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A DIPHASIC LOW MACH MODEL WITH PHASE CHANGE

THE L(OW) M(ACH) N(UCLEAR) C(ORE) MODEL

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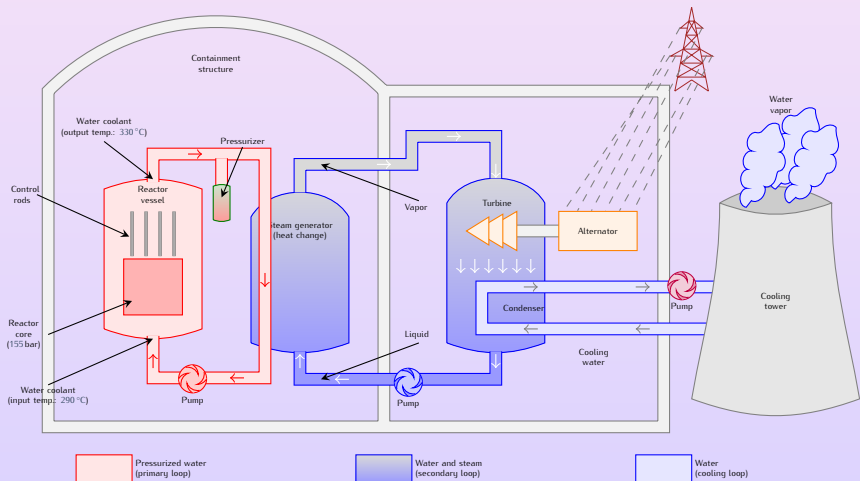
OUTLINE

- 1 Context
- 2 The L(ow) M(ach) N(uclear) C(ore) model
- 3 Equation of State
- 4 1D-model
- 5 2D-model
- 6 Conclusion & Perspectives

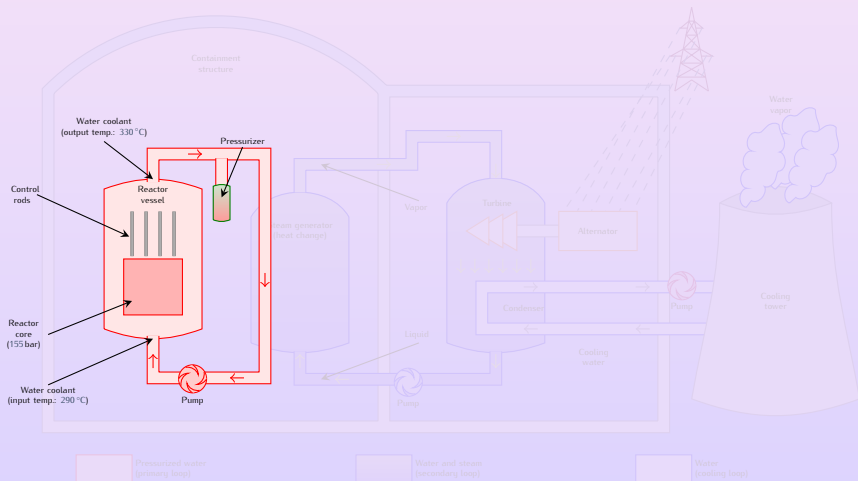
Section 1

CONTEXT

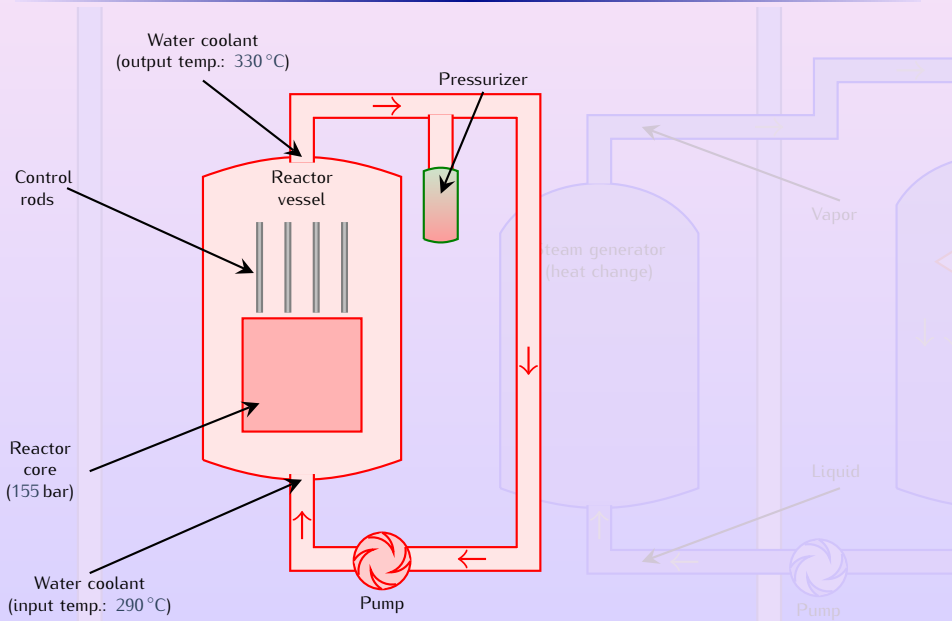
PRESSURIZED WATER REACTOR



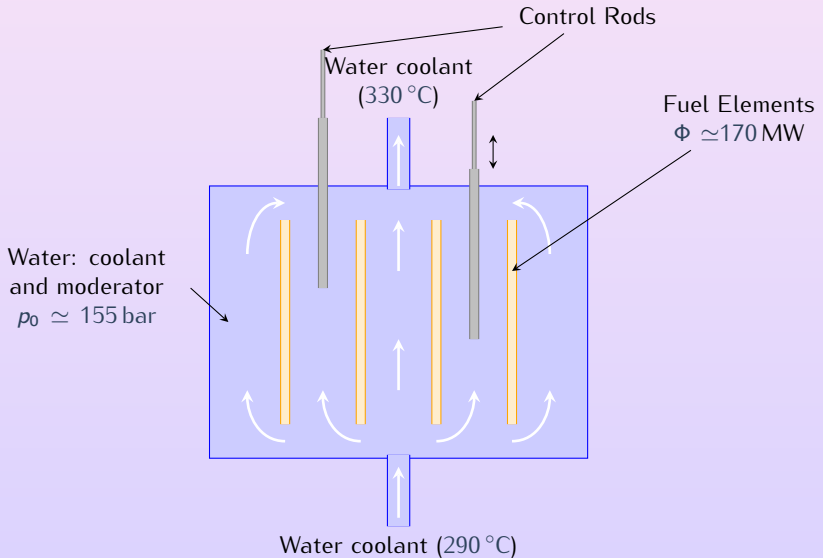
PRESSURIZED WATER REACTOR



PRESSURIZED WATER REACTOR



CORE OF A PRESSURIZED WATER REACTOR



Section 2

THE L(OW) M(ACH) N(UCLEAR) C(ORE) MODEL

COMPRESSIBLE NAVIER-STOKES SYSTEM

$$\begin{cases} \partial_t \varrho + \operatorname{div}(\varrho \mathbf{u}) = 0 \\ \partial_t(\varrho \mathbf{u}) + \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \operatorname{div}(\sigma(\mathbf{u})) + \varrho \mathbf{g} \\ \partial_t(\varrho h) + \operatorname{div}(\varrho h \mathbf{u}) = \partial_t p + \mathbf{u} \cdot \nabla p + \sigma(\mathbf{u}) : \nabla \mathbf{u} + \Phi \end{cases}$$

where

$$\sigma(\mathbf{u}) \stackrel{\text{def}}{=} \nu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- Unknowns
- Given quantities
- Equation Of State

COMPRESSIBLE NAVIER-STOKES SYSTEM

$$\begin{cases} \partial_t \varrho + \operatorname{div}(\varrho \mathbf{u}) = 0 \\ \partial_t(\varrho \mathbf{u}) + \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \operatorname{div}(\sigma(\mathbf{u})) + \varrho \mathbf{g} \\ \partial_t(\varrho h) + \operatorname{div}(\varrho h \mathbf{u}) = \partial_t p + \mathbf{u} \cdot \nabla p + \sigma(\mathbf{u}) : \nabla \mathbf{u} + \Phi \end{cases}$$

where

$$\sigma(\mathbf{u}) \stackrel{\text{def}}{=} \nu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

• Unknowns

- $(t, \mathbf{x}) \mapsto \mathbf{u}$ velocity,
- $(t, \mathbf{x}) \mapsto h$ enthalpy,
- $(t, \mathbf{x}) \mapsto p$ pressure;

• Given quantities

• Equation Of State

COMPRESSIBLE NAVIER-STOKES SYSTEM

$$\begin{cases} \partial_t \varrho + \operatorname{div}(\varrho \mathbf{u}) = 0 \\ \partial_t(\varrho \mathbf{u}) + \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \operatorname{div}(\sigma(\mathbf{u})) + \varrho \mathbf{g} \\ \partial_t(\varrho h) + \operatorname{div}(\varrho h \mathbf{u}) = \partial_t p + \mathbf{u} \cdot \nabla p + \sigma(\mathbf{u}) : \nabla \mathbf{u} + \Phi \end{cases}$$

where

$$\sigma(\mathbf{u}) \stackrel{\text{def}}{=} \nu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- **Unknowns**
- **Given quantities**
 - $(t, \mathbf{x}) \mapsto \Phi \geq 0$ power density,
 - \mathbf{g} gravity;
- **Equation Of State**

COMPRESSIBLE NAVIER-STOKES SYSTEM

$$\begin{cases} \partial_t \varrho + \operatorname{div}(\varrho \mathbf{u}) = 0 \\ \partial_t(\varrho \mathbf{u}) + \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \operatorname{div}(\sigma(\mathbf{u})) + \varrho \mathbf{g} \\ \partial_t(\varrho h) + \operatorname{div}(\varrho h \mathbf{u}) = \partial_t p + \mathbf{u} \cdot \nabla p + \sigma(\mathbf{u}) : \nabla \mathbf{u} + \Phi \end{cases}$$

where

$$\sigma(\mathbf{u}) \stackrel{\text{def}}{=} \nu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- **Unknowns**
- **Given quantities**
- **Equation Of State**
 - $(t, \mathbf{x}) \mapsto \nu$ viscosity and $\eta = -\frac{2}{3}\nu$,
 - $(h, p) \mapsto \varrho$ density.

COMPRESSIBLE NAVIER-STOKES SYSTEM → LMNC-MODEL

Compressible Navier-Stokes system (ρ)



Dimensionless compressible Navier-Stokes system



$$M = \frac{\text{speed of fluid}}{\text{speed of sound}} \ll 1$$



LMNC system (ρ_0 and \bar{p})

S. Dellacherie, *On A Low Mach Nuclear Core Model*, ESAIM: Proc., 35 (2012)

LMNC-MODEL

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \frac{\beta(h)}{\rho_0} \Phi, \\ \partial_t h + \mathbf{u} \cdot \nabla h = \frac{\Phi}{\varrho(h)}, \\ \varrho(h) \left(\partial_t(\mathbf{u}) + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) + \nabla \bar{p} = \operatorname{div}(\boldsymbol{\sigma}(\mathbf{u})) + \varrho(h) \mathbf{g}, \end{cases}$$

where

$$\boldsymbol{\sigma}(\mathbf{u}) \stackrel{\text{def}}{=} \nu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- Unknowns
- Given quantities
- Equation Of State

LMNC-MODEL

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \frac{\beta(h)}{p_0} \Phi, \\ \partial_t h + \mathbf{u} \cdot \nabla h = \frac{\Phi}{\varrho(h)}, \\ \varrho(h) \left(\partial_t(\mathbf{u}) + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) + \nabla \bar{p} = \operatorname{div}(\boldsymbol{\sigma}(\mathbf{u})) + \varrho(h) \mathbf{g}, \end{cases}$$

where

$$\boldsymbol{\sigma}(\mathbf{u}) \stackrel{\text{def}}{=} \nu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- **Unknowns**

- $(t, \mathbf{x}) \mapsto \mathbf{u}$ velocity,
- $(t, \mathbf{x}) \mapsto h$ enthalpy,
- $(t, \mathbf{x}) \mapsto \bar{p}$ dynamic pressure;

- **Given quantities**

- **Equation Of State**

LMNC-MODEL

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \frac{\beta(h)}{p_0} \Phi, \\ \partial_t h + \mathbf{u} \cdot \nabla h = \frac{\Phi}{\varrho(h)}, \\ \varrho(h) (\partial_t(\mathbf{u}) + (\mathbf{u} \cdot \nabla)\mathbf{u}) + \nabla \bar{p} = \operatorname{div}(\boldsymbol{\sigma}(\mathbf{u})) + \varrho(h)\mathbf{g}, \end{cases}$$

where

$$\boldsymbol{\sigma}(\mathbf{u}) \stackrel{\text{def}}{=} \nu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- **Unknowns**
- **Given quantities**
 - $(t, \mathbf{x}) \mapsto \Phi \geq 0$ power density,
 - \mathbf{g} gravity,
 - $p_0 > 0$ thermodynamic pressure (constant),
- **Equation Of State**

LMNC-MODEL

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \frac{\beta(h)}{\rho_0} \Phi, \\ \partial_t h + \mathbf{u} \cdot \nabla h = \frac{\Phi}{\varrho(h)}, \\ \varrho(h) (\partial_t(\mathbf{u}) + (\mathbf{u} \cdot \nabla)\mathbf{u}) + \nabla \bar{p} = \operatorname{div}(\boldsymbol{\sigma}(\mathbf{u})) + \varrho(h)\mathbf{g}, \end{cases}$$

where

$$\boldsymbol{\sigma}(\mathbf{u}) \stackrel{\text{def}}{=} \nu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \eta \nabla \mathbf{u}$$

- Unknowns
- Given quantities
- Equation Of State
 - $(t, \mathbf{x}) \mapsto \nu$ viscosity and $\eta = -\frac{2}{3}\nu$,
 - $h \mapsto \varrho$ density,
 - $h \mapsto \beta \stackrel{\text{def}}{=} -\frac{\rho_0}{\varrho^2(h)} \varrho'(h)$ compressibility coefficient.

