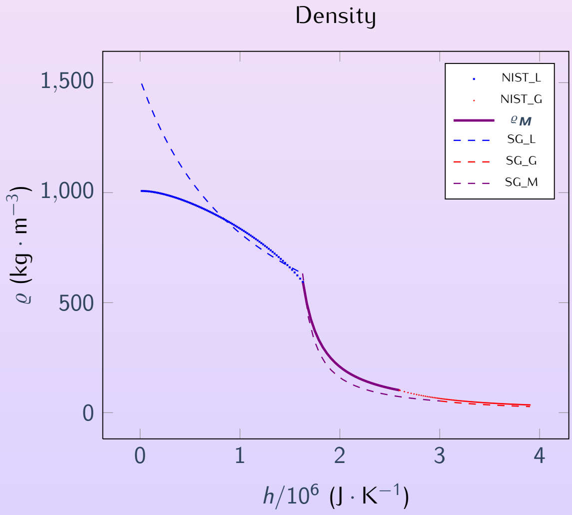


700
600
500
400
300
654 K
617 K
155 bar
T^s (K)
0
5
10
15
20
P (MPa)
NIST
SG

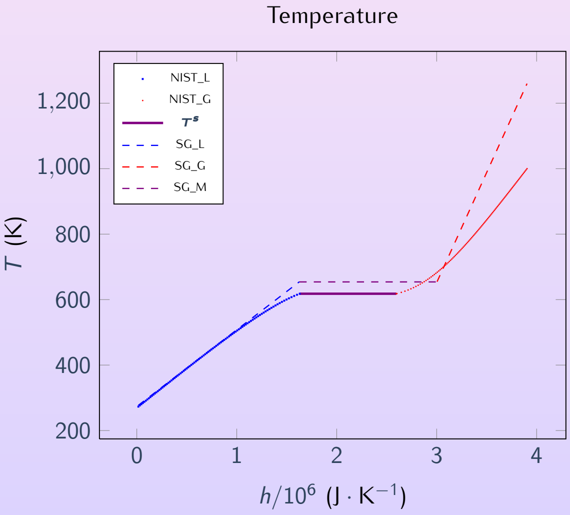
NIST vs SG

	NIST	SG
T^s	$617\,\mathrm{K}$	$654\,\mathrm{K}$
h^s_ℓ	$1.629 \times 10^6\,\mathrm{J}\cdot\mathrm{K}^{-1}$	$1.627 \times 10^6\,\mathrm{J}\cdot\mathrm{K}^{-1}$
h^s_g	$2.596 \times 10^6\,\mathrm{J}\cdot\mathrm{K}^{-1}$	$3.004 \times 10^6\,\mathrm{J}\cdot\mathrm{K}^{-1}$
ϱ^s_ℓ	$594.38\,\mathrm{kg}\cdot\mathrm{m}^{-3}$	$632.663\,\mathrm{kg}\cdot\mathrm{m}^{-3}$
ϱ^s_g	$101.93\,\mathrm{kg}\cdot\mathrm{m}^{-3}$	$52.937\,\mathrm{kg}\cdot\mathrm{m}^{-3}$

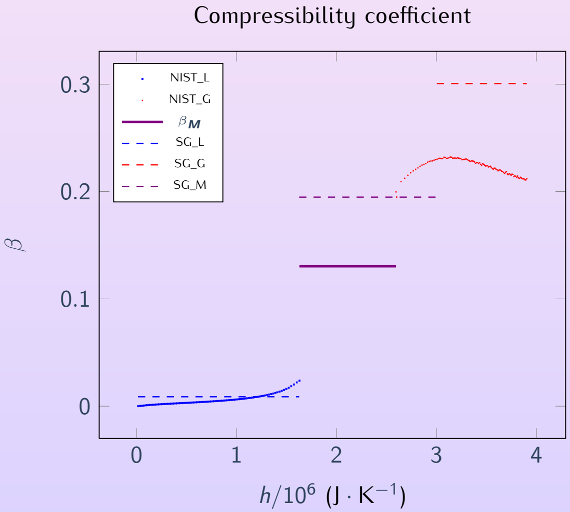
NIST vs SG: $h \mapsto \varrho$



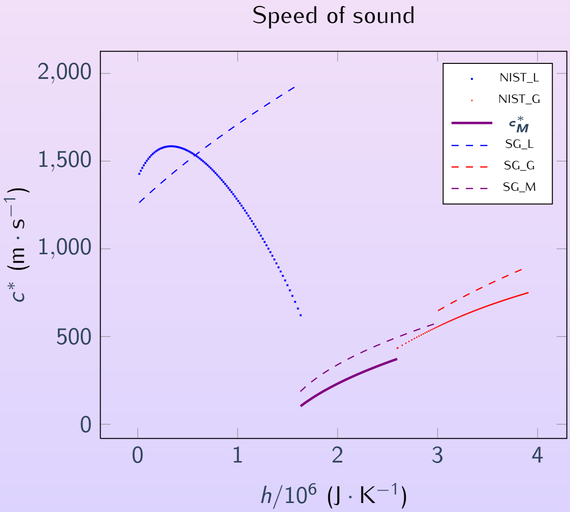
NIST vs SG: $h \mapsto T$



NIST vs SG: $h \mapsto \beta$



NIST vs SG: $h \mapsto c^*$



Pure phase EoS: Tabulated laws at $p = p_0$

κ	h [kJ/kg]	ϱ_κ [kg/m^3]	T_κ [K]	c^*_κ [m · s^{-1}]	β_κ
ℓ	15.608	1007.5	273.16	1427.4	✗
ℓ	30.678	1007.5	276.79	1445.0	✗
⋮	⋮	⋮	⋮	⋮	⋮
ℓ	1602.8	609.10	614.77	659.56	✗
ℓ	h^s_ℓ	594.38	T^s	621.43	✗
g	h^s_g	101.93	T^s	433.40	✗
g	2602.6	101.06	618.41	435.61	✗
⋮	⋮	⋮	⋮	⋮	⋮
g	2.5299	35.139	996.37	747.83	✗
g	2.5290	34.985	1000.0	749.37	✗

Source: http://webbook.nist.gov/chemistry/fluid/

Pure phase EoS: Tabulated laws at $p = p_0$

Liquid phase

- Discretization of the enthalpy interval $[1.56 \times 10^4; h_\ell^s]$:

$$h_i \simeq (1.56 + 1.68i) \times 10^4, \qquad i \in \mathfrak{I} = \{1, \dots, 96\}$$

- Approximation of $\beta_\ell(h_i) = -\frac{p_0}{\varrho_\ell^2(h_i)} \varrho_\ell'(h_i)$ by finite differences
- Least squares polynomial approximation over the set of discrete values $((\varrho_\ell, \beta_\ell, T_\ell, c_\ell^*)(h_i))_{i\in\mathfrak{I}}$:

$$(\varrho_\ell, \beta_\ell, T_\ell, c_\ell^*)\left(\frac{h}{10^6}\right) = \sum_{j=0}^{N} \left(\frac{h}{10^6}\right)^i a_j, \qquad N \leq 6$$

Pure phase EoS: Tabulated laws at $p = p_0$

Vapor phase

- Discretization of the enthalpy interval $[h_g^s; 25.29 \times 10^6]$:

$$h_i \simeq (2.596 + 0.0122i) \times 10^6, \qquad i \in \mathfrak{I} = \{1, \ldots, 107\}$$

- Approximation of $\beta_g(h_i) = -\frac{p_0}{\varrho_g^2(h_i)} \varrho_g'(h_i)$ by finite differences
- Least squares polynomial approximation over the set of discrete values $((\varrho_g, \beta_g, T_g, c_g^*)(h_i))_{i \in \mathfrak{I}}$:

$$(\varrho_g, \beta_g, T_g, c_g^*)\left(\frac{h}{10^6}\right) = \sum_{j=0}^{N} \left(\frac{h}{10^6}\right)^j a_j, \qquad N \leq 6$$

Section 4

1D-model

- Governing equations
- Analytical solutions
- Numerical schemes
- Numerical tests

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1D-model

- Governing equations
- Analytical solutions
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Governing equations

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho} \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu\partial_y v) = -\varrho g \end{cases}$$

- Unknowns
- Given quantities
- Equation Of State
- Boundary Conditions
- Initial Conditions

Governing equations

$$
\begin{cases}
\partial_y v = \dfrac{\beta}{p_0}\Phi \\
\partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho} \\
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\end{cases}
$$

- **Unknowns**
  - $(t, y) \mapsto v$ velocity,
  - $(t, y) \mapsto h$ enthalpy,
  - $(t, y) \mapsto \bar{p}$ dynamic pressure;
- **Given quantities**
- **Equation Of State**
- **Boundary Conditions**
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Governing equations

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- Unknowns
- Given quantities
  - $p_0 > 0$ thermodynamic pressure (constant),
  - $(t, y) \mapsto \Phi \geq 0$ power density,
  - g gravity.
- Equation Of State
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Governing equations

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho} \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu\partial_y v) = -\varrho g \end{cases}$$

- Unknowns
- Given quantities
- Equation Of State
  - $h \mapsto \mu$ viscosity (constant),
  - $h \mapsto \varrho$ density (stiffened gas or tabulated)
  - $h \mapsto \beta = -\frac{p_0}{\varrho^2(h)}\varrho'(h)$ compressibility coefficient.
- Boundary Conditions
- Initial Conditions

Governing equations

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho} \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu\partial_y v) = -\varrho g \end{cases}$$

- Unknowns
- Given quantities
- Equation Of State
- Boundary Conditions
  - top: dynamic pressure $\bar{p}(t, y = L) = p_0$
  - bottom:
    - entrance flow rate $(\varrho v)(t, y = 0) = D_e(t)$
    - entrance enthalpy $h(t, y = 0) = h_e(t)$
- Initial Conditions

Governing equations

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho} \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\varrho g \end{cases}$$

- Unknowns
- Given quantities
- Equation Of State
- Boundary Conditions
- Initial Conditions
  - $h(t=0,y) = h_0(y)$,
  - $v(t=0,y) = v_0(y) = v_e(0) + \frac{1}{p_0}\int_0^y \beta(h_0(z))\Phi(0,z)\,\mathrm{d}z$,
  - $\bar{p}(t=0,y) = p_0$.

Section 4

1D-MODEL

- Governing equations
- Analytical solutions
- Numerical schemes
- Numerical tests

1D-SG-MONOPHASIC

- Velocity
- Enthalpy
- Dynamic pressure

1D-SG-MONOPHASIC

- **Velocity**
  Direct integration of $\partial_y v = \frac{\beta}{p_0}\Phi$.

$$v(t,y) = v_e(t) + \frac{\beta}{p_0}\Psi(t,y), \qquad \Psi(t,y) \stackrel{\text{def}}{=} \int_0^y \Phi(t,z)\,\mathrm{d}z$$

- **Enthalpy**
- **Dynamic pressure**

1D-SG-MONOPHASIC

- Velocity
- Enthalpy
  Method of characteristics on $\partial_t h + v\partial_y h = \frac{\Phi}{\varrho(h)} = \frac{\beta\Phi}{p_0}(h - q)$.
- Dynamic pressure

1D-SG-MONOPHASIC

- ► Velocity
- ▼ Enthalpy

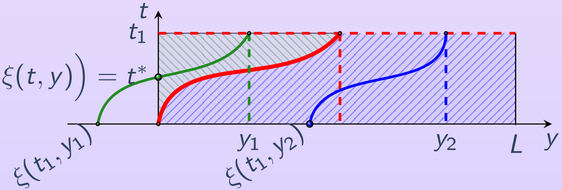
  Method of characteristics on $\partial_t h + v\partial_y h = \frac{\Phi}{\varrho(h)} = \frac{\beta\Phi}{p_0}(h-q)$.

  Example: if Φ and v_e are constant, then

$$h(t,y) = \begin{cases} q + (h_0(\xi(t,y)) - q)\, e^{\frac{\beta\Phi}{p_0}t} & \text{if } \xi(t,y) \geq 0, \\ h_e(t^*(t,y)) + \frac{\Phi}{D_e(t^*(t,y))}y & \text{if } \xi(t,y) < 0. \end{cases}$$

where $\xi(t,y) = \left(y + \frac{p_0}{\beta\Phi}v_e\right) e^{-\frac{\beta\Phi}{p_0}t} - \frac{p_0}{\beta\Phi}v_e$ and

$$-\frac{p_0}{\beta\Phi}\ln\left(1 + \frac{\beta\Phi}{p_0 v_e}\xi(t,y)\right) = t^*$$



- ► Dynamic pressure

1D-SG-monophasic

- Velocity
- Enthalpy
- Dynamic pressure
  Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_t(\varrho v) - \partial_y(\varrho v^2) - \varrho g$.

1D-SG-monophasic

- ► Velocity
- ► Enthalpy
- ▼ Dynamic pressure

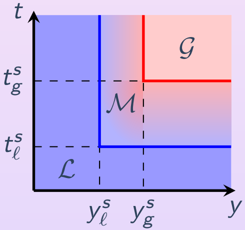
  Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_t(\varrho v) - \partial_y(\varrho v^2) - \varrho g$.