Section 3

EQUATION OF STATE

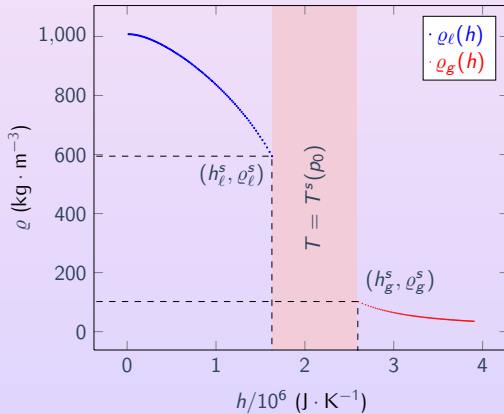
- Thermodynamic of the Phase Change
- Mixture
- Stiffened Gas EoS
- Tabulated EoS

Section 3

EQUATION OF STATE

- Thermodynamic of the Phase Change
- Mixture
- Stiffened Gas EoS
- Tabulated EoS

$h \mapsto \rho$ AT $p_0 = 155$ bar FROM NIST¹



¹Source: <http://webbook.nist.gov/chemistry/fluid/>

Section 3

EQUATION OF STATE

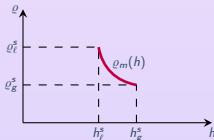
- Thermodynamic of the Phase Change
- **Mixture**
- Stiffened Gas EoS
- Tabulated EoS

MIXTURE

$$\begin{cases} \varrho = \alpha \varrho_g^s + (1 - \alpha) \varrho_\ell^s \\ \varrho h = \alpha \varrho_g^s h_g^s + (1 - \alpha) \varrho_\ell^s h_\ell^s \end{cases} \quad \text{for } h \in [h_\ell^s; h_g^s]$$

⇓

$$\varrho_m(h) = \frac{p_0 / \beta_m}{h - q_m}$$



where

$$\beta_m \stackrel{\text{def}}{=} -p_0 \frac{\varrho_g^s - \varrho_\ell^s}{\varrho_g^s \varrho_\ell^s (h_g^s - h_\ell^s)} = -\frac{p_0}{\varrho_m(h)} \varrho'_m(h)$$

$$q_m \stackrel{\text{def}}{=} \frac{\varrho_g^s h_g^s - \varrho_\ell^s h_\ell^s}{\varrho_g^s - \varrho_\ell^s}$$

Section 3

EQUATION OF STATE

- Thermodynamic of the Phase Change
- Mixture
- **Stiffened Gas EoS**
- Tabulated EoS

PURE PHASE EoS: STIFFENED GAS LAW

$$\varrho_{\kappa}(h) = \frac{\gamma_{\kappa}}{\gamma_{\kappa} - 1} \frac{p_0 + \pi_{\kappa}}{h - q_{\kappa}}$$

where

- $\gamma_{\kappa} > 1$ adiabatic coefficient,
- π_{κ} reference pressure,
- q_{κ} binding energy.

$$\beta_{\kappa} = -\frac{p_0}{\varrho_{\kappa}^2(h)} \varrho'_{\kappa}(h) = \frac{\gamma_{\kappa} - 1}{\gamma_{\kappa}} \frac{p_0}{p_0 + \pi_{\kappa}} \quad \text{constant}$$

$$\varrho_{\kappa}(h) = \frac{p_0 / \beta_{\kappa}}{h - q_{\kappa}}$$

PURE PHASE EoS: STIFFENED GAS LAW

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$$\varrho_{\kappa}(h) = \frac{p_0 / \beta_{\kappa}}{h - q_{\kappa}}$$

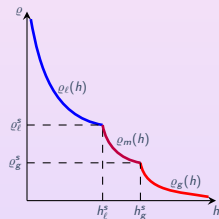
DIPHASIC STIFFENED GAS EOS WITH PHASE TRANSITION

$$\varrho(h) = \frac{p_0/\beta(h)}{h - q(h)}$$

where

$$\beta(h) = \begin{cases} \beta_\ell, & \text{if } h \leq h_\ell^s, \\ \beta_m & \text{if } h_\ell^s < h < h_g^s, \\ \beta_g, & \text{if } h \geq h_g^s, \end{cases}$$

$$q(h) = \begin{cases} q_\ell, & \text{if } h \leq h_\ell^s, \\ q_m & \text{if } h_\ell^s < h < h_g^s, \\ q_g, & \text{if } h \geq h_g^s, \end{cases}$$



Section 3

EQUATION OF STATE

- Thermodynamic of the Phase Change
- Mixture
- Stiffened Gas EoS
- **Tabulated EoS**

SATURATION VALUES

- Liquid $\kappa = \ell$ and vapor $\kappa = g$ are characterized by their EoS

$$(h, p) \mapsto \varrho_{\kappa} = \frac{\gamma_{\kappa}}{\gamma_{\kappa} - 1} \frac{p + \pi_{\kappa}}{h - q_{\kappa}}$$

(see [Le Metayer](#) and [Saurel](#) for parameters of liquid water and steam)

- Second principle of thermodynamics: when phases coexist, they have the same pressures p_0 , the same temperatures and their chemical potentials are equal:

$$T = T^s(p).$$

- We define saturation values at $p = p_0$:

$$h_{\kappa}^s \stackrel{\text{def}}{=} h_{\kappa}(p_0, T^s(p_0)), \quad \varrho_{\kappa}^s \stackrel{\text{def}}{=} \varrho_{\kappa}(h_{\kappa}^s, p_0).$$

SATURATION VALUES

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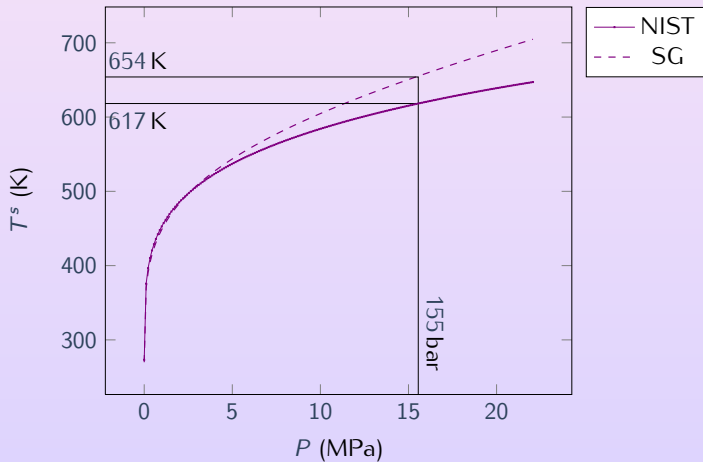
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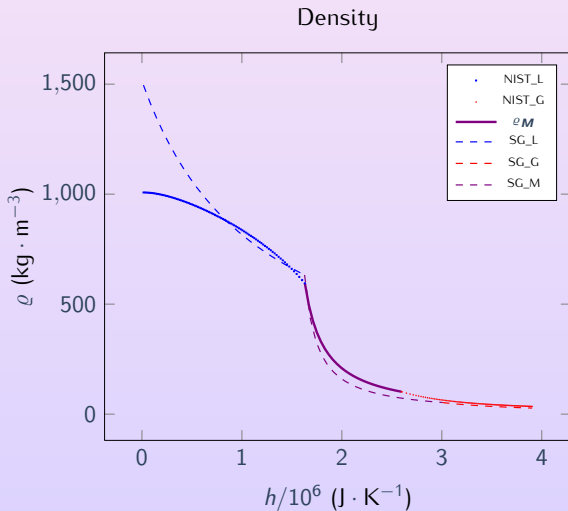
$$p \mapsto T^s$$



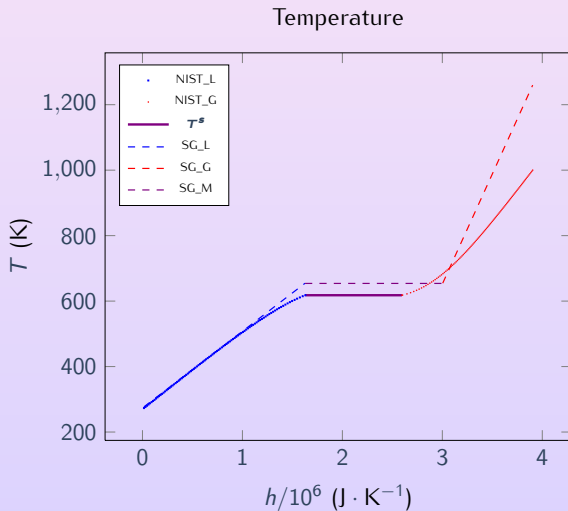
NIST vs SG

	NIST	SG
T^s	617 K	654 K
h_ℓ^s	$1.629 \times 10^6 \text{ J} \cdot \text{K}^{-1}$	$1.627 \times 10^6 \text{ J} \cdot \text{K}^{-1}$
h_g^s	$2.596 \times 10^6 \text{ J} \cdot \text{K}^{-1}$	$3.004 \times 10^6 \text{ J} \cdot \text{K}^{-1}$
ρ_ℓ^s	$594.38 \text{ kg} \cdot \text{m}^{-3}$	$632.663 \text{ kg} \cdot \text{m}^{-3}$
ρ_g^s	$101.93 \text{ kg} \cdot \text{m}^{-3}$	$52.937 \text{ kg} \cdot \text{m}^{-3}$

NIST vs SG: $h \mapsto \rho$

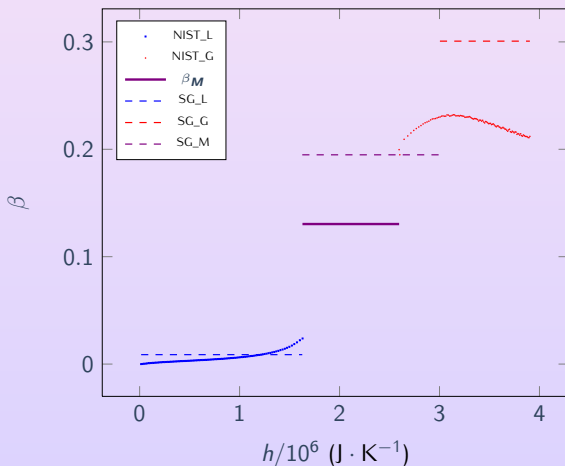


NIST vs SG: $h \mapsto T$



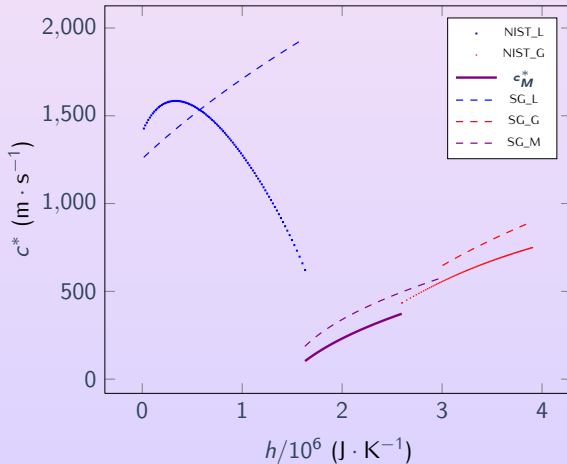
NIST vs SG: $h \mapsto \beta$

Compressibility coefficient



NIST vs SG: $h \mapsto c^*$

Speed of sound



PURE PHASE EoS: TABULATED LAWS AT $p = p_0$

κ	h [kJ/kg]	ρ_κ [kg/m ³]	T_κ [K]	c_κ^* [m · s ⁻¹]	β_κ
l	15.608	1007.5	273.16	1427.4	X
l	30.678	1007.5	276.79	1445.0	X
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
l	1602.8	609.10	614.77	659.56	X
l	h_ℓ^s	594.38	T^s	621.43	X
g	h_g^s	101.93	T^s	433.40	X
g	2602.6	101.06	618.41	435.61	X
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
g	2.5299	35.139	996.37	747.83	X
g	2.5290	34.985	1000.0	749.37	X