  Example: if Φ and v_e are constant, then

$$\bar{p}(t,y) = p_0 + \frac{\beta\Phi}{p_0}(\mu(y) - \mu(L)) + \frac{p_0(g + \frac{\beta\Phi}{p_0} v_e)}{\beta} \int_y^L \frac{1}{h(t,z) - q} \, dz + \beta\Phi^2 \int_y^L \frac{z}{h(t,z) - q} \, dz$$

1D-SG-diphasic

Φ, v_e, h_e, h_0: constant; IC and BC: liquid phase.

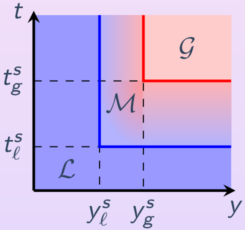


$$y_\ell^s = \frac{D_e}{\Phi}(h_\ell^s - h_e)$$
$$y_g^s = \frac{D_e}{\Phi}(h_g^s - h_e)$$
$$t_\ell^s = \frac{p_0}{\beta_\ell \Phi} \ln\left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell}\right)$$
$$t_g^s = t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln\left(\frac{h_g^s - q_m}{h_\ell^s - q_m}\right)$$

- Velocity
- Enthalpy

1D-SG-DIPHASIC

Φ, v_e, h_e, h_0: constant; IC and BC: liquid phase.



$$
\begin{aligned}
y_\ell^s &= \frac{D_e}{\Phi}(h_\ell^s - h_e)\\
y_g^s &= \frac{D_e}{\Phi}(h_g^s - h_e)\\
t_\ell^s &= \frac{p_0}{\beta_\ell \Phi} \ln\left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell}\right)\\
t_g^s &= t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln\left(\frac{h_g^s - q_m}{h_\ell^s - q_m}\right)
\end{aligned}
$$

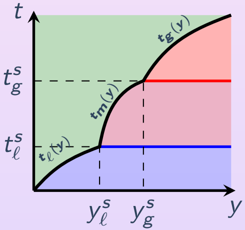
▼ **Velocity**: direct integration of $\partial_y v = \frac{\beta(h)}{p_0}\Phi$.

$$
v(t,y) = \begin{cases} v_e + \frac{\beta_\ell \Phi}{p_0} y & \text{if } (t,y) \in \mathcal{L},\\ v_e + \frac{\beta_\ell \Phi}{p_0} y_\ell^s + \frac{\beta_m \Phi}{p_0}(y - y_\ell^s) & \text{if } (t,y) \in \mathcal{M},\\ v_e + \frac{\beta_\ell \Phi}{p_0} y_\ell^s + \frac{\beta_m \Phi}{p_0}(y_g^s - y_\ell^s) + \frac{\beta_g \Phi}{p_0}(y - y_g^s) & \text{if } (t,y) \in \mathcal{G}, \end{cases}
$$

► **Enthalpy**

1D-SG-diphasic

Φ, v_e, h_e, h_0: constant; IC and BC: liquid phase.



$$y_\ell^s = \frac{D_e}{\Phi}(h_\ell^s - h_e)$$
$$y_g^s = \frac{D_e}{\Phi}(h_g^s - h_e)$$
$$t_\ell^s = \frac{p_0}{\beta_\ell \Phi} \ln\left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell}\right)$$
$$t_g^s = t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln\left(\frac{h_g^s - q_m}{h_\ell^s - q_m}\right)$$

- ► **Velocity**
- ▼ **Enthalpy**: method of characteristics on $\partial_t h + v\partial_y h = \frac{\beta(h)\Phi}{p_0}(h - q(h))$.

$$h(t,y) = \begin{cases} q_\ell + (h_0 - q_\ell)e^{\frac{\beta_\ell \Phi}{p_0} t} & \text{if } (t,y) \in \mathcal{L} \text{ and } t < t_\ell(y), \\ q_m + (h_\ell^s - q_m)e^{\frac{\beta_m \Phi}{p_0}(t - t_\ell^s)} & \text{if } (t,y) \in \mathcal{M} \text{ and } t < t_m(y), \\ q_g + (h_g^s - q_g)e^{\frac{\beta_g \Phi}{p_0}(t - t_g^s)} & \text{if } (t,y) \in \mathcal{G} \text{ and } t < t_g(y), \\ h_e + \frac{\Phi}{D_e} y & \text{otherwise.} \end{cases}$$

1D-TAB-DIPHASIC

$$(h_e^\infty, D_e^\infty > 0, \Phi^\infty(y)) \stackrel{\text{def}}{=} \lim_{t\to+\infty} (h_e(t), D_e(t), \Phi(t,y))$$

1. **Enthalpy**
   Using $\partial_y(\varrho^\infty v^\infty) = 0$ we have $\partial_y h^\infty = \frac{\Phi^\infty}{D_e^\infty}$.

$$h^\infty(y) = h_e^\infty + \frac{\Psi(y)}{D_e^\infty}, \qquad \Psi(y) \stackrel{\text{def}}{=} \int_0^y \Phi^\infty(z)\, dz$$

2. **Velocity**

$$v^\infty(y) = \frac{D_e^\infty}{\varrho(h^\infty(y))}$$

3. **Dynamic pressure**
   Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_y(\varrho v^2) - \varrho g$.

Section 4

1D-MODEL

- Governing equations
- Analytical solutions
- Numerical schemes
- Numerical tests

MOC-scheme (SG & TAB)

- Enthalpy
- Velocity

MOC-scheme (SG & TAB)

▼ **Enthalpy** - key idea:

$$\partial_t h(t^{n+1}, y_i) + v(t^{n+1}, y_i)\partial_y h(t^{n+1}, y_i) = \frac{\Phi(t^{n+1}, y_i)}{\varrho(h(t^{n+1}, y_i))}$$

$$\rightsquigarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau) = \frac{\Phi\big(\tau, \chi(\tau; t^{n+1}, y_i)\big)}{\varrho(\tilde{h}_i^{n+1}(\tau))}$$

where $\bar{t} \in [t^n; t^{n+1}[$, $\tau \mapsto \tilde{h}_i^{n+1}(\tau) \stackrel{\text{def}}{=} h\big(\tau, \chi(\tau; t^{n+1}, y_i)\big)$ and χ is the characteristic flow defined as the solution of

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}\tau}\chi(\tau; t^{n+1}, y_i) = v\left(\tau, \chi(\tau; t^{n+1}, y_i)\right), & \tau \le t^{n+1}, \\ \chi(t^{n+1}; t^{n+1}, y_i) = y_i. \end{cases}$$

► **Velocity**

MOC-scheme (SG & TAB)

▼ **Enthalpy** - key idea:

$$\partial_t h(t^{n+1},y_i)+v(t^{n+1},y_i)\partial_y h(t^{n+1},y_i)=\frac{\Phi(t^{n+1},y_i)}{\varrho(h(t^{n+1},y_i))}$$

$$\rightsquigarrow$$

$$\int_{\bar{t}}^{t^{n+1}}\frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau)\,\mathrm{d}\tau=\int_{\bar{t}}^{t^{n+1}}\frac{\Phi(\tau,\chi(\tau;t^{n+1},y_i))}{\varrho(\tilde{h}_i^{n+1}(\tau))}\,\mathrm{d}\tau$$

where $\bar{t}\in[t^n;t^{n+1}[$, $\tau\mapsto\tilde{h}_i^{n+1}(\tau)\stackrel{\text{def}}{=}h\big(\tau,\chi(\tau;t^{n+1},y_i)\big)$ and χ is the characteristic flow defined as the solution of

$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}\tau}\chi(\tau;t^{n+1},y_i)=v\left(\tau,\chi(\tau;t^{n+1},y_i)\right), & \tau\le t^{n+1},\\ \chi(t^{n+1};t^{n+1},y_i)=y_i.\end{cases}$$

► **Velocity**

MOC-scheme (SG & TAB)

- **Enthalpy** - key idea:

$$\partial_t h(t^{n+1}, y_i) + v(t^{n+1}, y_i)\partial_y h(t^{n+1}, y_i) = \frac{\Phi(t^{n+1}, y_i)}{\varrho(h(t^{n+1}, y_i))}$$

$$\rightsquigarrow$$

$$h(t^{n+1}, y_i) - \tilde{h}_i^{n+1}(\bar{t}) = \int_{\bar{t}}^{t^{n+1}} \frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau)\,\mathrm{d}\tau = \int_{\bar{t}}^{t^{n+1}} \frac{\Phi(\tau, \chi(\tau; t^{n+1}, y_i))}{\varrho(\tilde{h}_i^{n+1}(\tau))}\,\mathrm{d}\tau$$

where $\bar{t} \in [t^n; t^{n+1}[$, $\tau \mapsto \tilde{h}_i^{n+1}(\tau) \stackrel{\text{def}}{=} h\big(\tau, \chi(\tau; t^{n+1}, y_i)\big)$ and χ is the characteristic flow defined as the solution of

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}\tau}\chi(\tau; t^{n+1}, y_i) = v\left(\tau, \chi(\tau; t^{n+1}, y_i)\right), & \tau \le t^{n+1}, \\ \chi(t^{n+1}; t^{n+1}, y_i) = y_i. \end{cases}$$

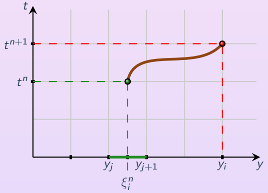
- **Velocity**

MOC-scheme (SG & TAB)

▼ **Enthalpy** - scheme: let $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.

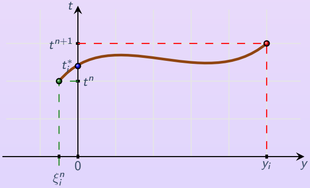
- If $\xi_i^n > 0$, let $\hat{h}_i^n \approx \tilde{h}_i^{n+1}(t^n)$ (at order 1 or higher) and then $\bar{t} = t^n$ and

$$h_i^{n+1} = \hat{h}_i^n + \Delta t \frac{\Phi(t^n, \xi_i^n)}{\varrho(\hat{h}_i^n)}$$



- If $\xi_i^n \leq 0$, let $t_i^* = t^{n+1} - y_i/v_i^n \approx \tau$ such that $\chi(\tau; t^{n+1}, y_i) = 0$ and then $\bar{t} = t_i^*$ and

$$h_i^{n+1} = h_e(t_i^*) + (t^{n+1} - t_i^*) \frac{\Phi(t^*, 0)}{\varrho(h_e(t_i^*))}$$



► **Velocity**

MOC-scheme (SG & TAB)

- **Enthalpy**
- **Velocity** : $\partial_y v = \frac{\beta(h)\Phi}{p_0}$

$$v_i^{n+1} = v_{i-1}^{n+1} + \frac{1}{p_0}\int_{y_{i-1}}^{y_i} \beta(h(t^{n+1},z))\Phi(t^{n+1},z)\,\mathrm{d}z$$
$$\approx v_{i-1}^{n+1} + \frac{\Delta y}{p_0}\beta(h_{i-1}^{n+1})\Phi(t^{n+1},y_{i-1}).$$

β is discontinuous at phase change points, so that if $h_\kappa^s \in (h_{i-1}^{n+1}, h_i^{n+1})$, let $y^* = y_{i-1} + \Delta y \frac{h_\kappa^s - h_{i-1}^{n+1}}{h_i^{n+1} - h_{i-1}^{n+1}}$ and then

$$\int_{y_{i-1}}^{y_i} \beta(h(t^{n+1},z))\Phi(t^{n+1},z)\,\mathrm{d}z$$
$$\approx (y^* - y_{i-1})\beta(h_{i-1}^{n+1})\Phi(t^{n+1},y_{i-1})\,\mathrm{d}y + (y_i - y^*)\beta(h_i^{n+1})\Phi(t^{n+1},y_i)\,\mathrm{d}y$$

INTMOC-scheme (SG)

- **Enthalpy** - key idea:

$$\frac{\frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau)}{\beta(\tilde{h}_i^{n+1}(\tau))\left(\tilde{h}_i^{n+1}(\tau)-q(\tilde{h}_i^{n+1}(\tau))\right)}=\frac{\Phi\big(\tau,\chi(\tau;t^{n+1},y_i)\big)}{p_0}$$

$$\Downarrow$$

$$\int_{\tilde{h}_i^{n+1}(\bar{t})}^{\tilde{h}_i^{n+1}(t^{n+1})}\frac{1}{\beta(h)(h-q(h))}\,\mathrm{d}h=\frac{1}{p_0}\int_{\bar{t}}^{t^{n+1}}\Phi\big(\tau,\chi(\tau;t^{n+1},y_i)\big)\,\mathrm{d}\tau$$

so that

$$\tilde{h}_i^{n+1}(t^{n+1})=R^{-1}\left(R(\tilde{h}_i^{n+1}(\bar{t}))+\frac{1}{p_0}\int_{\bar{t}}^{t^{n+1}}\Phi\big(\tau,\chi(\tau;t^{n+1},y_i)\big)d\tau\right)$$

where

$$R(h)\stackrel{\text{def}}{=}\int_0^{\tilde{h}}\frac{1}{\beta(h)(h-q(h))}\,\mathrm{d}h$$