Source: <http://webbook.nist.gov/chemistry/fluid/>

PURE PHASE EoS: TABULATED LAWS AT $p = p_0$

Liquid phase

- Discretization of the enthalpy interval $[1.56 \times 10^4; h_\ell^s]$:

$$h_i \simeq (1.56 + 1.68i) \times 10^4, \quad i \in \mathcal{I} = \{1, \dots, 96\}$$

- Approximation of $\beta_\ell(h_i) = -\frac{p_0}{\varrho_\ell^2(h_i)} \varrho'_\ell(h_i)$ by finite differences
- Least squares polynomial approximation over the set of discrete values $((\varrho_\ell, \beta_\ell, T_\ell, c_\ell^*)(h_i))_{i \in \mathcal{I}}$:

$$(\varrho_\ell, \beta_\ell, T_\ell, c_\ell^*) \left(\frac{h}{10^6} \right) = \sum_{j=0}^N \left(\frac{h}{10^6} \right)^j a_j, \quad N \leq 6$$

PURE PHASE EoS: TABULATED LAWS AT $p = p_0$

Vapor phase

- Discretization of the enthalpy interval $[h_g^s; 25.29 \times 10^6]$:

$$h_i \simeq (2.596 + 0.0122i) \times 10^6, \quad i \in \mathcal{J} = \{1, \dots, 107\}$$

- Approximation of $\beta_g(h_i) = -\frac{p_0}{\varrho_g^2(h_i)} \varrho_g'(h_i)$ by finite differences
- Least squares polynomial approximation over the set of discrete values $((\varrho_g, \beta_g, T_g, c_g^*)(h_i))_{i \in \mathcal{J}}$:

$$(\varrho_g, \beta_g, T_g, c_g^*) \left(\frac{h}{10^6} \right) = \sum_{j=0}^N \left(\frac{h}{10^6} \right)^j a_j, \quad N \leq 6$$

Section 4

1D-MODEL

- Governing equations
- Analytical solutions
- Numerical schemes
- Numerical tests

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1D-MODEL

- Governing equations
- Analytical solutions
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GOVERNING EQUATIONS

$$\begin{cases} \partial_y v = \frac{\beta}{\rho_0} \Phi \\ \partial_t h + v \partial_y h = \frac{\Phi}{\rho} \\ \partial_t(\rho v) + \partial_y(\rho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\rho g \end{cases}$$

- Unknowns
- Given quantities
- Equation Of State
- Boundary Conditions
- Initial Conditions

GOVERNING EQUATIONS

$$\begin{cases} \partial_y \mathbf{v} = \frac{\beta}{\rho_0} \Phi \\ \partial_t h + \mathbf{v} \partial_y h = \frac{\Phi}{\rho} \\ \partial_t (\rho \mathbf{v}) + \partial_y (\rho \mathbf{v}^2 + \bar{p}) - \partial_y (\mu \partial_y \mathbf{v}) = -\rho g \end{cases}$$

• Unknowns

- $(t, y) \mapsto v$ velocity,
- $(t, y) \mapsto h$ enthalpy,
- $(t, y) \mapsto \bar{p}$ dynamic pressure;

• Given quantities

• Equation Of State

• Boundary Conditions

• Initial Conditions

GOVERNING EQUATIONS

$$\begin{cases} \partial_y v = \frac{\beta}{\rho_0} \Phi \\ \partial_t h + v \partial_y h = \frac{\Phi}{\rho} \\ \partial_t(\rho v) + \partial_y(\rho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\rho g \end{cases}$$

- **Unknowns**
- **Given quantities**
 - $\rho_0 > 0$ thermodynamic pressure (constant),
 - $(t, y) \mapsto \Phi \geq 0$ power density,
 - g gravity.
- **Equation Of State**
- **Boundary Conditions**
- **Initial Conditions**

GOVERNING EQUATIONS

$$\begin{cases} \partial_y v = \frac{\beta}{\rho_0} \Phi \\ \partial_t h + v \partial_y h = \frac{\Phi}{\rho} \\ \partial_t(\rho v) + \partial_y(\rho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\rho g \end{cases}$$

- **Unknowns**
- **Given quantities**
- **Equation Of State**
 - $(t, y) \mapsto \mu$ viscosity,
 - $h \mapsto \rho$ density (stiffened gas or tabulated)
 - $h \mapsto \beta = -\frac{p_0}{\rho^2(h)} \rho'(h)$ compressibility coefficient.
- **Boundary Conditions**
- **Initial Conditions**

GOVERNING EQUATIONS

$$\begin{cases} \partial_y v = \frac{\beta}{\rho_0} \Phi \\ \partial_t h + v \partial_y h = \frac{\Phi}{\rho} \\ \partial_t(\rho v) + \partial_y(\rho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\rho g \end{cases}$$

- **Unknowns**
- **Given quantities**
- **Equation Of State**
- **Boundary Conditions**
 - top: dynamic pressure $\bar{p}(t, y = L) = p_0$
 - bottom:
 - entrance flow rate $(\rho v)(t, y = 0) = D_e(t)$
 - entrance enthalpy $h(t, y = 0) = h_e(t)$
- **Initial Conditions**

GOVERNING EQUATIONS

$$\begin{cases} \partial_y v = \frac{\beta}{\rho_0} \Phi \\ \partial_t h + v \partial_y h = \frac{\Phi}{\rho} \\ \partial_t(\rho v) + \partial_y(\rho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\rho g \end{cases}$$

- Unknowns
- Given quantities
- Equation Of State
- Boundary Conditions
- Initial Conditions
 - $h(t = 0, y) = h_0(y),$
 - $v(t = 0, y) = v_0(y) = v_e(0) + \frac{1}{\rho_0} \int_0^y \beta(h_0(z)) \Phi(0, z) dz,$
 - $\bar{p}(t = 0, y) = p_0.$

Section 4

1D-MODEL

- Governing equations
- **Analytical solutions**
- Numerical schemes
- Numerical tests

1D-SG-MONOPHASIC

- 1 Velocity
- 2 Enthalpy
- 3 Dynamic pressure

1D-SG-MONOPHASIC

1 Velocity

Direct integration of $\partial_y v = \frac{\beta}{p_0} \Phi$.

$$v(t, y) = v_e(t) + \frac{\beta}{p_0} \Psi(t, y), \quad \Psi(t, y) \stackrel{\text{def}}{=} \int_0^y \Phi(t, z) \, dz$$

2 Enthalpy

3 Dynamic pressure

1D-SG-MONOPHASIC

1 Velocity

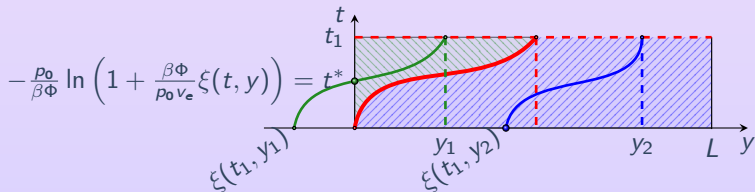
2 Enthalpy

Method of characteristics on $\partial_t h + v \partial_y h = \frac{\Phi}{\rho(h)} = \frac{\beta\Phi}{\rho_0} (h - q)$.

Example: if Φ and v_e are constant, then

$$h(t, y) = \begin{cases} q + (h_0(\xi(t, y)) - q) e^{\frac{\beta\Phi}{\rho_0} t} & \text{if } \xi(t, y) \geq 0, \\ h_e(t^*(t, y)) + \frac{\Phi}{D_e(t^*(t, y))} y & \text{if } \xi(t, y) < 0. \end{cases}$$

where $\xi(t, y) = \left(y + \frac{\rho_0}{\beta\Phi} v_e\right) e^{-\frac{\beta\Phi}{\rho_0} t} - \frac{\rho_0}{\beta\Phi} v_e$ and



3 Dynamic pressure

1D-SG-MONOPHASIC

- 1 Velocity
- 2 Enthalpy
- 3 Dynamic pressure

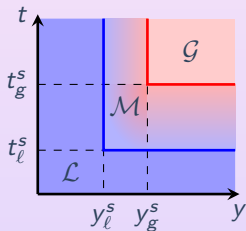
Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_t(\rho v) - \partial_y(\rho v^2) - \rho g$.

Example: if Φ and v_e are constant, then

$$\begin{aligned} \bar{p}(t, y) = & p_0 + \frac{\beta \Phi}{\rho_0} (\mu(y) - \mu(L)) \\ & + \frac{\rho_0 (g + \frac{\beta \Phi}{\rho_0} v_e)}{\beta} \int_y^L \frac{1}{h(t, z) - q} dz \\ & + \beta \Phi^2 \int_y^L \frac{z}{h(t, z) - q} dz \end{aligned}$$

1D-SG-DIPHASIC

Let Φ , v_e , h_e and h_0 be constant and suppose liquid phase in entrance.



$$y_l^s = \frac{D_e}{\Phi} (h_l^s - h_e)$$

$$y_g^s = \frac{D_e}{\Phi} (h_g^s - h_e)$$

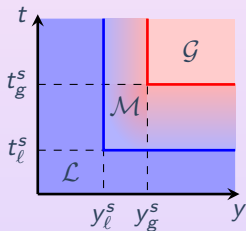
$$t_l^s = \frac{p_0}{\beta_l \Phi} \ln \left(\frac{h_l^s - q_l}{h_0 - q_l} \right)$$

$$t_g^s = t_l^s + \frac{p_0}{\beta_m \Phi} \ln \left(\frac{h_g^s - q_m}{h_l^s - q_m} \right)$$

- 1 Velocity
- 2 Enthalpy

1D-SG-DIPHASIC

Let Φ , v_e , h_e and h_0 be constant and suppose liquid phase in entrance.



$$y_\ell^s = \frac{D_e}{\Phi} (h_\ell^s - h_e)$$

$$y_g^s = \frac{D_g}{\Phi} (h_g^s - h_e)$$

$$t_\ell^s = \frac{p_0}{\beta_\ell \Phi} \ln \left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell} \right)$$

$$t_g^s = t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln \left(\frac{h_g^s - q_m}{h_\ell^s - q_m} \right)$$

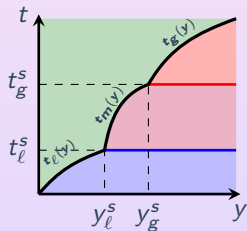
- ① **Velocity:** direct integration of $\partial_y v = \frac{\beta(h)}{\rho_0} \Phi$.

$$v(t, y) = \begin{cases} v_e + \frac{\beta_\ell \Phi}{\rho_0} y & \text{if } (t, y) \in \mathcal{L}, \\ v_e + \frac{\beta_\ell \Phi}{\rho_0} y_\ell^s + \frac{\beta_m \Phi}{\rho_0} (y - y_\ell^s) & \text{if } (t, y) \in \mathcal{M}, \\ v_e + \frac{\beta_\ell \Phi}{\rho_0} y_\ell^s + \frac{\beta_m \Phi}{\rho_0} (y_g^s - y_\ell^s) + \frac{\beta_g \Phi}{\rho_0} (y - y_g^s) & \text{if } (t, y) \in \mathcal{G}, \end{cases}$$

- ② **Enthalpy**

1D-SG-DIPHASIC

Let Φ , v_e , h_e and h_0 be constant and suppose liquid phase in entrance.



$$y_l^s = \frac{D_e}{\Phi} (h_l^s - h_e)$$

$$y_g^s = \frac{D_e}{\Phi} (h_g^s - h_e)$$

$$t_l^s = \frac{p_0}{\beta_l \Phi} \ln \left(\frac{h_l^s - q_l}{h_0 - q_l} \right)$$

$$t_g^s = t_l^s + \frac{p_0}{\beta_m \Phi} \ln \left(\frac{h_g^s - q_m}{h_l^s - q_m} \right)$$

1 Velocity

2 Enthalpy: method of characteristics on $\partial_t h + v \partial_y h = \frac{\beta(h)\Phi}{\rho_0} (h - q(h))$.

$$h(t, y) = \begin{cases} q_l + (h_0 - q_l) e^{\frac{\beta_l \Phi}{\rho_0} t} & \text{if } (t, y) \in \mathcal{L} \text{ and } t < t_l(y), \\ q_m + (h_l^s - q_m) e^{\frac{\beta_m \Phi}{\rho_0} (t - t_l^s)} & \text{if } (t, y) \in \mathcal{M} \text{ and } t < t_m(y), \\ q_g + (h_g^s - q_g) e^{\frac{\beta_g \Phi}{\rho_0} (t - t_g^s)} & \text{if } (t, y) \in \mathcal{G} \text{ and } t < t_g(y), \\ h_e + \frac{\Phi}{D_e} y & \text{otherwise.} \end{cases}$$

1D-TAB-DIPHASIC

$$(h_e^\infty, D_e^\infty > 0, \Phi^\infty(y)) \stackrel{\text{def}}{=} \lim_{t \rightarrow +\infty} (h_e(t), D_e(t), \Phi(t, y))$$

1 Enthalpy

Using $\partial_y(\rho^\infty v^\infty) = 0$ we have $\partial_y h^\infty = \frac{\Phi^\infty}{D_e^\infty}$.

$$h^\infty(y) = h_e^\infty + \frac{\Psi(y)}{D_e^\infty}, \quad \Psi(y) \stackrel{\text{def}}{=} \int_0^y \Phi^\infty(z) dz$$

2 Velocity

$$v^\infty(y) = \frac{D_e^\infty}{\rho(h^\infty(y))}$$

3 Dynamic pressure

Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_y(\rho v^2) - \rho g$.

Section 4

1D-MODEL

- Governing equations
- Analytical solutions
- **Numerical schemes**
- Numerical tests

MOC-SCHEME (SG & TAB)

- 1 Enthalpy
- 2 Velocity

MOC-SCHEME (SG & TAB)

1 Enthalpy - key idea:

$$\partial_t h(t^{n+1}, y_i) + v(t^{n+1}, y_i) \partial_y h(t^{n+1}, y_i) = \frac{\Phi(t^{n+1}, y_i)}{\rho(h(t^{n+1}, y_i))}$$

$$\Downarrow$$

$$\frac{d}{d\tau} \tilde{h}_i^{n+1}(\tau) = \frac{\Phi(\tau, \chi(\tau; t^{n+1}, y_i))}{\rho(\tilde{h}_i^{n+1}(\tau))}$$

where $\bar{t} \in [t^n; t^{n+1}[$, $\tau \mapsto \tilde{h}_i^{n+1}(\tau) \stackrel{\text{def}}{=} h(\tau, \chi(\tau; t^{n+1}, y_i))$ and χ is the solution of

$$\begin{cases} \frac{d}{d\tau} \chi(\tau; t^{n+1}, y_i) = v(\tau, \chi(\tau; t^{n+1}, y_i)), & \tau \leq t^{n+1}, \\ \chi(t^{n+1}; t^{n+1}, y_i) = y_i. \end{cases}$$

2 Velocity

MOC-SCHEME (SG & TAB)

1 Enthalpy - key idea:

$$\partial_t h(t^{n+1}, y_i) + v(t^{n+1}, y_i) \partial_y h(t^{n+1}, y_i) = \frac{\Phi(t^{n+1}, y_i)}{\rho(h(t^{n+1}, y_i))}$$

$$\Downarrow$$

$$\int_{\bar{t}}^{t^{n+1}} \frac{d}{d\tau} \tilde{h}_i^{n+1}(\tau) d\tau = \int_{\bar{t}}^{t^{n+1}} \frac{\Phi(\tau, \chi(\tau; t^{n+1}, y_i))}{\rho(\tilde{h}_i^{n+1}(\tau))} d\tau$$

where $\bar{t} \in [t^n; t^{n+1}[$, $\tau \mapsto \tilde{h}_i^{n+1}(\tau) \stackrel{\text{def}}{=} h(\tau, \chi(\tau; t^{n+1}, y_i))$ and χ is the solution of

$$\begin{cases} \frac{d}{d\tau} \chi(\tau; t^{n+1}, y_i) = v(\tau, \chi(\tau; t^{n+1}, y_i)), & \tau \leq t^{n+1}, \\ \chi(t^{n+1}; t^{n+1}, y_i) = y_i. \end{cases}$$

2 Velocity

MOC-SCHEME (SG & TAB)

1 Enthalpy - key idea:

$$\partial_t h(t^{n+1}, y_i) + v(t^{n+1}, y_i) \partial_y h(t^{n+1}, y_i) = \frac{\Phi(t^{n+1}, y_i)}{\rho(h(t^{n+1}, y_i))}$$

$$h(t^{n+1}, y_i) - \tilde{h}_i^{n+1}(\bar{t}) = \int_{\bar{t}}^{t^{n+1}} \frac{d}{d\tau} \tilde{h}_i^{n+1}(\tau) d\tau = \int_{\bar{t}}^{t^{n+1}} \frac{\Phi(\tau, \chi(\tau; t^{n+1}, y_i))}{\rho(\tilde{h}_i^{n+1}(\tau))} d\tau$$

where $\bar{t} \in [t^n; t^{n+1}]$, $\tau \mapsto \tilde{h}_i^{n+1}(\tau) \stackrel{\text{def}}{=} h(\tau, \chi(\tau; t^{n+1}, y_i))$ and χ is the solution of

$$\begin{cases} \frac{d}{d\tau} \chi(\tau; t^{n+1}, y_i) = v(\tau, \chi(\tau; t^{n+1}, y_i)), & \tau \leq t^{n+1}, \\ \chi(t^{n+1}; t^{n+1}, y_i) = y_i. \end{cases}$$

2 Velocity

MOC-SCHEME (SG & TAB)

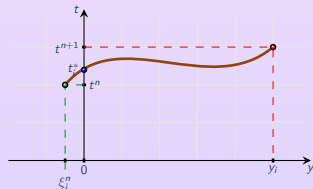
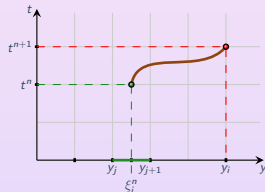
① **Enthalpy** - scheme: let $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.

- If $\xi_i^n > 0$, let $\hat{h}_i^n \approx \tilde{h}_i^{n+1}(t^n)$ (at order 1 or higher) and then $\bar{t} = t^n$ and

$$h_i^{n+1} = \hat{h}_i^n + \Delta t \frac{\Phi(t^n, \xi_i^n)}{\varrho(\hat{h}_i^n)}$$

- If $\xi_i^n \leq 0$, let $t_i^* = t^{n+1} - y_i/v_i^n \approx \tau$ such that $\chi(\tau; t^{n+1}, y_i) = 0$ and then $\bar{t} = t_i^*$ and

$$h_i^{n+1} = h_e(t_i^*) + (t^{n+1} - t_i^*) \frac{\Phi(t_i^*, 0)}{\varrho(h_e(t_i^*))}$$



② **Velocity**

MOC-SCHEME (SG & TAB)

1 Enthalpy

2 Velocity : $\partial_y v = \frac{\beta(h)\Phi}{\rho_0}$

$$v_i^{n+1} = v_{i-1}^{n+1} + \frac{1}{\rho_0} \int_{y_{i-1}}^{y_i} \beta(h(t^{n+1}, z)) \Phi(t^{n+1}, z) dz$$

$$\approx v_{i-1}^{n+1} + \frac{\Delta y}{\rho_0} \beta(h_{i-1}^{n+1}) \Phi(t^{n+1}, y_{i-1}).$$

β is discontinuous at phase change points, so that if $h_{\kappa}^s \in (h_{i-1}^{n+1}, h_i^{n+1})$, let $y^* = y_{i-1} + \Delta y \frac{h_{\kappa}^s - h_{i-1}^{n+1}}{h_i^{n+1} - h_{i-1}^{n+1}}$ and then

$$\int_{y_{i-1}}^{y_i} \beta(h(t^{n+1}, z)) \Phi(t^{n+1}, z) dz$$

$$\approx (y^* - y_{i-1}) \beta(h_{i-1}^{n+1}) \Phi(t^{n+1}, y_{i-1}) dy + (y_i - y^*) \beta(h_i^{n+1}) \Phi(t^{n+1}, y_i) dy$$

INTMOC-SCHEME (SG)

1 Enthalpy - key idea:

$$\frac{\frac{d}{d\tau} \tilde{h}_i^{n+1}(\tau)}{\beta(\tilde{h}_i^{n+1}(\tau)) \left(\tilde{h}_i^{n+1}(\tau) - q(\tilde{h}_i^{n+1}(\tau)) \right)} = \frac{\Phi(\tau, \chi(\tau; t^{n+1}, y_i))}{p_0}$$

$$\int_{\tilde{h}_i^{n+1}(\bar{t})}^{\tilde{h}_i^{n+1}(t^{n+1})} \frac{1}{\beta(h)(h-q(h))} dh = \frac{1}{p_0} \int_{\bar{t}}^{t^{n+1}} \Phi(\tau, \chi(\tau; t^{n+1}, y_i)) d\tau$$

so that

$$\tilde{h}_i^{n+1}(t^{n+1}) = R^{-1} \left(R(\tilde{h}_i^{n+1}(\bar{t})) + \frac{1}{p_0} \int_{\bar{t}}^{t^{n+1}} \Phi(\tau, \chi(\tau; t^{n+1}, y_i)) d\tau \right)$$

where

$$R(h) \stackrel{\text{def}}{=} \int_0^{\tilde{h}} \frac{1}{\beta(h)(h-q(h))} dh$$

INTMOC-SCHEME (SG)

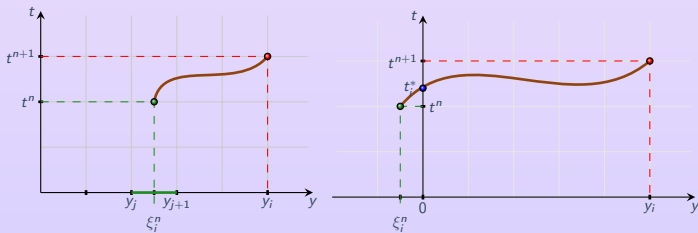
① **Enthalpy** - scheme: let $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.

- If $\xi_i^n > 0$, let $\hat{h}_i^n \approx \tilde{h}_i^{n+1}(t^n)$ (at order 1 or 2) and then $\bar{t} = t^n$ and

$$h_i^{n+1} = R^{-1} \left(R(\hat{h}_i^n) + \frac{\Delta t}{\rho_0} \frac{\Phi(t^n, \xi_i^n) + \Phi(t^{n+1}, y_i)}{2} \right)$$

- If $\xi_i^n \leq 0$, let $t_i^* = t^{n+1} - y_i/v_i^n \approx \tau$ such that $\chi(\tau; t^{n+1}, y_i) = 0$ and then $\bar{t} = t_i^*$ and

$$h_i^{n+1} = R^{-1} \left(R(h_e(t_i^*)) + \frac{t^{n+1} - t_i^*}{\rho_0} \frac{\Phi(t_i^*, 0) + \Phi(t^{n+1}, y_i)}{2} \right)$$

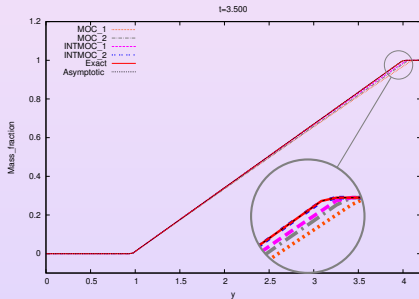
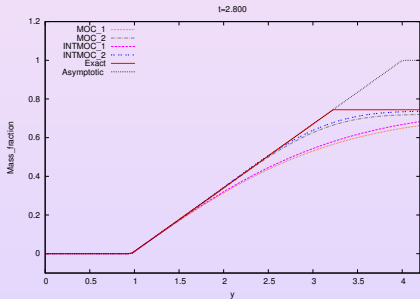


Section 4

1D-MODEL

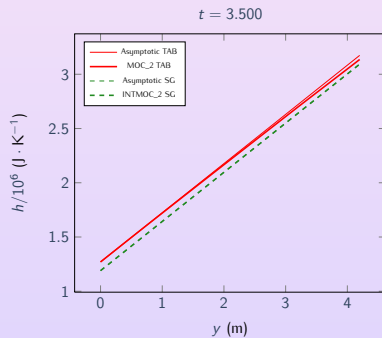
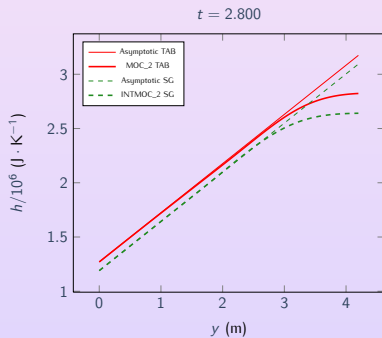
- Governing equations
- Analytical solutions
- Numerical schemes
- Numerical tests

SG: MOC (ORDER 1 OR 2) VS INTMOC (ORDER 1 OR 2)

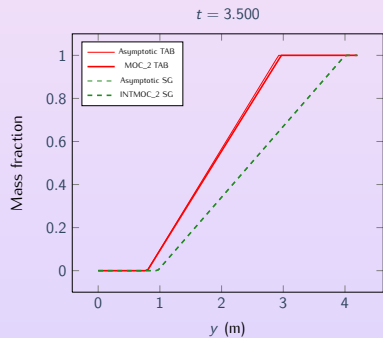
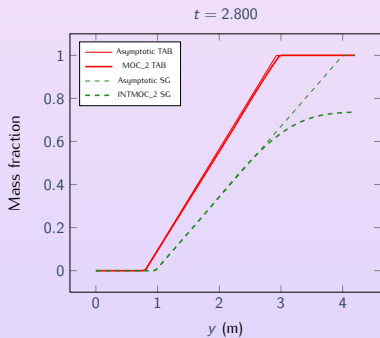


- Initially the domain is filled with liquid phase
- At $t = 1.769$ s mixture appears for $y > y_\ell^s \simeq 0.964$ m
- At $t = 2.929$ s pure vapor phase appears for $y > y_g^s \simeq 4.002$ m
- The asymptotic state is reached at $t = 2.957$ s