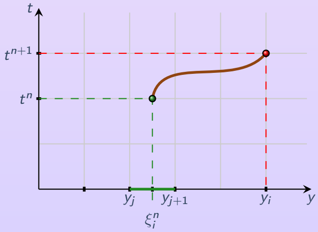
INTMOC-SCHEME (SG)

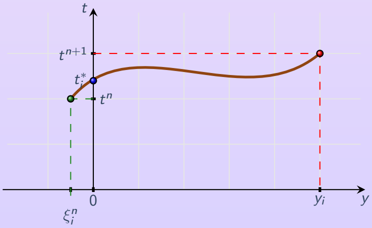
- **Enthalpy** - scheme: let $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.
  - If $\xi_i^n > 0$, let $\hat{h}_i^n \approx \tilde{h}_i^{n+1}(t^n)$ (at order 1 or 2) and then $\bar{t} = t^n$ and

$$h_i^{n+1} = R^{-1}\left(R(\hat{h}_i^n) + \frac{\Delta t}{p_0}\frac{\Phi(t^n, \xi_i^n) + \Phi(t^{n+1}, y_j)}{2}\right)$$

  - If $\xi_i^n \le 0$, let $t_i^* = t^{n+1} - y_i/v_i^n \approx \tau$ such that $\chi(\tau; t^{n+1}, y_i) = 0$ and then $\bar{t} = t_i^*$ and

$$h_i^{n+1} = R^{-1}\left(R(h_e(t_i^*)) + \frac{t^{n+1} - t_i^*}{p_0}\frac{\Phi(t_i^*, 0) + \Phi(t^{n+1}, y_i)}{2}\right)$$



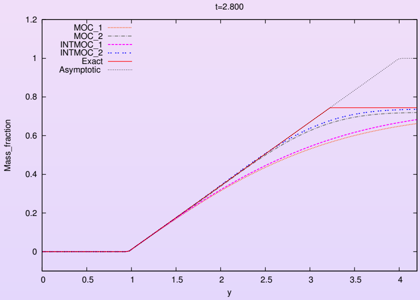


Section 4

1D-model

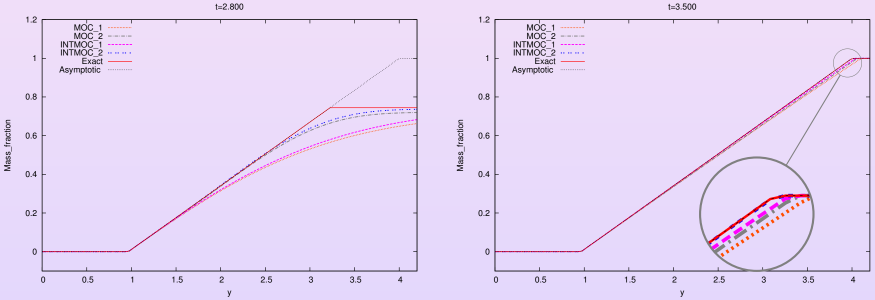
- Governing equations
- Analytical solutions
- Numerical schemes
- Numerical tests

SG: MOC (order 1 or 2) vs INTMOC (order 1 or 2)



- Initially the domain is filled with liquid phase
- At $t = 1.769\,\mathrm{s}$ mixture appears for $y > y_\ell^s \simeq 0.964\,\mathrm{m}$
- At $t = 2.929\,\mathrm{s}$ pure vapor phase appears for $y > y_g^s \simeq 4.002\,\mathrm{m}$
- The asymptotic state is reached at $t = 2.957\,\mathrm{s}$

SG: MOC (order 1 or 2) vs INTMOC (order 1 or 2)



- Initially the domain is filled with liquid phase
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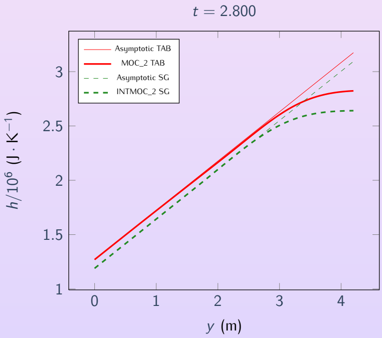
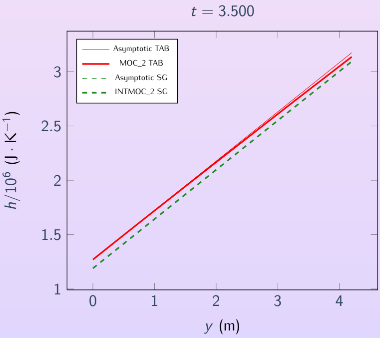
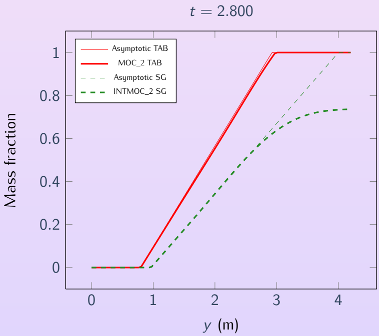
- At $t = 2.929\,\text{s}$ pure vapor phase appears for $y > y_g^s \simeq 4.002\,\text{m}$
- The asymptotic state is reached at $t = 2.957\,\text{s}$

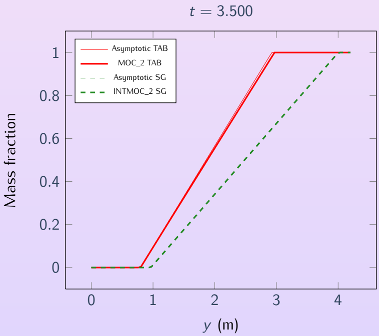
SG (INTMOC 2) vs TAB (MOC 2)



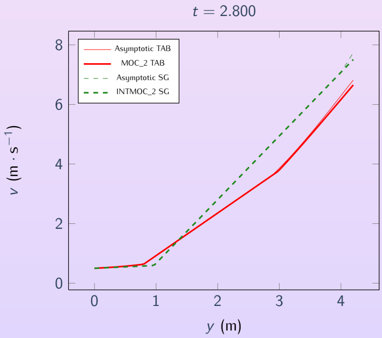


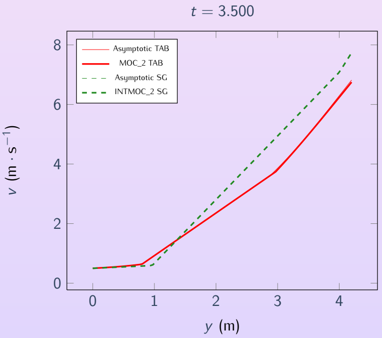
SG (INTMOC 2) vs TAB (MOC 2)



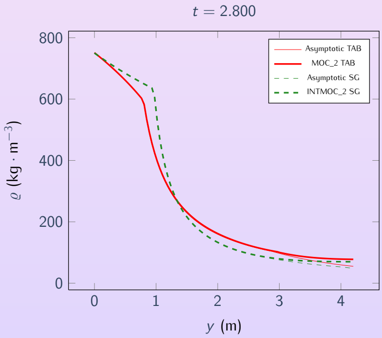


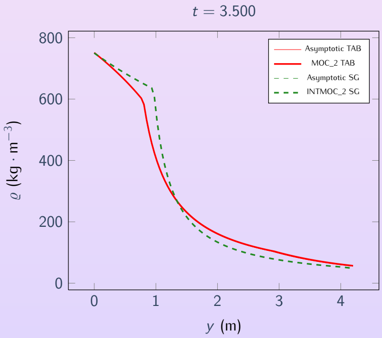
SG (INTMOC 2) vs TAB (MOC 2)





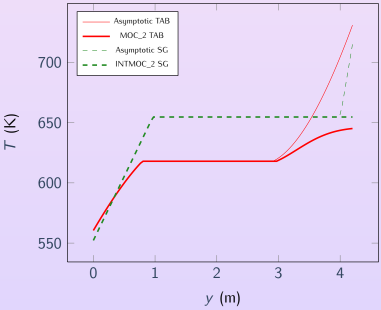
SG (INTMOC 2) vs TAB (MOC 2)



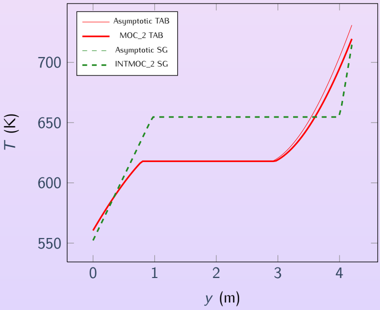


SG (INTMOC 2) vs TAB (MOC 2)

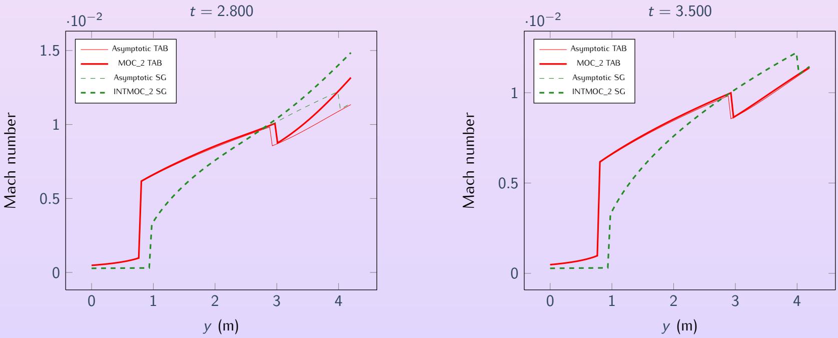
$t = 2.800$



$t = 3.500$



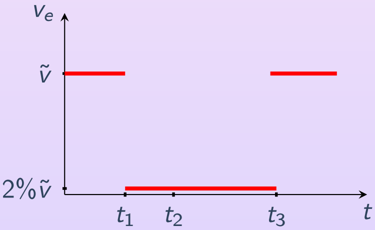
SG (INTMOC 2) vs TAB (MOC 2)

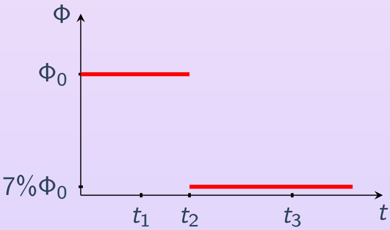


Loss of Flow

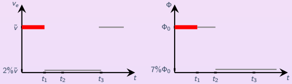
$$v_e(t) = \begin{cases} \tilde{v} & \text{if } 0 \le t < t_1, \\ 2\%\tilde{v} & \text{if } t_1 \le t < t_3, \\ \tilde{v} & \text{if } t \ge t_3, \end{cases}$$

$$\Phi(t) = \begin{cases} \Phi_0 & \text{if } 0 \le t < t_2, \\ 7\%\Phi_0 & \text{if } t \ge t_2. \end{cases}$$



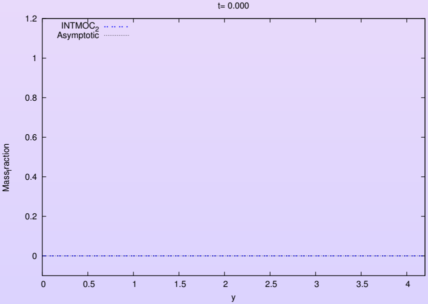


Loss of Flow

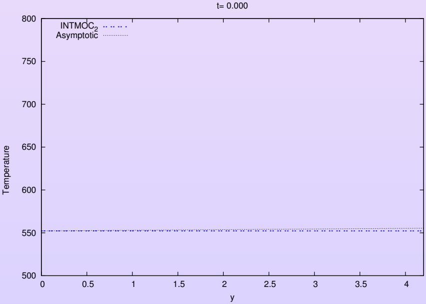


Initial data: $v_e(t) = \tilde{v}$ and $\Phi(t, y) = \Phi_0$.

Mass fraction

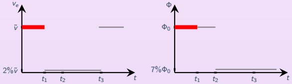


Temperature



Description | $[t_0 - t_1]$ | $[t_1 - t_2]$ | $[t_2 - t_3]$ | $t > t_3$ | Fin

Loss of Flow



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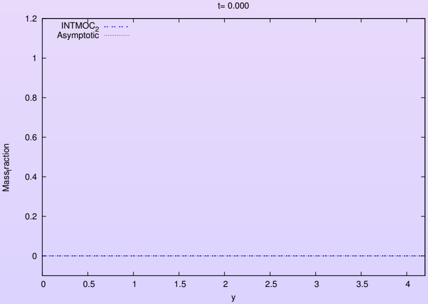
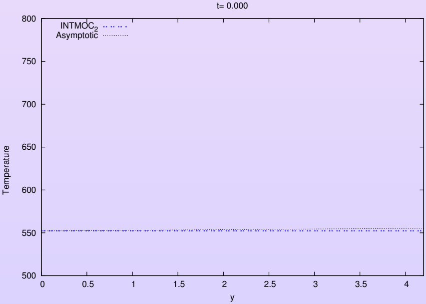
Mass fraction



Temperature



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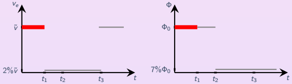

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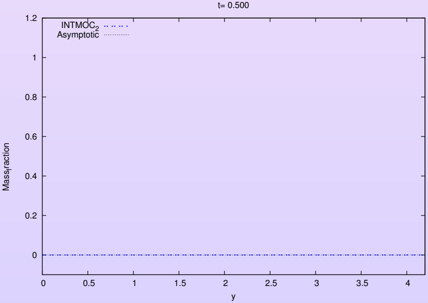
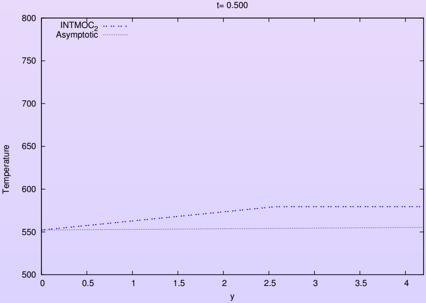
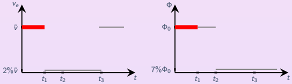


Temperature

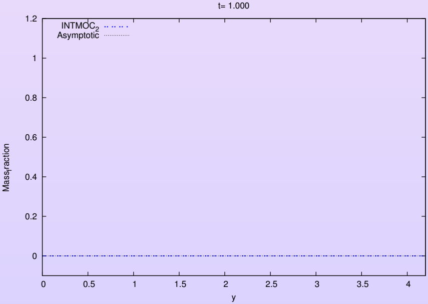


Loss of Flow

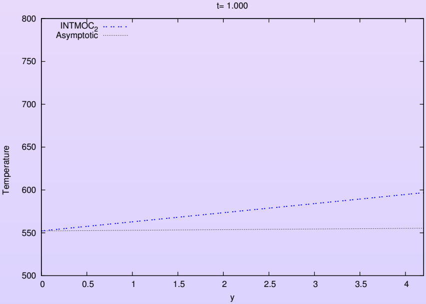


Initial data: $v_e(t) = \tilde{v}$ and $\Phi(t, y) = \Phi_0$.

Mass fraction

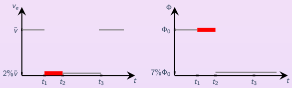


Temperature



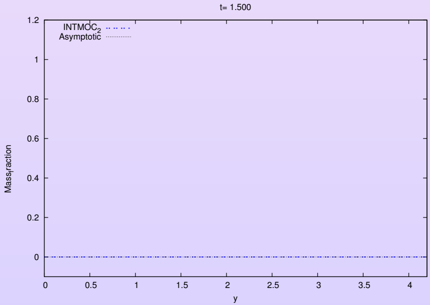
Description $[t_0 - t_1]$ $[t_1 - t_2]$ $[t_2 - t_3]$ $t > t_3$ Fin

Loss of Flow

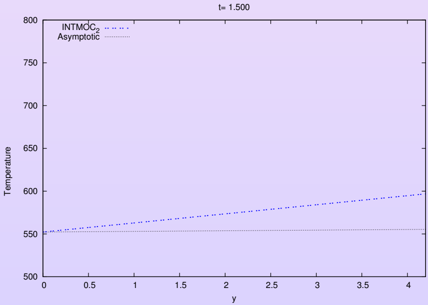


At t_1 most of the pumps stop $\implies v_e(t) \searrow$.

Mass fraction



Temperature



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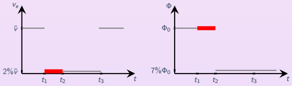
Loss of Flow



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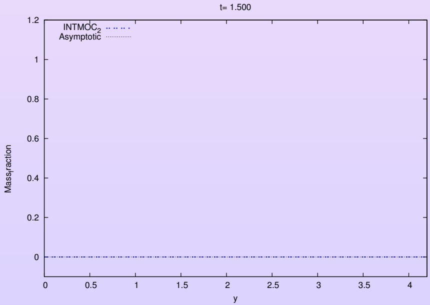
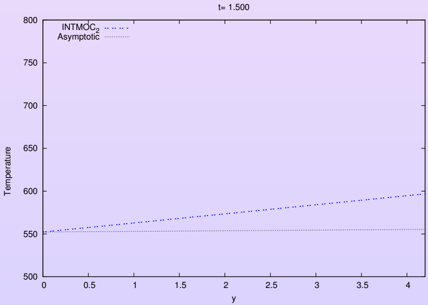
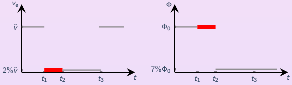


Temperature



◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

Loss of Flow



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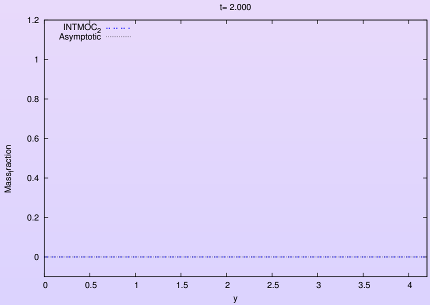
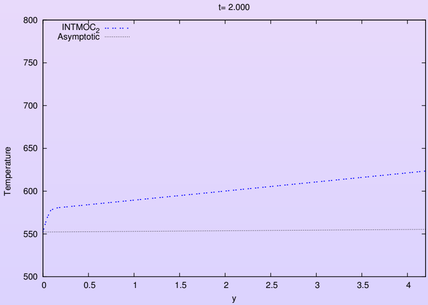


Temperature



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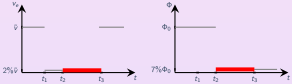
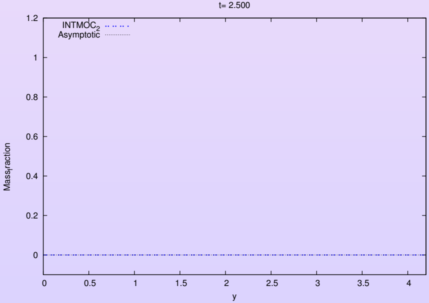
Loss of Flow

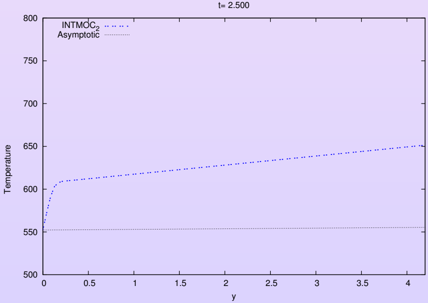


At t_2 the security system drops control rods into the core
$\implies \Phi(t) \searrow 7\%\Phi_0$.

Mass fraction

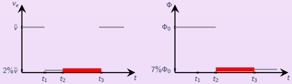


Temperature



Description [t_0 — t_1] [t_1 — t_2] [t_2 — t_3] $t > t_3$ Fin

Loss of Flow



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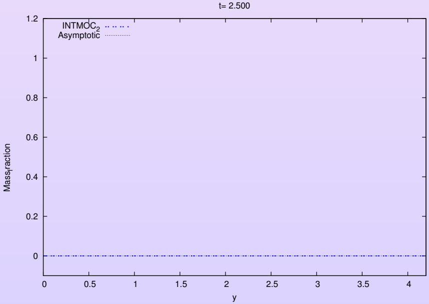
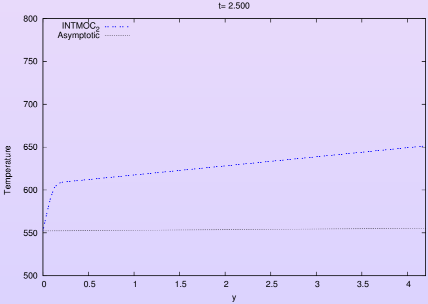
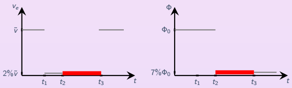


Temperature



Description $[t_0 - t_1]$ $[t_1 - t_2]$ $[t_2 - t_3]$ $t > t_3$ Fin

Loss of Flow



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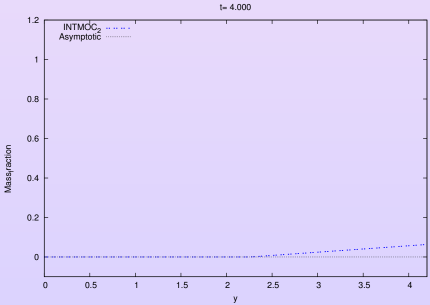
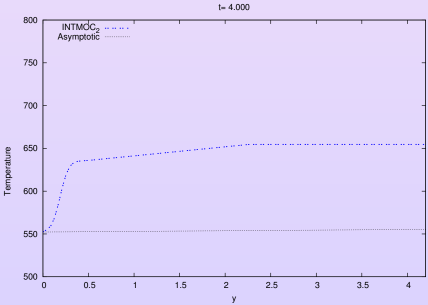
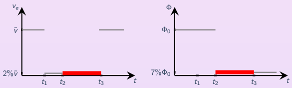


Temperature



Loss of Flow



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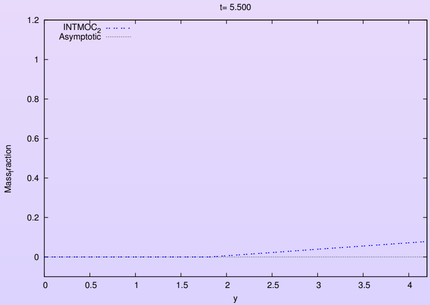
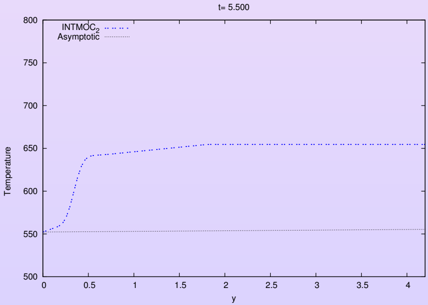
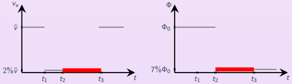


Temperature



◀ Description ▶ $[t_0 - t_1]$ ▶ $[t_1 - t_2]$ ▶▶ $[t_2 - t_3]$ ▶▶ $t > t_3$ ▶▶ Fin

Loss of Flow



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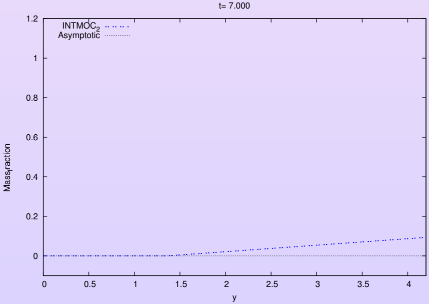
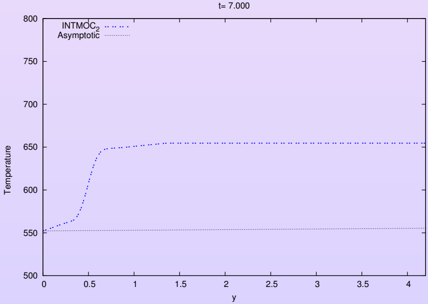
Mass fraction



Temperature



◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

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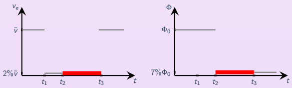

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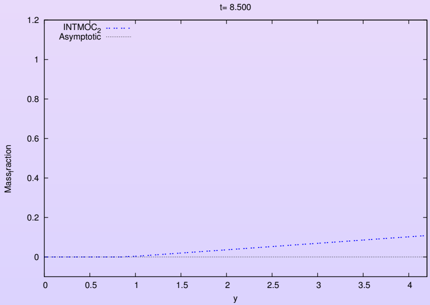
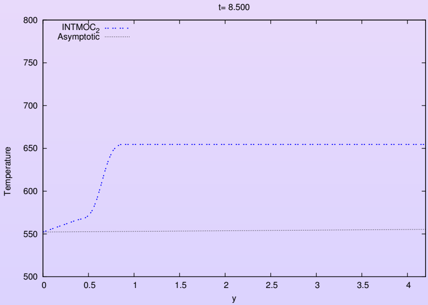


Temperature



◀ Description ▶ $[t_0 - t_1]$ ▶ $[t_1 - t_2]$ ⏩ $[t_2 - t_3]$ ⏩ $t > t_3$ ⏩ Fin

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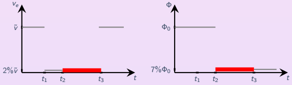

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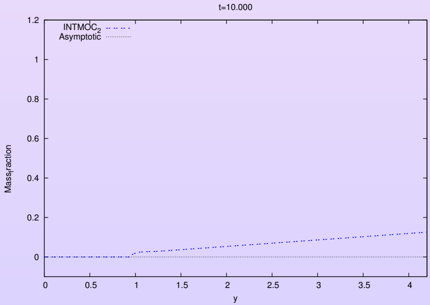