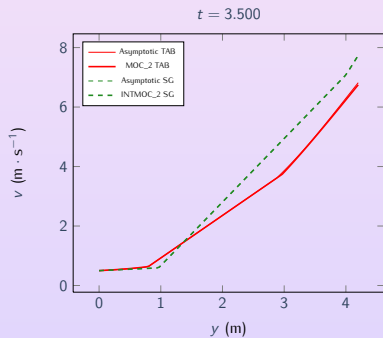
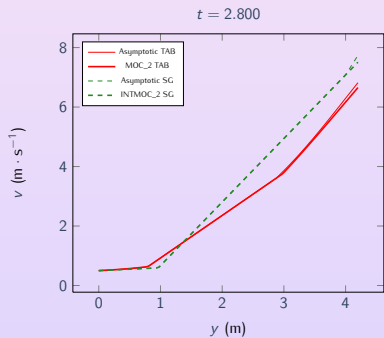
SG (INTMOC 2) vs TAB (MOC 2)



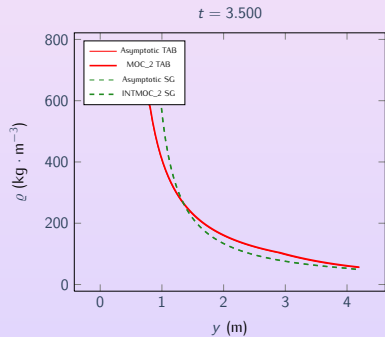
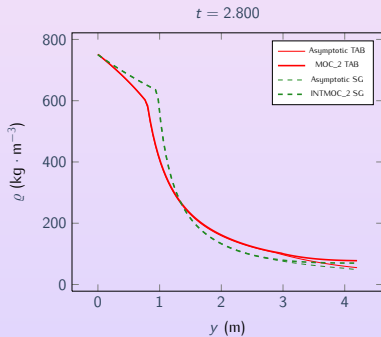
SG (INTMOC 2) vs TAB (MOC 2)



SG (INTMOC 2) vs TAB (MOC 2)

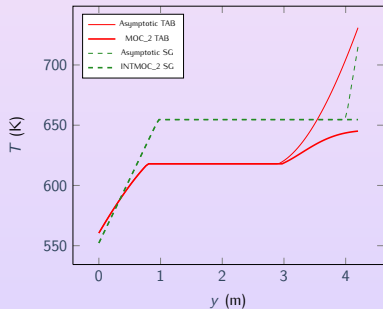


SG (INTMOC 2) vs TAB (MOC 2)

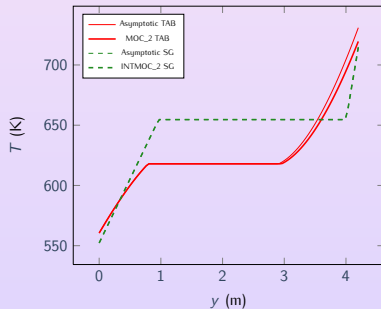


SG (INTMOC 2) vs TAB (MOC 2)

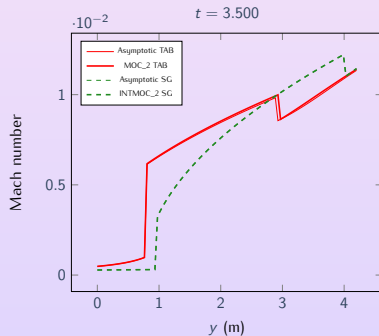
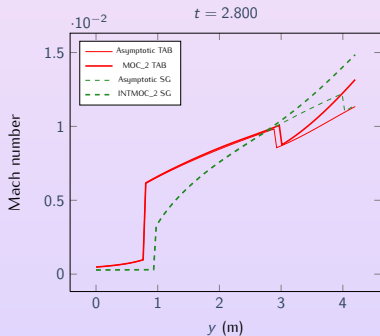
$t = 2.800$



$t = 3.500$



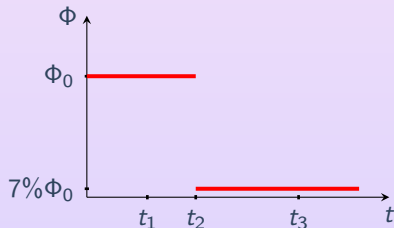
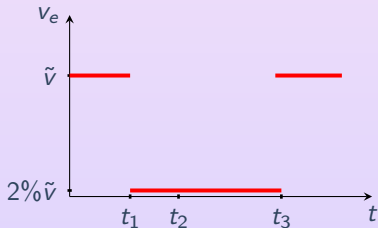
SG (INTMOC 2) vs TAB (MOC 2)



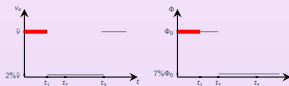
LOSS OF FLOW

$$v_e(t) = \begin{cases} \tilde{v} & \text{if } 0 \leq t < t_1, \\ 2\% \tilde{v} & \text{if } t_1 \leq t < t_3, \\ \tilde{v} & \text{if } t \geq t_3, \end{cases}$$

$$\Phi(t) = \begin{cases} \Phi_0 & \text{if } 0 \leq t < t_2, \\ 7\% \Phi_0 & \text{if } t \geq t_2. \end{cases}$$

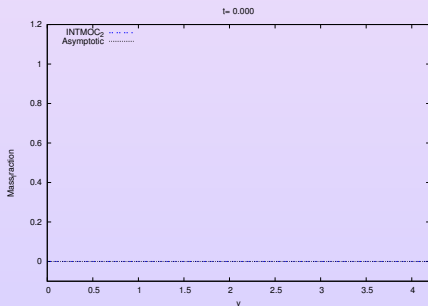


LOSS OF FLOW

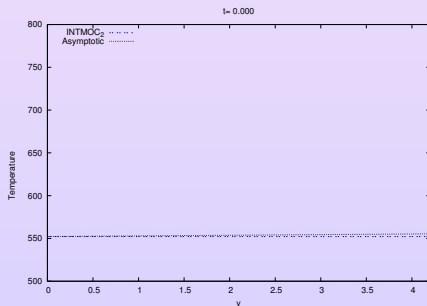


Initial data: $v_e(t) = \tilde{v}$ and $\Phi(t, y) = \Phi_0$.

Mass fraction



Temperature



◀ Description

▶ $[t_0 - t_1]$

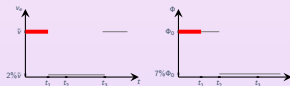
▶ $[t_1 - t_2]$

▶▶ $[t_2 - t_3]$

▶▶▶ $t > t_3$

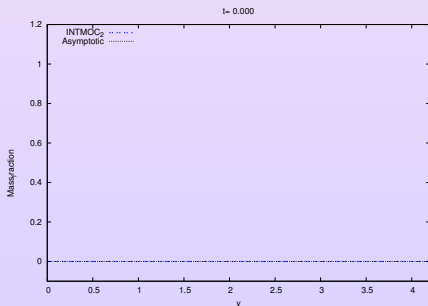
▶▶▶ Fin

LOSS OF FLOW

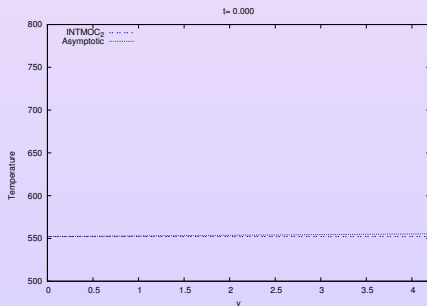


Initial data: $v_e(t) = \tilde{v}$ and $\Phi(t, y) = \Phi_0$.

Mass fraction



Temperature



◀ Description

▶ $[t_0 - t_1]$

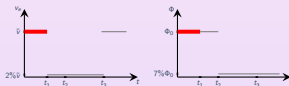
▶ $[t_1 - t_2]$

▶▶ $[t_2 - t_3]$

▶▶▶ $t > t_3$

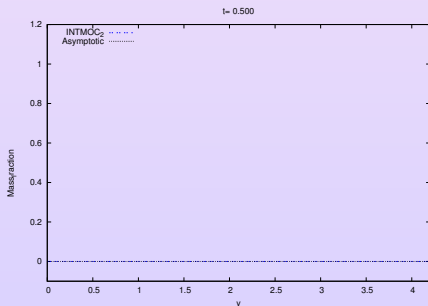
▶▶▶ Fin

LOSS OF FLOW

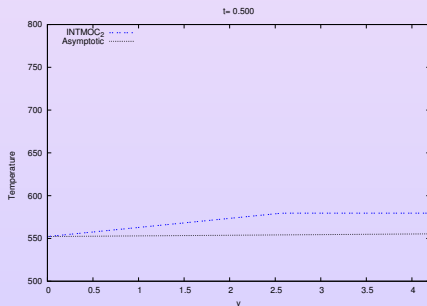


Initial data: $v_e(t) = \tilde{v}$ and $\Phi(t, y) = \Phi_0$.

Mass fraction



Temperature



◀ Description

▶ [t₀ - t₁]

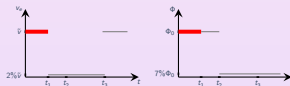
▶ [t₁ - t₂]

▶▶ [t₂ - t₃]

▶▶▶ t > t₃

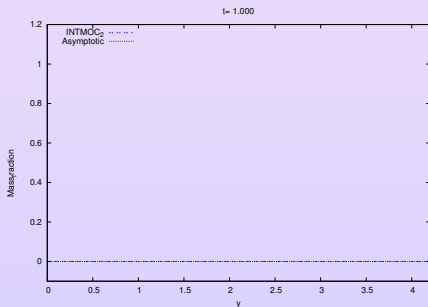
▶▶▶ Fin

LOSS OF FLOW

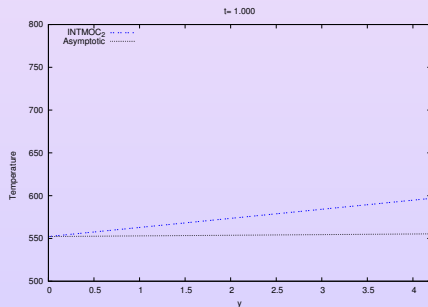


Initial data: $v_e(t) = \tilde{v}$ and $\Phi(t, y) = \Phi_0$.

Mass fraction



Temperature



◀ Description

▶ $[t_0 - t_1]$

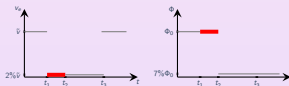
▶ $[t_1 - t_2]$

▶▶ $[t_2 - t_3]$

▶▶▶ $t > t_3$

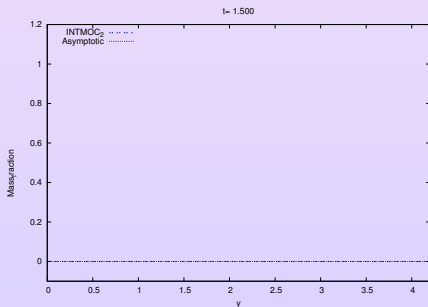
▶▶▶ Fin

LOSS OF FLOW

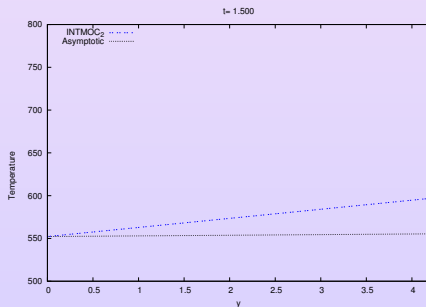


At t_1 most of the pumps stop $\implies v_e(t) \searrow$.