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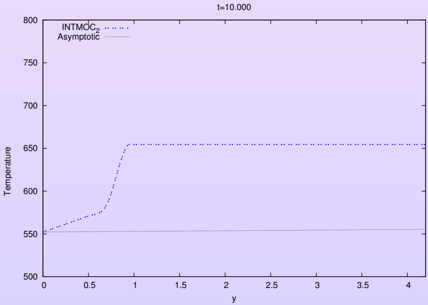

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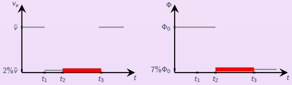

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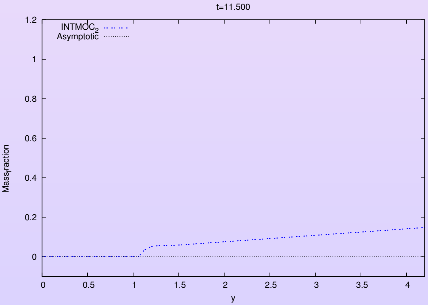
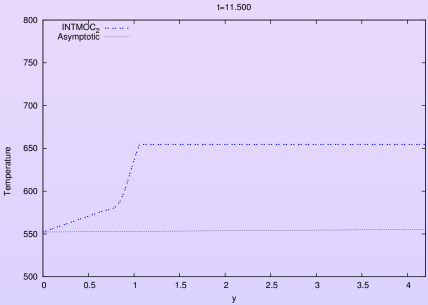


Temperature



◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

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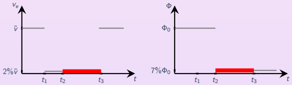

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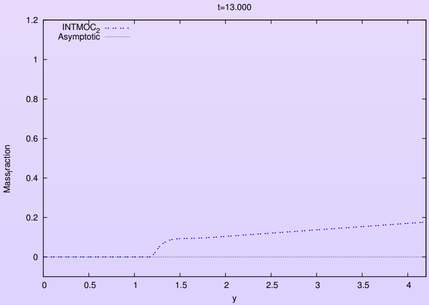

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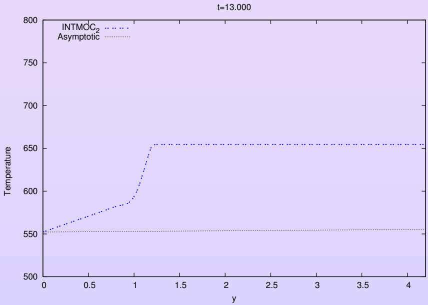

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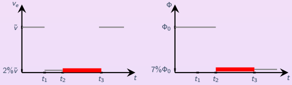

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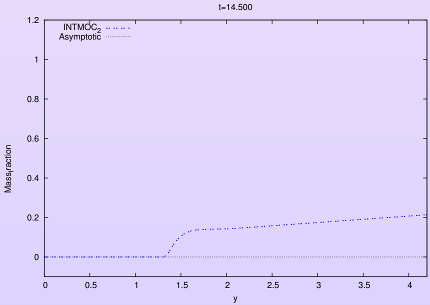
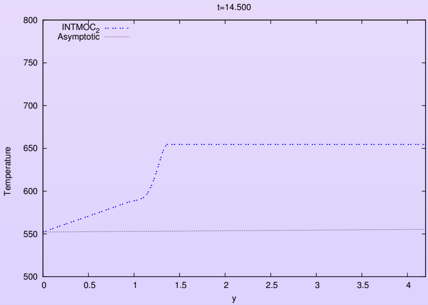


Temperature



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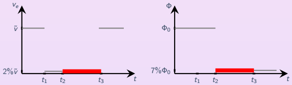

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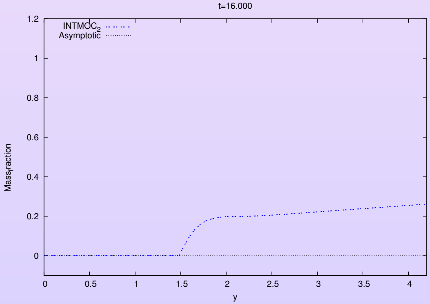

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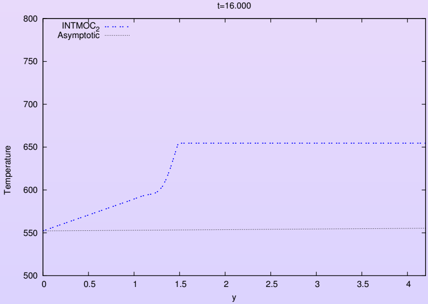

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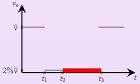

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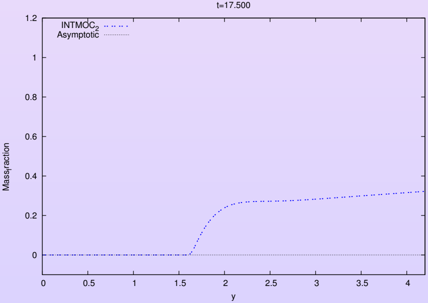
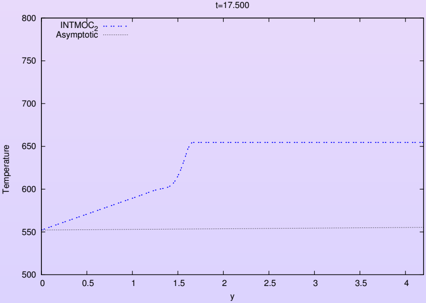


Temperature



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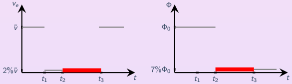

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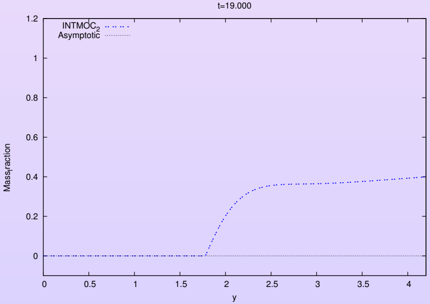
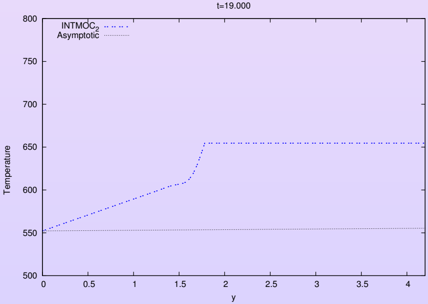
Mass fraction

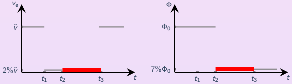


Temperature



◀ Description ▶ $[t_0 - t_1]$ ▶ $[t_1 - t_2]$ ⏩ $[t_2 - t_3]$ ⏩ $t > t_3$ ⏩ Fin

Loss of Flow



At t_2 the security system drops control rods into the core

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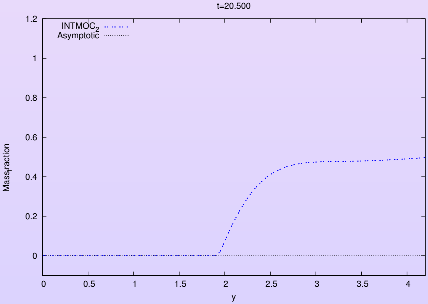
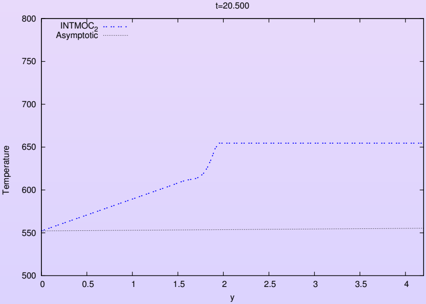
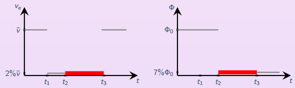


Temperature



◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

Loss of Flow



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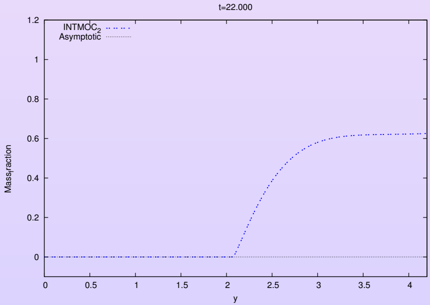
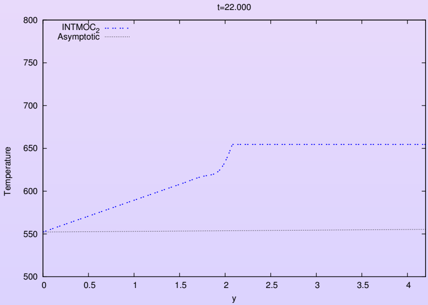


Temperature



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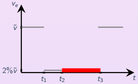
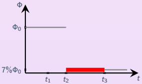
Loss of Flow





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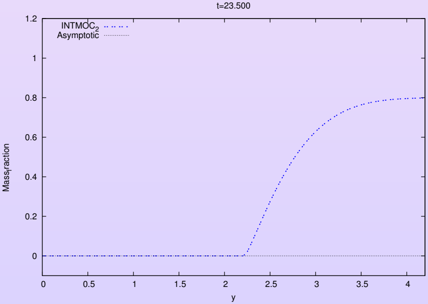
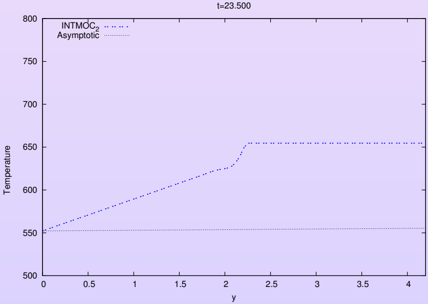
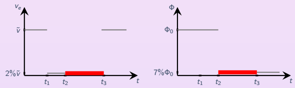


Temperature



Description $[t_0 - t_1]$ $[t_1 - t_2]$ $[t_2 - t_3]$ $t > t_3$ Fin

Loss of Flow



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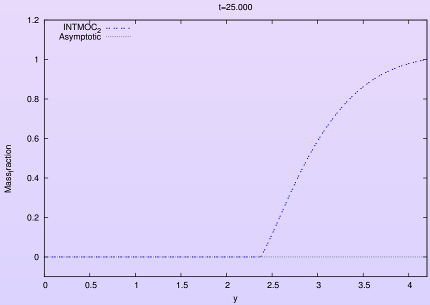
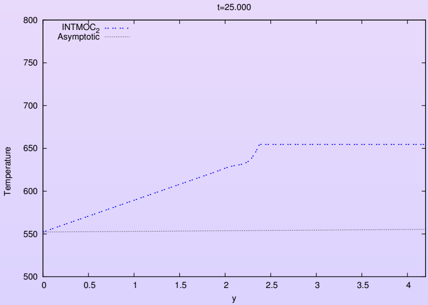
Mass fraction

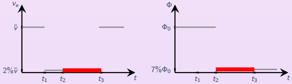


Temperature



Description | $[t_0 - t_1]$ | $[t_1 - t_2]$ | $[t_2 - t_3]$ | $t > t_3$ | Fin

Loss of Flow



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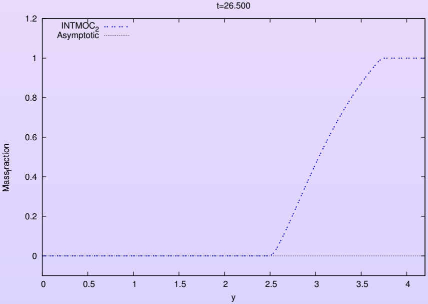
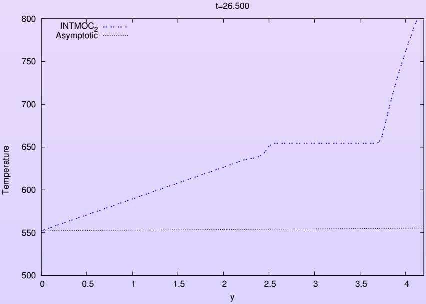


Temperature



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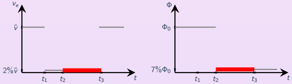

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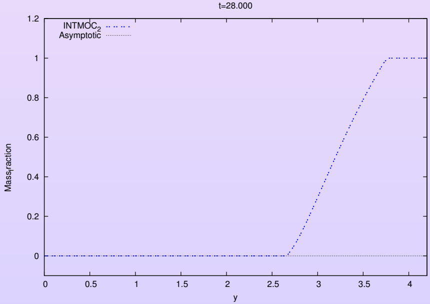
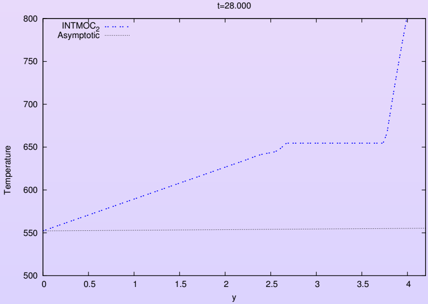


Temperature



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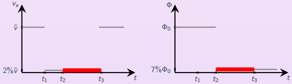