Mass fraction



Temperature



◀ Description

▶ $[t_0 - t_1]$

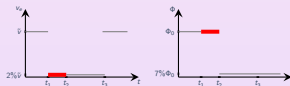
▶ $[t_1 - t_2]$

▶▶ $[t_2 - t_3]$

▶▶▶ $t > t_3$

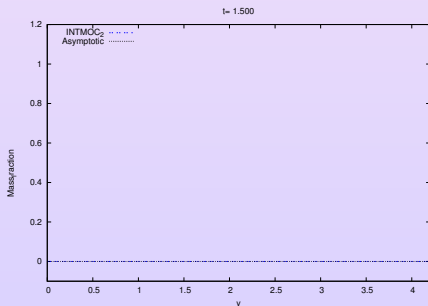
▶▶▶ Fin

LOSS OF FLOW

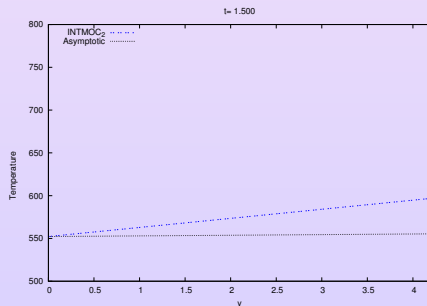


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Mass fraction



Temperature



◀ Description

▶ $[t_0 - t_1]$

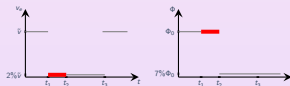
▶ $[t_1 - t_2]$

▶▶ $[t_2 - t_3]$

▶▶▶ $t > t_3$

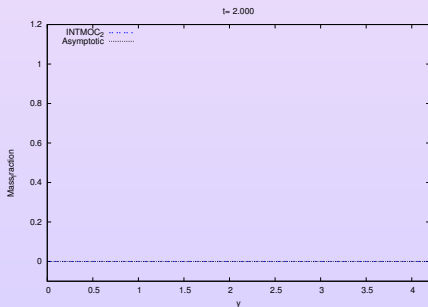
▶▶▶ Fin

LOSS OF FLOW

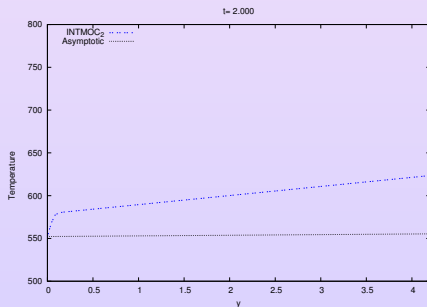


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Temperature



◀ Description

▶ $[t_0 - t_1]$

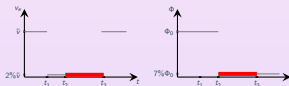
▶ $[t_1 - t_2]$

▶▶ $[t_2 - t_3]$

▶▶▶ $t > t_3$

▶▶▶ Fin

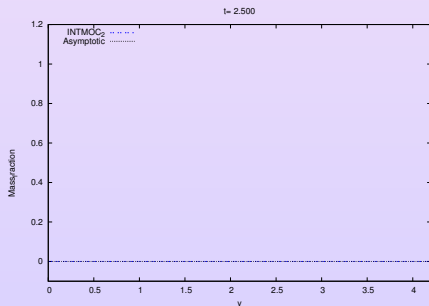
LOSS OF FLOW



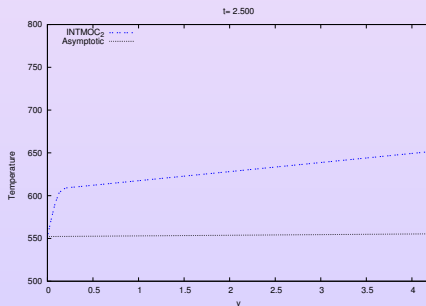
At t_2 the security system drops control rods into the core

$$\Rightarrow \Phi(t) \searrow 7\% \Phi_0.$$

Mass fraction



Temperature



◀ Description

▶ $[t_0 - t_1]$

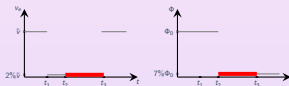
▶ $[t_1 - t_2]$

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▶▶▶ $t > t_3$

▶▶▶ Fin

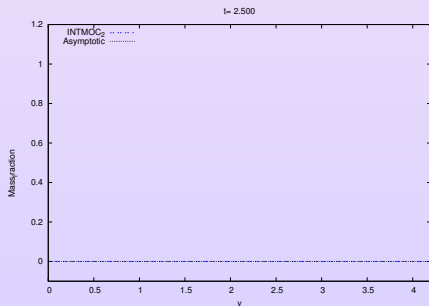
LOSS OF FLOW



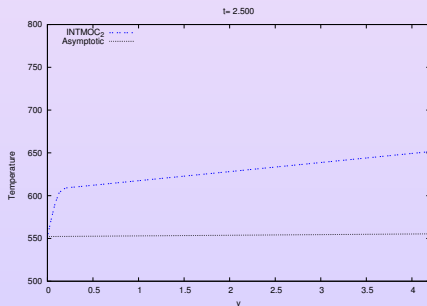
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Temperature



◀ Description

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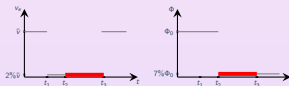
▶ $[t_1 - t_2]$

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▶▶▶ Fin

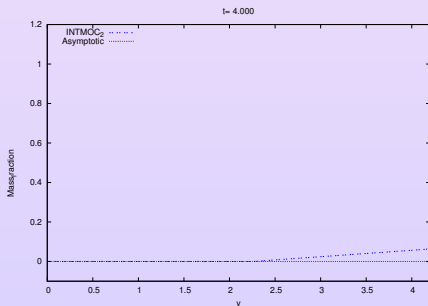
LOSS OF FLOW



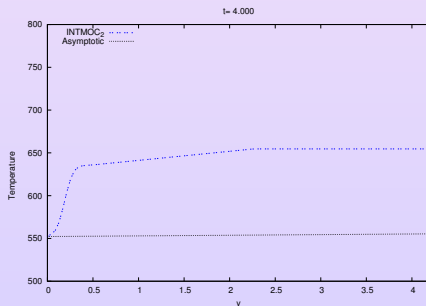
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Temperature



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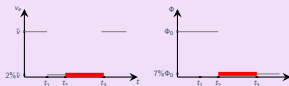
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▶▶▶ Fin

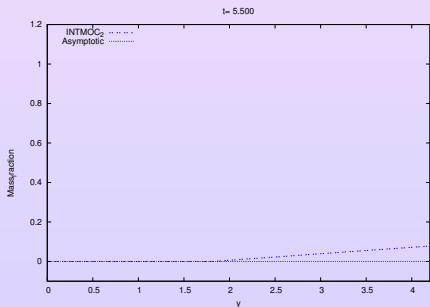
LOSS OF FLOW



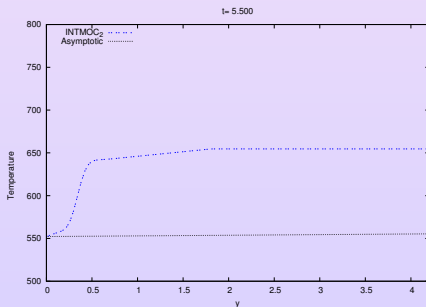
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Mass fraction



Temperature



◀ Description

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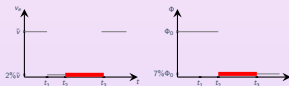
▶ $[t_1 - t_2]$

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▶▶▶ Fin

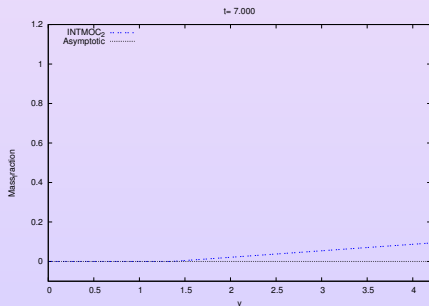
LOSS OF FLOW



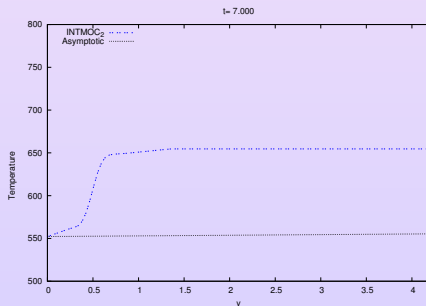
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Mass fraction



Temperature



◀ Description

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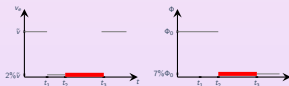
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▶▶▶ Fin

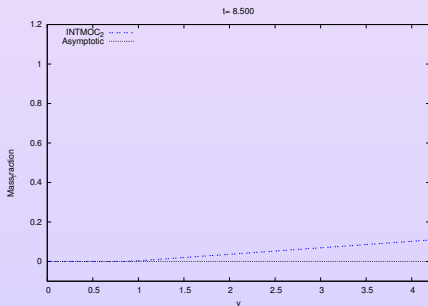
LOSS OF FLOW



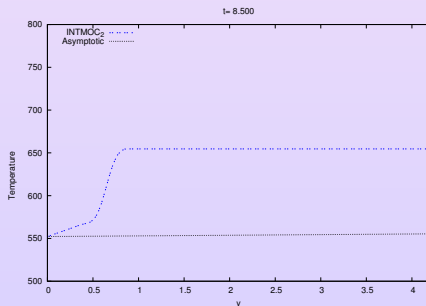
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Temperature



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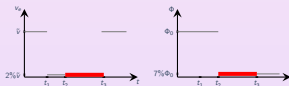
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▶▶▶ Fin

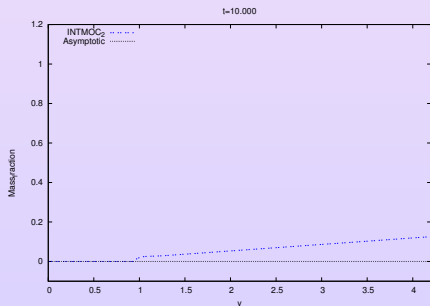
LOSS OF FLOW



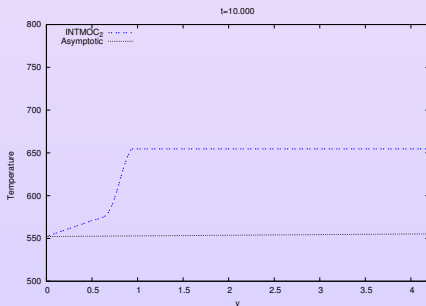
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Mass fraction



Temperature



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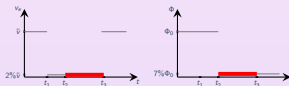
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▶▶▶ Fin

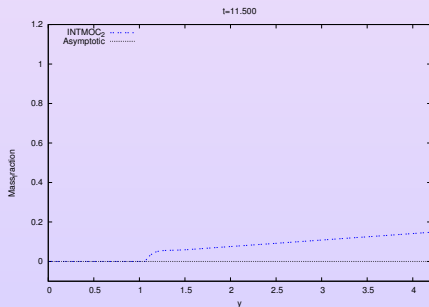
LOSS OF FLOW



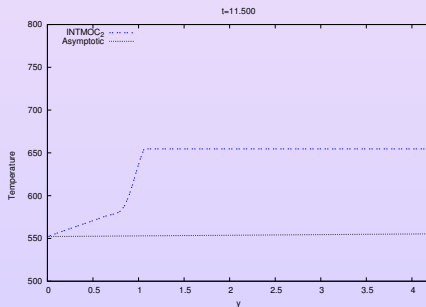
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Temperature



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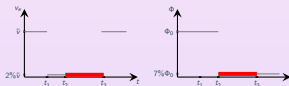
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▶▶▶ Fin

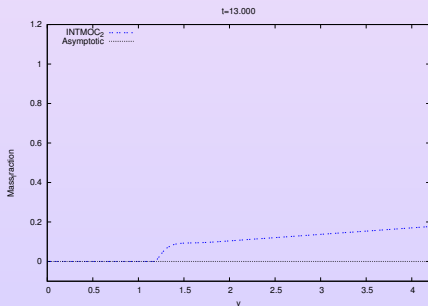
LOSS OF FLOW



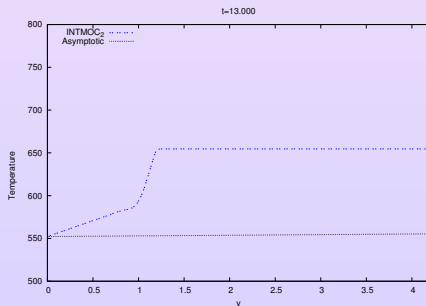
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Temperature



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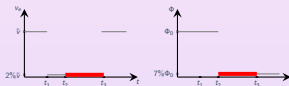
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▶▶▶ Fin

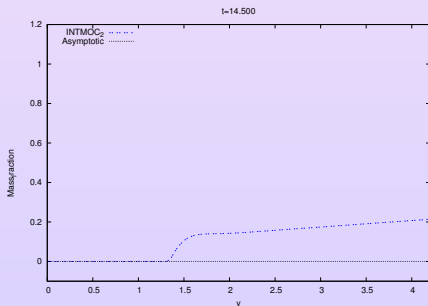
LOSS OF FLOW



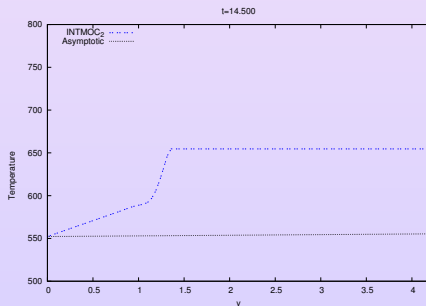
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Mass fraction



Temperature



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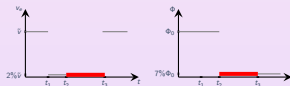
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▶▶▶ Fin

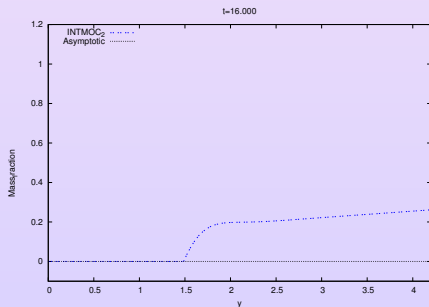
LOSS OF FLOW



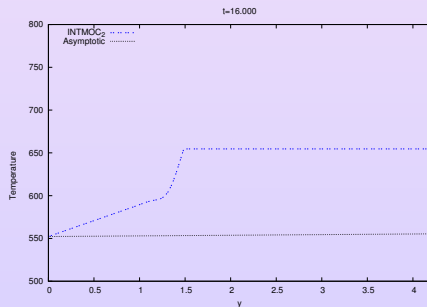
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Temperature