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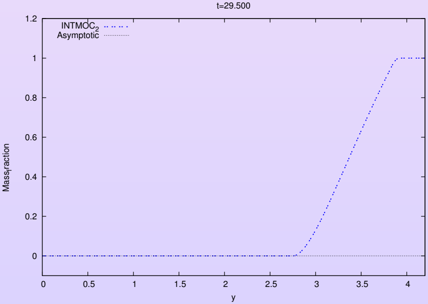
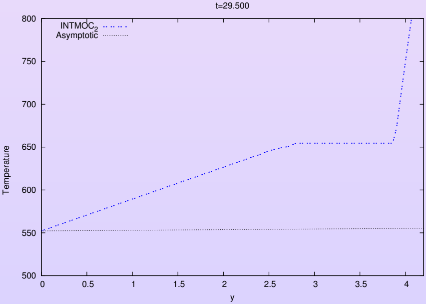
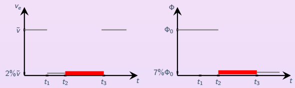


Temperature



◀ Description ▶ $[t_0 - t_1]$ ▶ $[t_1 - t_2]$ ▶▶ $[t_2 - t_3]$ ▶▶ $t > t_3$ ▶▶ Fin

Loss of Flow



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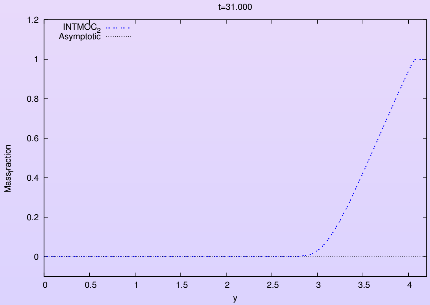
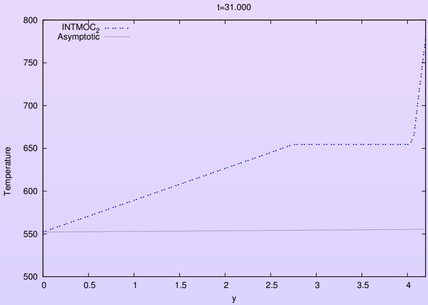


Temperature



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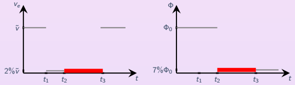
Loss of Flow



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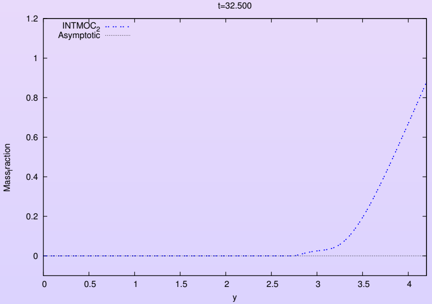
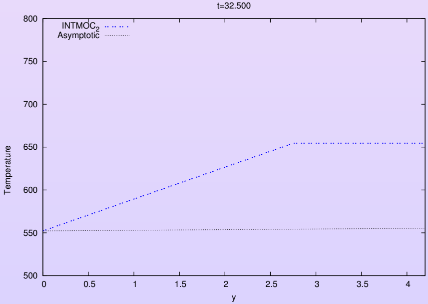
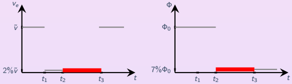


Temperature



Loss of Flow



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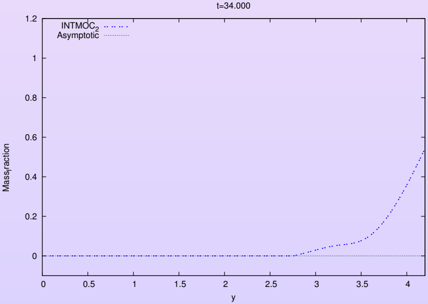

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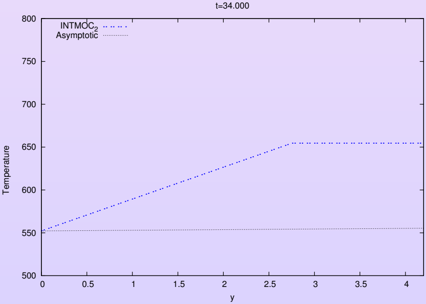
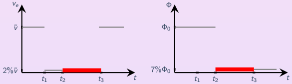


◀ Description ▶ $[t_0 - t_1]$ ▶ $[t_1 - t_2]$ ⏩ $[t_2 - t_3]$ ⏩ $t > t_3$ ⏩ Fin

Loss of Flow



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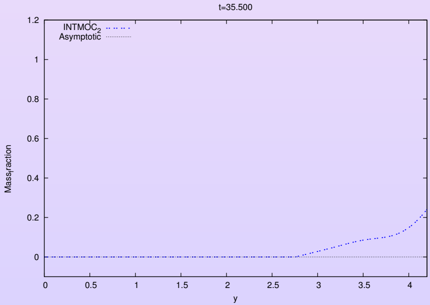
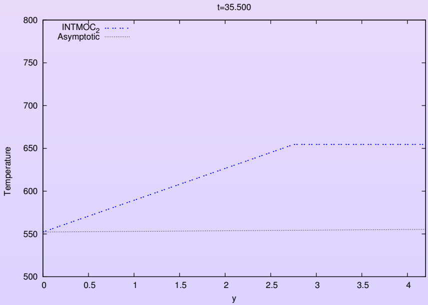


Temperature



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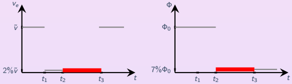

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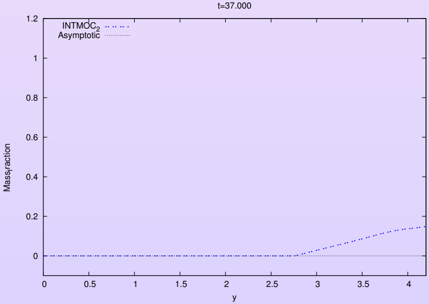
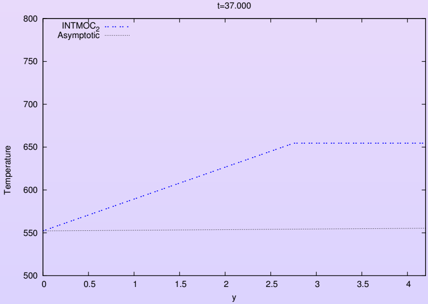


Temperature



◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

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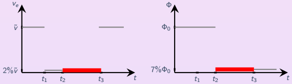

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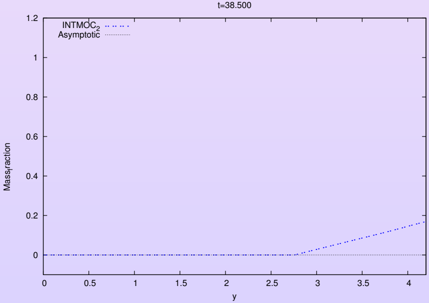
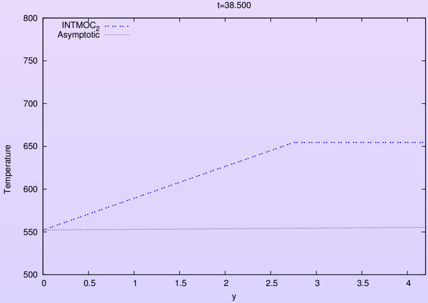


Temperature



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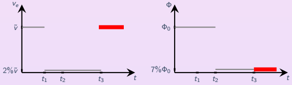
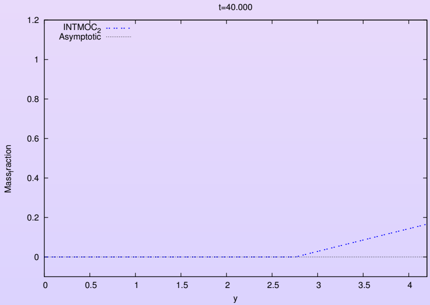


At t_3 the security pumps are turned on $\implies$ $v_e(t) \nearrow$ and the fluid comes back to the liquid phase.

Mass fraction



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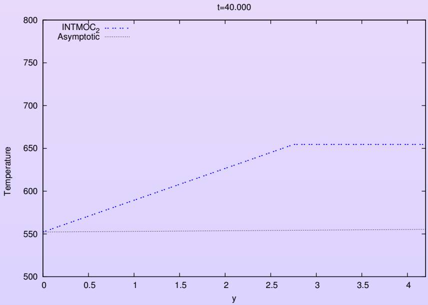

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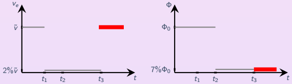

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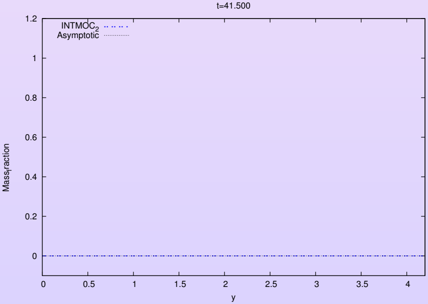
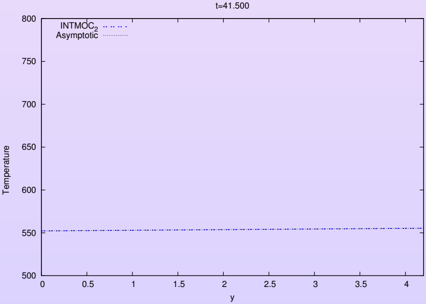


Temperature



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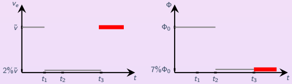

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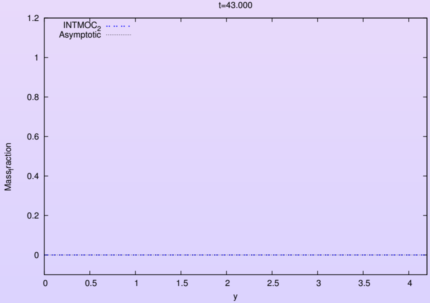
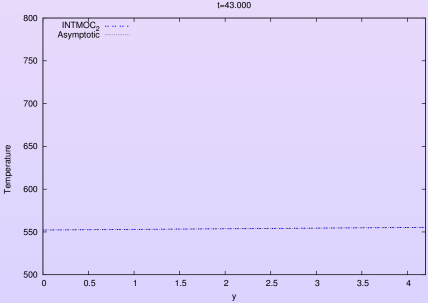


Temperature



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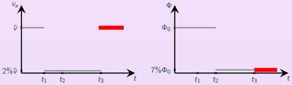

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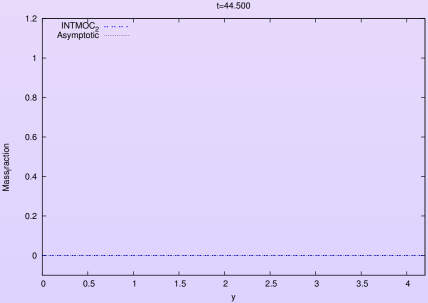
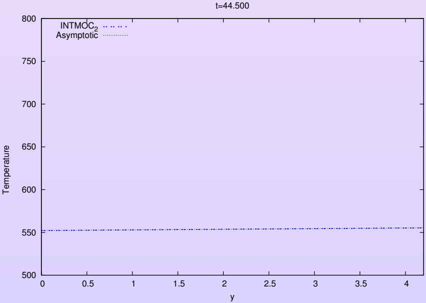


Temperature



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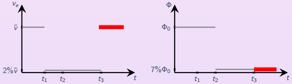

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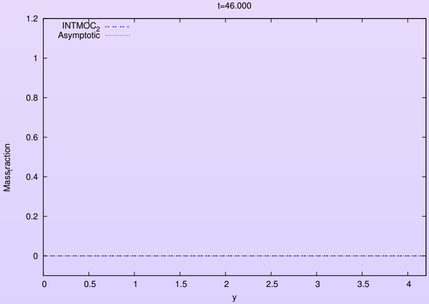

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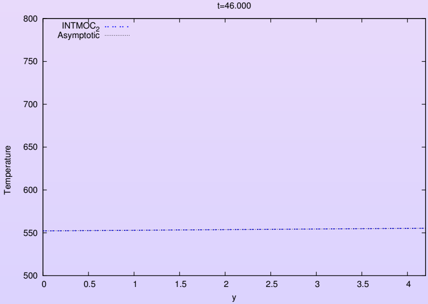

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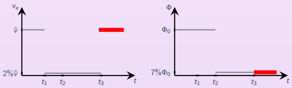

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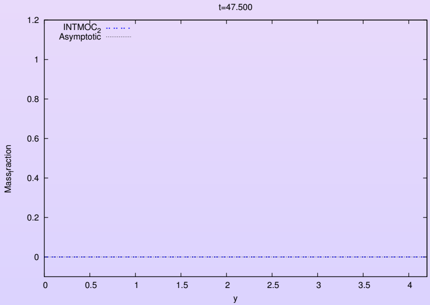

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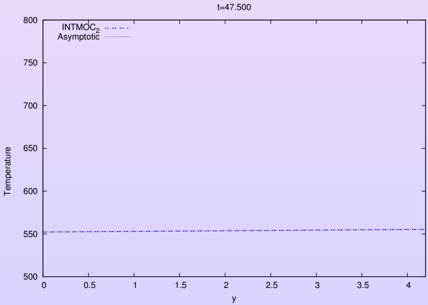
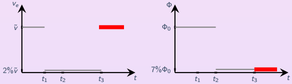


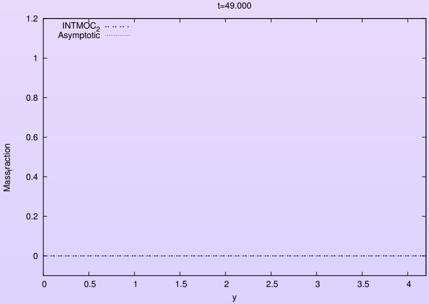
◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

Loss of Flow

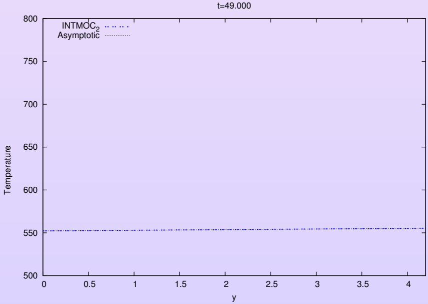


At t_3 the security pumps are turned on $\implies v_e(t) \nearrow$ and the fluid comes back to the liquid phase.

Mass fraction

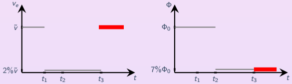


Temperature



Description | $[t_0 - t_1]$ | $[t_1 - t_2]$ | $[t_2 - t_3]$ | $t > t_3$ | Fin

Loss of Flow



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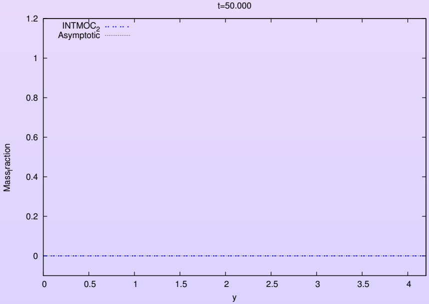
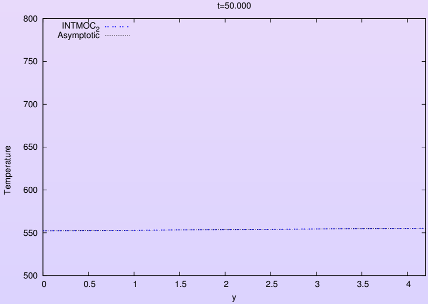


Temperature



Description $[t_0 - t_1]$ $[t_1 - t_2]$ $[t_2 - t_3]$ $t > t_3$ Fin

Section 5

2D-model

- Governing equations
- Analytical solutions
- Numerical scheme
- Numerical tests

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2D-model

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- Analytical solutions
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Governing equations

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \dfrac{\beta(h)}{p_0}\Phi \\ \partial_t h + \mathbf{u}\cdot\nabla h = \dfrac{\Phi}{\varrho(h)} \\ \varrho(h)\Big(\partial_t(\mathbf{u}) + (\mathbf{u}\cdot\nabla)\mathbf{u}\Big) + \nabla\bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h)\mathbf{g} \end{cases}$$

where

$$\sigma(\mathbf{u}) = \mu(h)\begin{pmatrix} 2\partial_x u & \partial_y u + \partial_x v \\ \partial_y u + \partial_x v & 2\partial_y v \end{pmatrix} + \eta(h)\begin{pmatrix} \partial_x u + \partial_y v & 0 \\ 0 & \partial_x u + \partial_y v \end{pmatrix}$$

- Unknowns
- Given quantities
- Equation Of State

Governing equations

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \dfrac{\beta(h)}{p_0}\Phi \\ \partial_t h + \mathbf{u}\cdot\nabla h = \dfrac{\Phi}{\varrho(h)} \\ \varrho(h)\Big(\partial_t(\mathbf{u}) + (\mathbf{u}\cdot\nabla)\mathbf{u}\Big) + \nabla\bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h)\mathbf{g} \end{cases}$$

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▼ **Unknowns**

- $(t,x,y)\mapsto \mathbf{u} \stackrel{\text{def}}{=} (u,v)$ velocity,
- $(t,x,y)\mapsto h$ enthalpy,
- $(t,x,y)\mapsto \bar{p}$ dynamic pressure;

► **Given quantities**

► **Equation Of State**

Governing equations

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \dfrac{\beta(h)}{p_0}\Phi \\ \partial_t h + \mathbf{u}\cdot\nabla h = \dfrac{\Phi}{\varrho(h)} \\ \varrho(h)\Big(\partial_t(\mathbf{u}) + (\mathbf{u}\cdot\nabla)\mathbf{u}\Big) + \nabla\bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h)\mathbf{g} \end{cases}$$

where

$$\sigma(\mathbf{u}) = \mu(h)\begin{pmatrix} 2\partial_x u & \partial_y u + \partial_x v \\ \partial_y u + \partial_x v & 2\partial_y v \end{pmatrix} + \eta(h)\begin{pmatrix} \partial_x u + \partial_y v & 0 \\ 0 & \partial_x u + \partial_y v \end{pmatrix}$$

- ► Unknowns
- ▼ Given quantities
  - $(t, x, y) \mapsto \Phi \geq 0$ power density,
  - $\mathbf{g} \stackrel{\text{def}}{=} (0, -g)$ gravity,
  - $p_0 > 0$ thermodynamic pressure (constant),
- ► Equation Of State

Governing equations

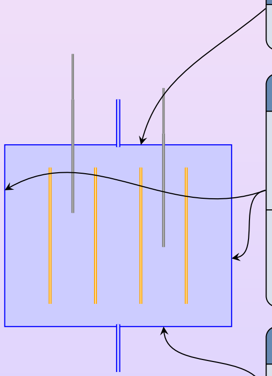
$$
\begin{cases}
\operatorname{div}(\mathbf{u}) = \dfrac{\beta(h)}{p_0}\Phi \\
\partial_t h + \mathbf{u}\cdot\nabla h = \dfrac{\Phi}{\varrho(h)} \\
\varrho(h)\Big(\partial_t(\mathbf{u}) + (\mathbf{u}\cdot\nabla)\mathbf{u}\Big) + \nabla\bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h)\mathbf{g}
\end{cases}
$$

where

$$
\sigma(\mathbf{u}) = \mu(h)\begin{pmatrix} 2\partial_x u & \partial_y u + \partial_x v \\ \partial_y u + \partial_x v & 2\partial_y v \end{pmatrix} + \eta(h)\begin{pmatrix} \partial_x u + \partial_y v & 0 \\ 0 & \partial_x u + \partial_y v \end{pmatrix}
$$

- Unknowns
- Given quantities
- Equation Of State
  - $h \mapsto \mu, \eta$ such that $2\mu + 3\eta > 0$,
  - $h \mapsto \varrho$ density (stiffened gas or tabulated),
  - $h \mapsto \beta \stackrel{\text{def}}{=} -\frac{p_0}{\varrho^2(h)}\varrho'(h)$ compressibility coefficient.

Boundary conditions



Free outflow

$\sigma(\mathbf{u})\cdot\mathbf{n}-\bar{p}\mathbf{n}=\mathbf{0}$

Robin

$$\begin{cases}\mathbf{u}\cdot\mathbf{n}=0\\(\omega(\sigma(\mathbf{u})\cdot\mathbf{n}-\bar{p}\mathbf{n})+(1-\omega)C_\tau\mathbf{u})\cdot\boldsymbol{\tau}=0\end{cases}$$

- Free-slip: $\omega=1$
- No-slip: $\omega=0$

Inflow

$$\begin{cases}h=h_e(t,x),\\(\varrho\mathbf{u})=(0,D_e(t,x))\end{cases}$$

Section 5

2D-model

- Governing equations
- Analytical solutions
- Numerical scheme
- Numerical tests

VERTICAL FLOW (*i.e.* $\mathbf{u} = (0, v)$)

- Let v_e, D_e and Φ constant. With free-slip conditions

$$(u, v, h, \bar{p})(t, x, y) = (0, v_{1D}(t, y), h_{1D}(t, y), \bar{p}_{1D}(t, y))$$

- With no-slip conditions an asymptotic solution with $u^{\infty}(x, y) = 0$ does not exist.

Section 5

2D-model

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FreeFem++ (1)

- Let ξ^n the foot at time t^n of the characteristic issuing from $\mathbf{x}$ at time t^{n+1}, then the convective part of the system can be approximated by

$$[\partial_t \star + (\mathbf{u} \cdot \nabla)\star](t^{n+1}, \mathbf{x}) \approx \frac{\star(t^{n+1}, \mathbf{x}) - \star(t^n, \xi^n)}{\Delta t}, \qquad \star = \mathbf{u} \text{ or } h$$

- Weak formulation of a semi-implicit temporal discretization: at time t^{n+1} find $(\mathbf{u}^{n+1}, \bar{p}^{n+1}, h^{n+1}) \in (\mathbf{u}_e + \mathcal{U}) \times \mathcal{P} \times (h_e + \mathcal{H})$ defined by
  - $\mathcal{U} = \left\{ \mathbf{v} \in (H^1(\Omega))^2 | \mathbf{v}(x,0) = \mathbf{0}, \mathbf{v} \cdot \mathbf{n}(0,y) = \mathbf{v} \cdot \mathbf{n}(L_x, y) = 0 \right\}$
  - $\mathcal{P} = L^2_0(\Omega) = \left\{ q \in L^2(\Omega) | \int_\Omega q(\mathbf{x}) \, d\mathbf{x} = 0 \right\}$
  - $\mathcal{H} = \left\{ k \in H^1(\Omega) | k(x,0) = 0 \right\}$

  such that …

FreeFem++ (2)

- $\forall \mathbf{u}_{\text{test}} \in \mathcal{U}$

$$\begin{aligned}\frac{1}{\Delta t}\int_\Omega \varrho(h^n)(\mathbf{u}^{n+1}-\mathbf{u}^n(\xi^n))\cdot\mathbf{u}_{\text{test}}\,\mathrm{d}\mathbf{x}\\ +\int_\Omega \mu(h^n)((\nabla\mathbf{u}^{n+1}+(\nabla\mathbf{u}^{n+1})^T)\colon\nabla(\mathbf{u}_{\text{test}}))\,\mathrm{d}\mathbf{x}\\ +\int_\Omega \eta(h^n)\,\mathrm{div}(\mathbf{u}^{n+1})\,\mathrm{div}(\mathbf{u}_{\text{test}})\,\mathrm{d}\mathbf{x}-\int_\Omega \bar{p}^{n+1}\,\mathrm{div}(\mathbf{u}_{\text{test}})\,\mathrm{d}\mathbf{x}\\ =\int_\Omega \varrho(h^n)\mathbf{g}\cdot\mathbf{u}_{\text{test}}\,\mathrm{d}\mathbf{x}\end{aligned}$$

- $\forall p_{\text{test}} \in \mathcal{P}$

$$\int_\Omega \mathrm{div}(\mathbf{u}^{n+1})p_{\text{test}}\,\mathrm{d}\mathbf{x}=\frac{1}{p_0}\int_\Omega \beta(h^n)\Phi(t^{n+1})p_{\text{test}}\,\mathrm{d}\mathbf{x}$$

- $\forall h_{\text{test}} \in \mathcal{H}$

$$\frac{1}{\Delta t}\int_\Omega (h^{n+1}-h^n(\xi^n))h_{\text{test}}\,\mathrm{d}\mathbf{x}=\int_\Omega \frac{\Phi(t^{n+1})}{\varrho(h^n)}h_{\text{test}}\,\mathrm{d}\mathbf{x}$$

Section 5

2D-model

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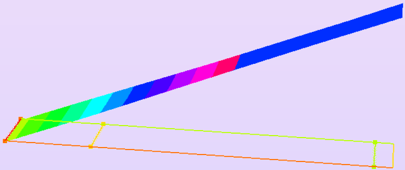
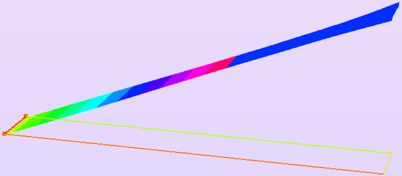
SG vs TAB

Enthalpy



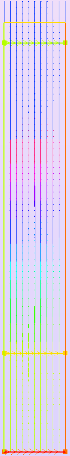
$h \in [1.19 \times 10^6; 3.16367 \times 10^6]$