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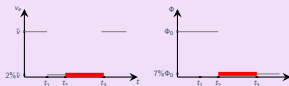
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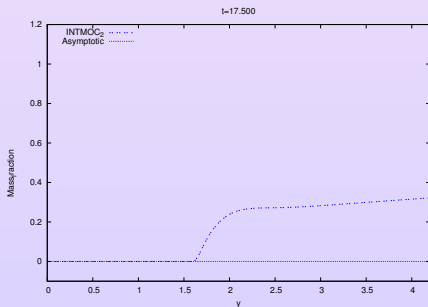
LOSS OF FLOW



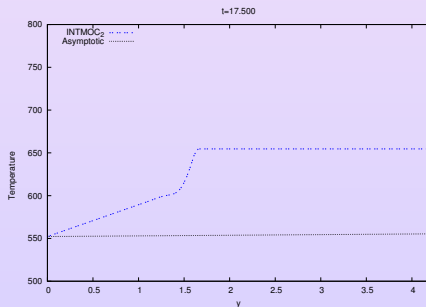
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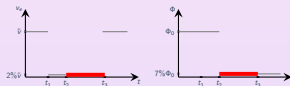
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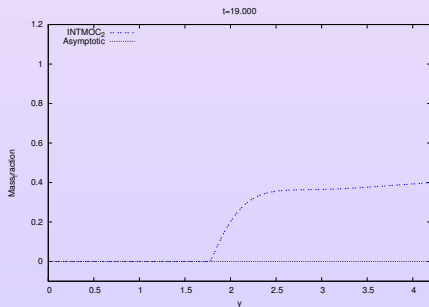
LOSS OF FLOW



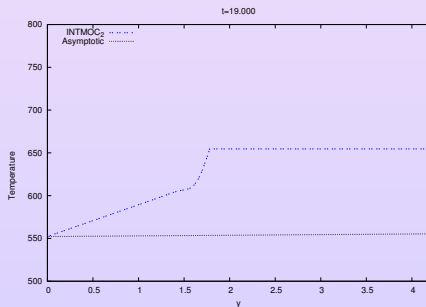
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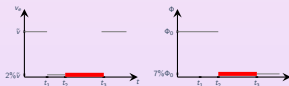
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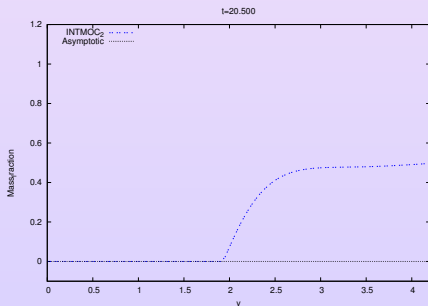
LOSS OF FLOW



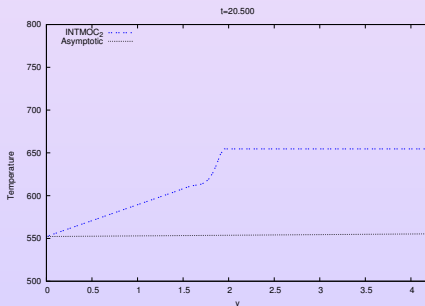
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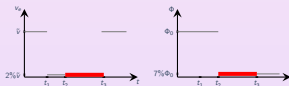
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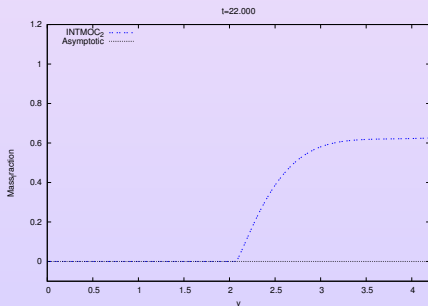
LOSS OF FLOW



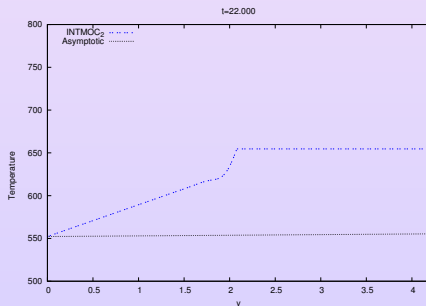
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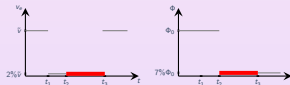
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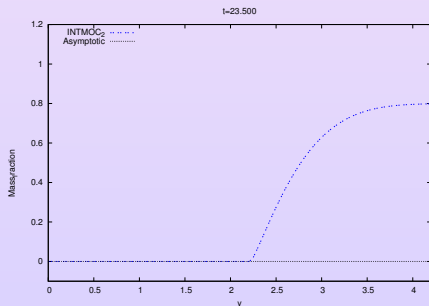
LOSS OF FLOW



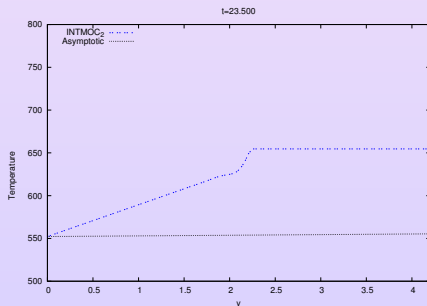
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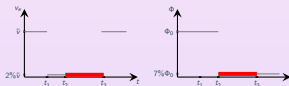
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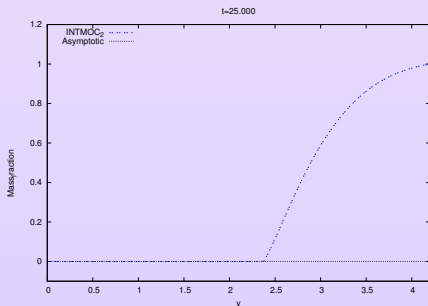
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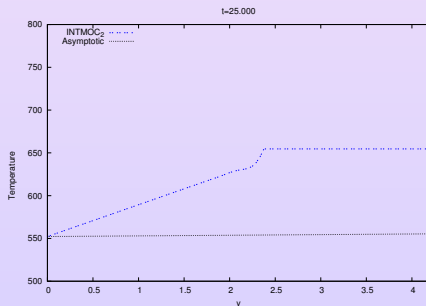
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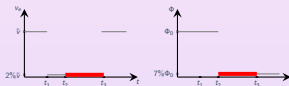
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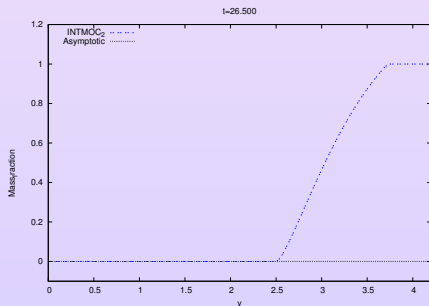
LOSS OF FLOW



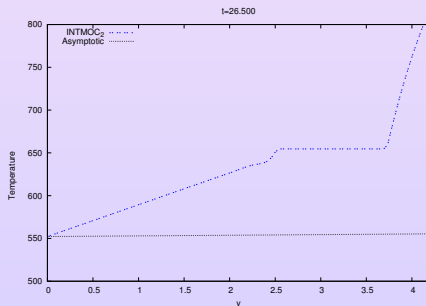
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Temperature



◀ Description

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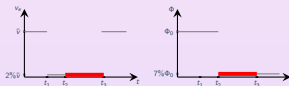
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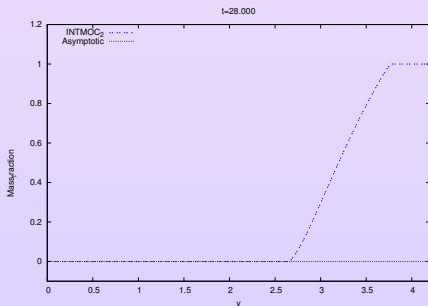
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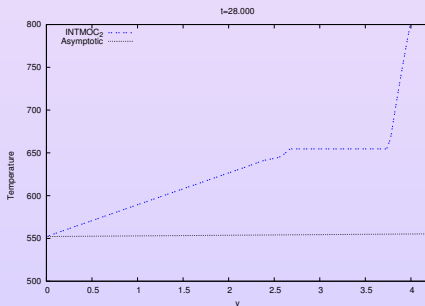
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◀ Description

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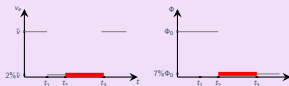
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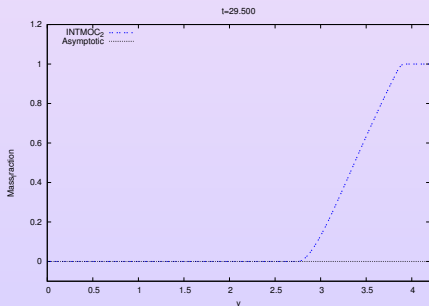
LOSS OF FLOW



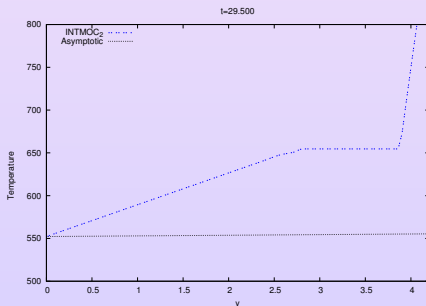
At t_2 the security system drops control rods into the core