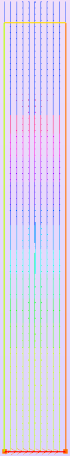


$h \in [1.19 \times 10^6; 3.08409 \times 10^6]$

SG vs TAB

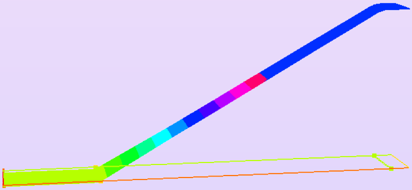
u

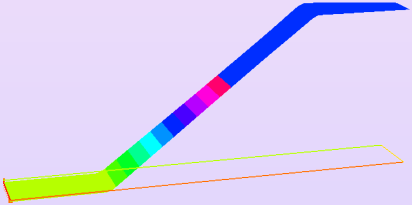




SG vs TAB

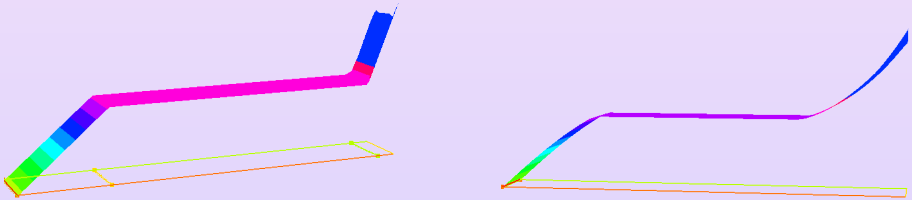
Mass fraction





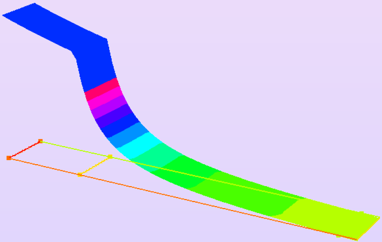
SG vs TAB

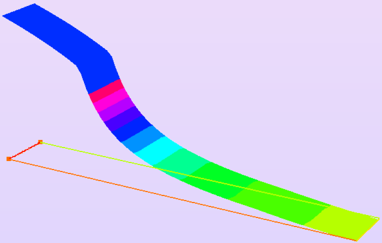
Temperature



SG vs TAB

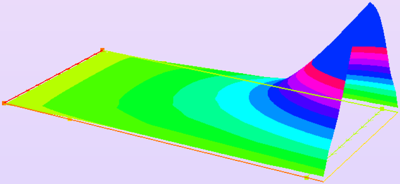
Density



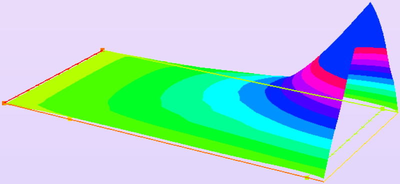


Free-slip vs No-slip

Enthalpy



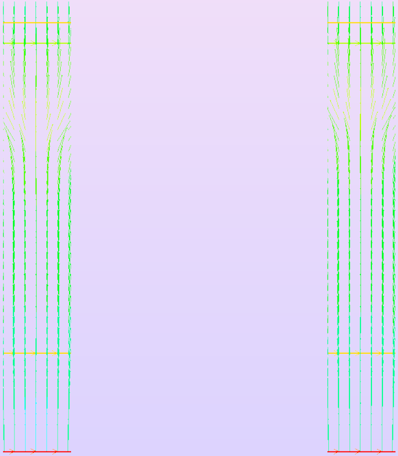
$h \in [1.19 \times 10^6; 2.73224 \times 10^6]$



$h \in [1.19 \times 10^6; 2.54928 \times 10^6]$

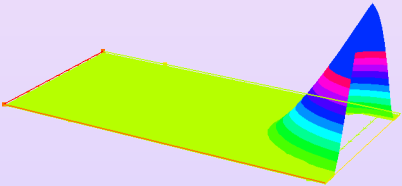
Free-slip vs No-slip

u

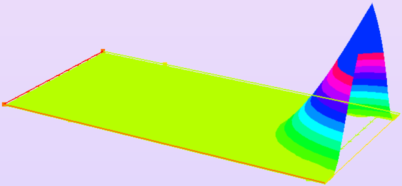


Free-slip vs No-slip

Mass fraction



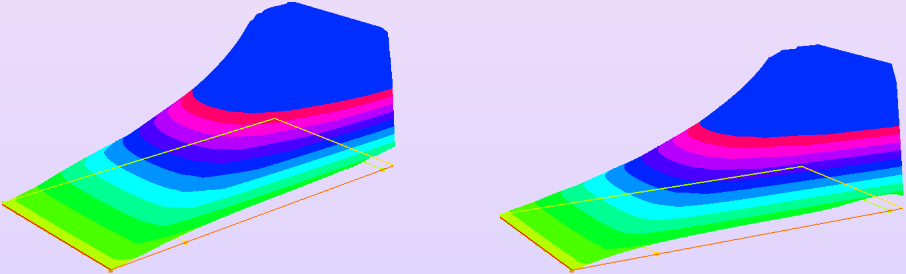
[0; 0.803611]



[0; 0.670913]

Free-slip vs No-slip

Temperature



Free-slip vs No-slip

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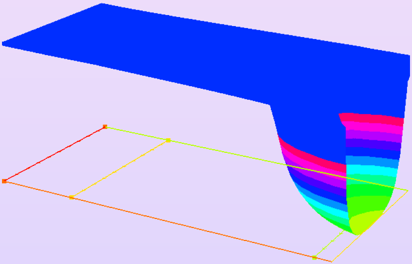
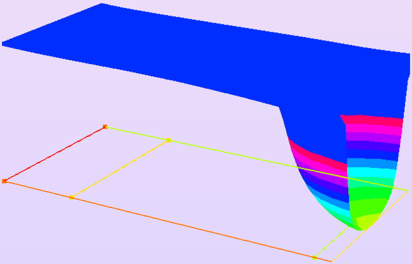


$\varrho \in [64.6185; 749.97]$



$\varrho \in [75.8681; 749.97]$

Section 6

Conclusion & Perspectives

Summary & Perspectives

- Model
  - ✓ mono/diphasic low Mach model with phase transition (stiffened gas & tabulated EoS),

- Theoretical study
  - ✓ unsteady exact solutions on some cases (1D-SG-diphasic), steady exact solutions on all 1D cases (also with tabulated EOS),

- Numerical Method
  - ✓ preliminary results: 1D & 2D

Summary & Perspectives

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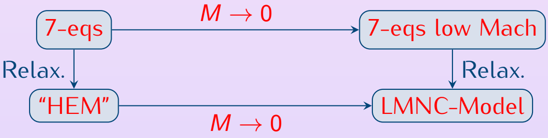
  - ✗ Heat diffusion,
  - ✗ $t \mapsto p_0(t)$,
  - ✗



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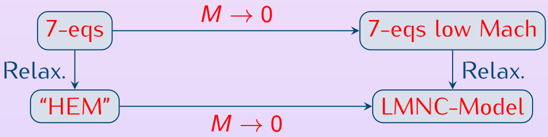

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  - ✓ unsteady exact solutions on some cases (1D-SG-diphasic), steady exact solutions on all 1D cases (also with tabulated EOS),

- Numerical Method
  - ✓ preliminary results: 1D & 2D
  - ✗ quantitative simulations: comparison with compressible model and experimental data,
  - ✗ 2D (C. Calgaro, E. Creusé, T. Goudon).

Appendix

- References
- MOC

References

- S. Dellacherie.
  On a low Mach nuclear core model.
  ESAIM Proc., 35:79–106, 2012.

- M. Bernard, S. Dellacherie, G. Faccanoni, B. Grec, O. Lafitte, T.-T. Nguyen and Y. Penel.
  Study of low Mach nuclear core model for single-phase flow.
  ESAIM Proc., 38:118–134, 2012.

- M. Bernard, S. Dellacherie, G. Faccanoni, B. Grec and Y. Penel.
  Study of low Mach nuclear core model for two-phase flows with phase transition I: stiffened gas law.
  Submitted.

- S. Dellacherie, G. Faccanoni, B. Grec, E. Nayir and Y. Penel.
  2D numerical simulation of a low Mach nuclear core model with stiffened gas using FreeFem++
  Submitted

- S. Dellacherie, G. Faccanoni, B. Grec and Y. Penel.
  Study of low Mach nuclear core model for two-phase flows with phase transition II: tabulated EOS.
  In preparation.

MOC scheme details

1. Foot of the characteristic $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.
2. $\hat{h}_i^n \approx h(t^n, \xi_i^n) \approx \tilde{h}_i^{n+1}(t^n)$.

MOC scheme details

1. Foot of the characteristic $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.
   This approximation is computed either at order one or two:
   1. at order one in time we have $\xi(t^n, y_i) \approx y_i - \Delta t \cdot v(t^n, y_i)$ so that we set
   $$\xi_i^n = y_i - \Delta t \cdot v_i^n,$$
   2. at order two in time we have
   $$\xi(t^n, y_i) \approx y_i - \Delta t \cdot v(t^n, y_i) - \frac{1}{2}\Delta t^2 \left(\partial_t v(t^n, y_i) - \frac{\beta(h(t^n, y_i))}{p_0} v(t^n, y_i)\Phi(t^n, y_i)\right)$$
   so that we set
   $$\xi_i^n = y_i - \Delta t \left(\frac{3}{2} v_i^n - \frac{1}{2} v_i^{n-1}\right) + \frac{\Delta t^2}{2}\frac{\beta(h_i^n)}{p_0} v_i^n \Phi(t^n, y_i).$$
2. $\hat{h}_i^n \approx h(t^n, \xi_i^n) \approx \tilde{h}_i^{n+1}(t^n)$.

MOC scheme details

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2. $\hat{h}_i^n \approx h(t^n, \xi_i^n) \approx \tilde{h}_i^{n+1}(t^n)$.
   If $\xi_i^n > 0$, let j be the index such that $\xi_i^n \in [y_j, y_{j+1})$ and $\theta_{ij}^n \stackrel{\text{def}}{=} \frac{y_{j+1}-\xi_i^n}{\Delta x}$.
   1. At order one $\hat{h}_i^n = \theta_{ij}^n h_j^n + (1-\theta_{ij}^n) h_{j+1}^n$.
   2. At order two $\hat{h}_i^n = \lambda_i^n h_j^- + (1-\lambda_i^n) h_j^+$ where

$$\lambda_i^n \stackrel{\text{def}}{=} \begin{cases} \frac{1+\theta_{ij}^n}{3}, & \text{if } \mathcal{P}_j^+(\theta_{ij}^n) \geq 0 \text{ and } \mathcal{P}_j^-(\theta_{ij}^n) \geq 0, \\ 0, & \text{if } \mathcal{P}_j^+(\theta_{ij}^n) \geq 0 \text{ and } \mathcal{P}_j^-(\theta_{ij}^n) < 0, \\ 1, & \text{if } \mathcal{P}_j^+(\theta_{ij}^n) < 0 \text{ and } \mathcal{P}_j^-(\theta_{ij}^n) \geq 0, \\ \theta_{ij}^n, & \text{otherwise,} \end{cases}$$

$$h_j^- \stackrel{\text{def}}{=} \begin{cases} h_j^n, \text{ if } \mathcal{P}_j^+(\theta_{ij}^n) < 0 \text{ and } \mathcal{P}_j^-(\theta_{ij}^n) < 0, \\ \dfrac{(\theta_{ij}^n)^2}{2}\left(h_{j-1}^n - 2h_j^n + h_{j+1}^n\right) - \dfrac{\theta_{ij}^n}{2}\left(h_{j-1}^n - 4h_j^n + 3h_{j+1}^n\right) + h_{j+1}^n, \text{ otherwise,} \end{cases}$$

$$h_j^+ \stackrel{\text{def}}{=} \begin{cases} h_{j+1}^n, \text{ if } \mathcal{P}_j^+(\theta_{ij}^n) < 0 \text{ and } \mathcal{P}_j^-(\theta_{ij}^n) < 0, \\ \dfrac{(\theta_{ij}^n)^2}{2}\left(h_{j+2}^n - 2h_{j+1}^n + h_j^n\right) - \dfrac{\theta_{ij}^n}{2}\left(h_{j+2}^n - h_j^n\right) + h_{j+1}^n, \text{ otherwise,} \end{cases}$$

and $\mathcal{P}_j^{\pm}(\theta) \stackrel{\text{def}}{=} (\theta - \delta_j^{\pm})(\theta - \delta_{j+1}^{\pm})$ where

$$\delta_j^- \stackrel{\text{def}}{=} \frac{2(h_{j+1}^n - h_j^n)}{h_{j-1}^n - 2h_j^n + h_{j+1}^n}, \qquad \delta_j^+ \stackrel{\text{def}}{=} \frac{2(h_{j+1}^n - h_j^n)}{h_j^n - 2h_{j+1}^n + h_{j+2}^n},$$

$$\delta_{j+1}^- \stackrel{\text{def}}{=} \frac{h_{j-1}^n - 4h_j^n + 3h_{j+1}^n}{h_{j-1}^n - 2h_j^n + h_{j+1}^n}, \qquad \delta_{j+1}^+ \stackrel{\text{def}}{=} \frac{h_{j+2}^n - h_j^n}{h_j^n - 2h_{j+1}^n + h_{j+2}^n}.$$