$$\Rightarrow \Phi(t) \searrow 7\% \Phi_0.$$

Mass fraction



Temperature



◀ Description

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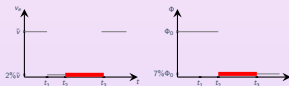
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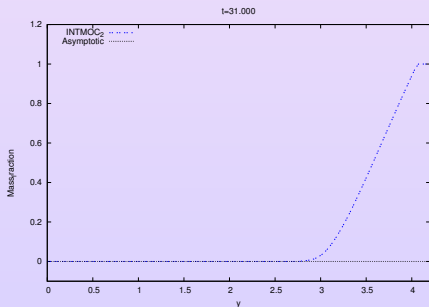
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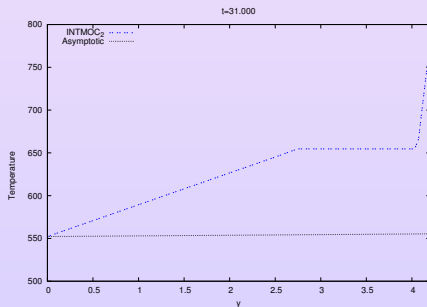
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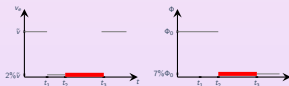
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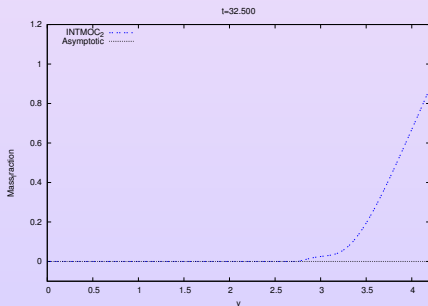
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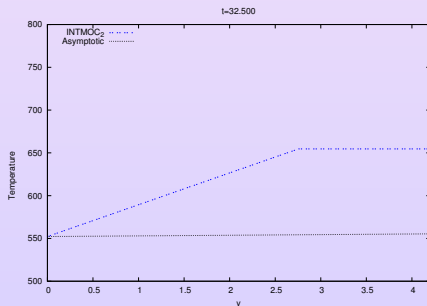
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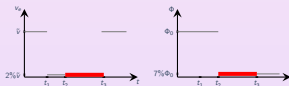
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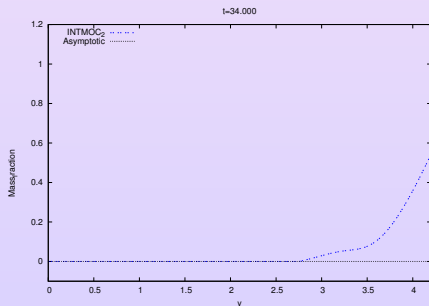
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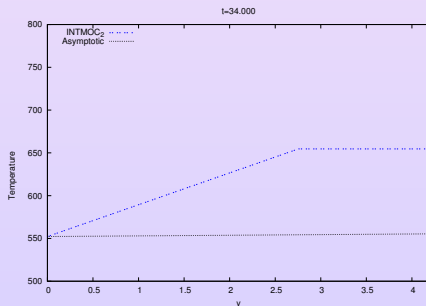
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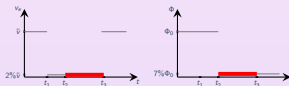
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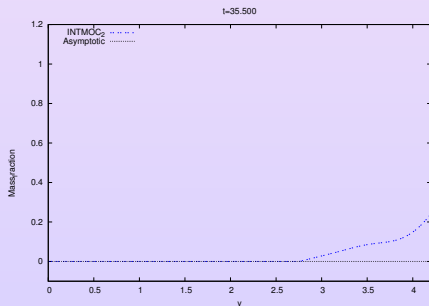
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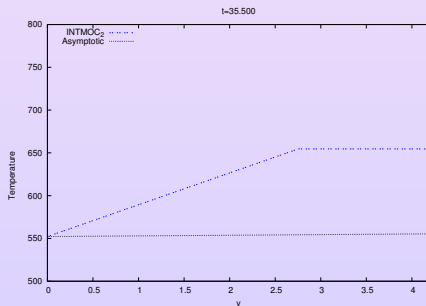
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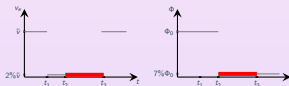
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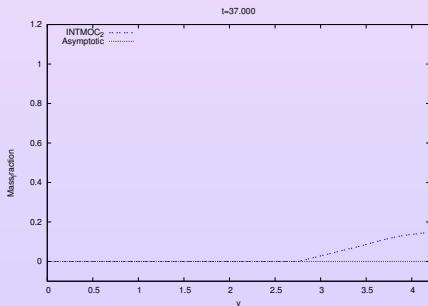
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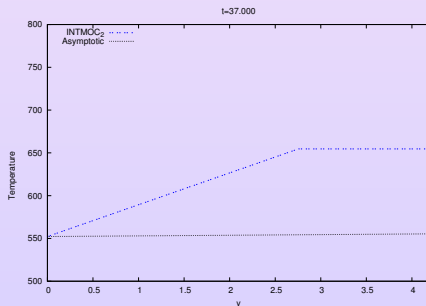
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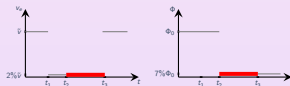
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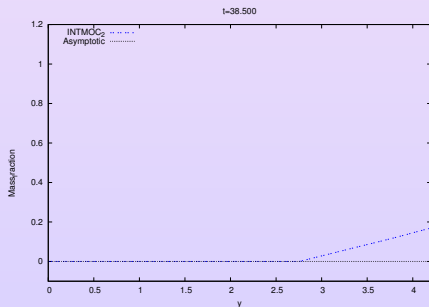
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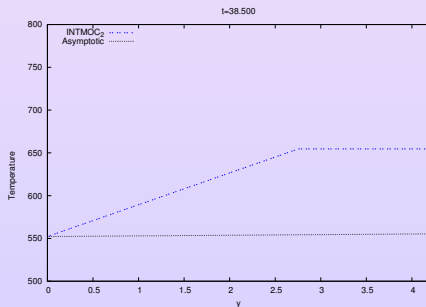
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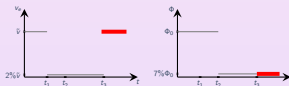
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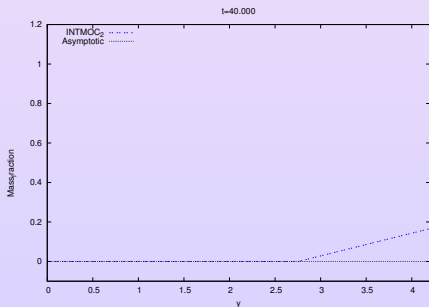
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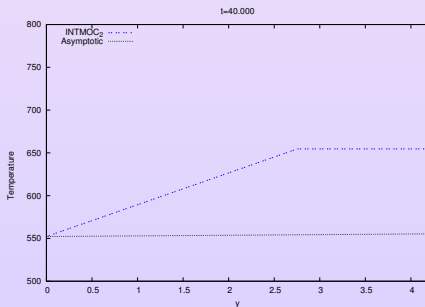


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Mass fraction



Temperature



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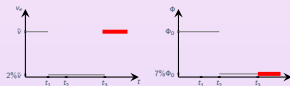
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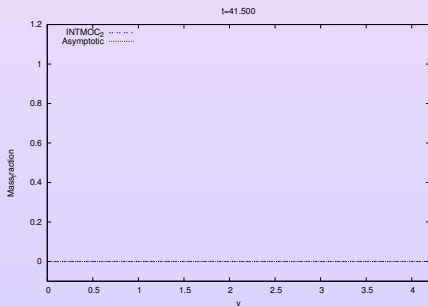
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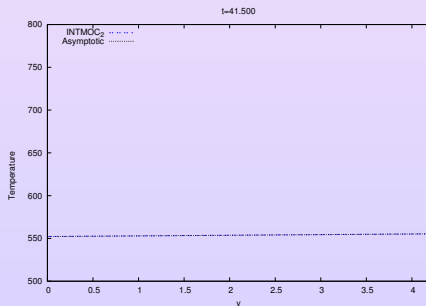


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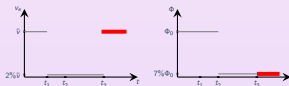
Mass fraction



Temperature

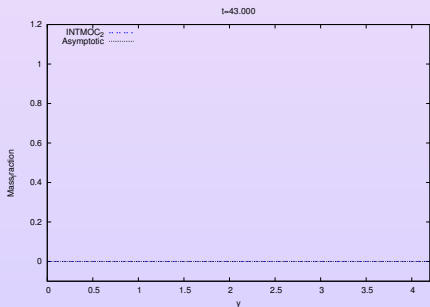

[◀ Description](#)
[▶ \[t₀ - t₁\]](#)
[▶ \[t₁ - t₂\]](#)
[▶▶ \[t₂ - t₃\]](#)
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LOSS OF FLOW

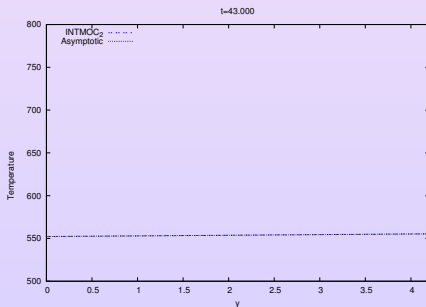


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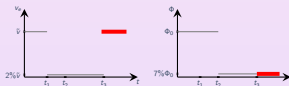
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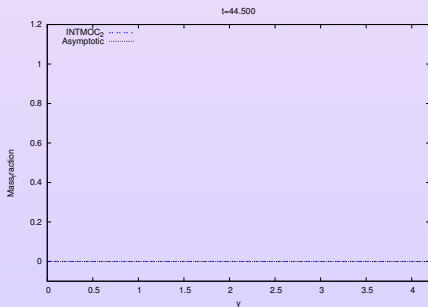
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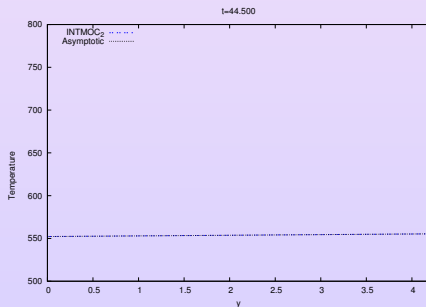


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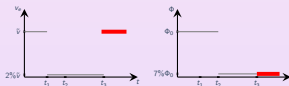
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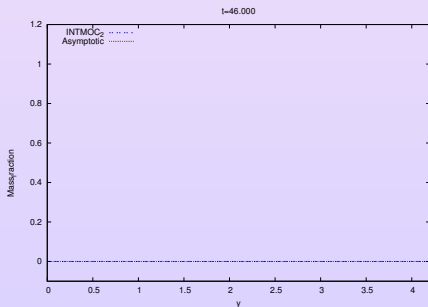
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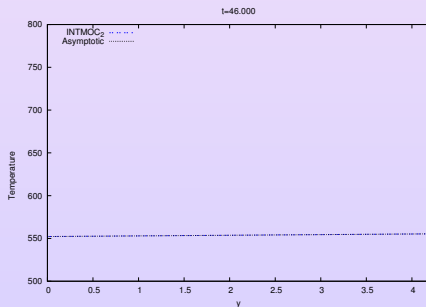


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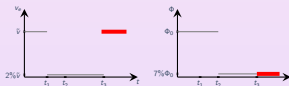
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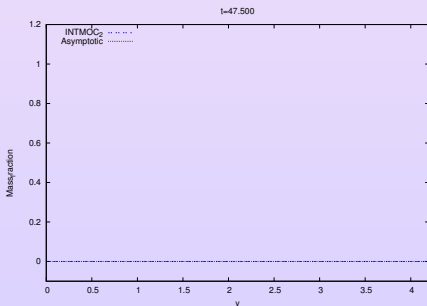

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LOSS OF FLOW

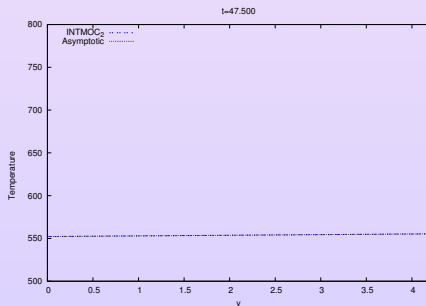


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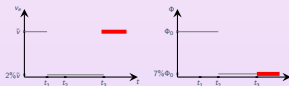
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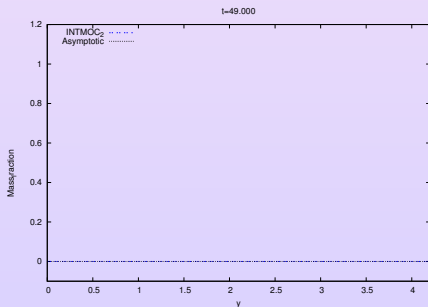
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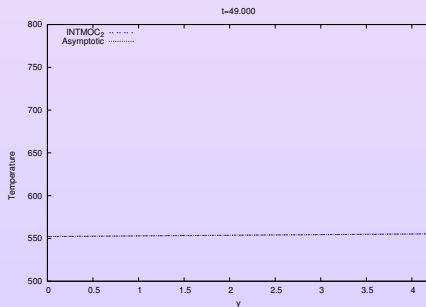


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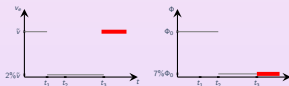
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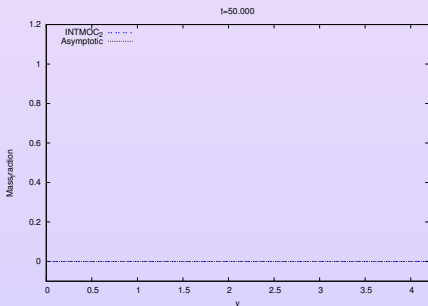
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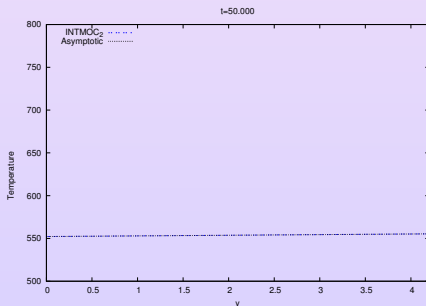


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Section 5

2D-MODEL

- Governing equations
- Analytical solutions
- Numerical scheme
- Numerical tests

E. Najir (Stage Master 1)

Section 5

2D-MODEL

- **Governing equations**
- Analytical solutions
- Numerical scheme
- Numerical tests

GOVERNING EQUATIONS

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \frac{\beta(h)}{\rho_0} \Phi \\ \partial_t h + \mathbf{u} \cdot \nabla h = \frac{\Phi}{\varrho(h)} \\ \varrho(h) \left(\partial_t(\mathbf{u}) + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) + \nabla \bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h) \mathbf{g} \end{cases}$$

where

$$\sigma(\mathbf{u}) = \mu \begin{pmatrix} 2\partial_x u & \partial_y u + \partial_x v \\ \partial_y u + \partial_x v & 2\partial_y v \end{pmatrix} + \eta \begin{pmatrix} \partial_x u + \partial_y v & 0 \\ 0 & \partial_x u + \partial_y v \end{pmatrix}$$

- Unknowns
- Given quantities
- Equation Of State

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• Unknowns

- $(t, x, y) \mapsto \mathbf{u} \stackrel{\text{def}}{=} (u, v)$ velocity,
- $(t, x, y) \mapsto h$ enthalpy,
- $(t, x, y) \mapsto \bar{p}$ dynamic pressure;

• Given quantities

• Equation Of State

GOVERNING EQUATIONS

$$\begin{cases} \operatorname{div}(\mathbf{u}) = \frac{\beta(h)}{p_0} \Phi \\ \partial_t h + \mathbf{u} \cdot \nabla h = \frac{\Phi}{\varrho(h)} \\ \varrho(h) \left(\partial_t(\mathbf{u}) + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) + \nabla \bar{p} = \operatorname{div}(\sigma(\mathbf{u})) + \varrho(h) \mathbf{g} \end{cases}$$

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- **Unknowns**

- **Given quantities**

- $(t, x, y) \mapsto \Phi \geq 0$ power density,
- $\mathbf{g} \stackrel{\text{def}}{=} (0, -g)$ gravity,
- $p_0 > 0$ thermodynamic pressure (constant),

- **Equation Of State**

GOVERNING EQUATIONS

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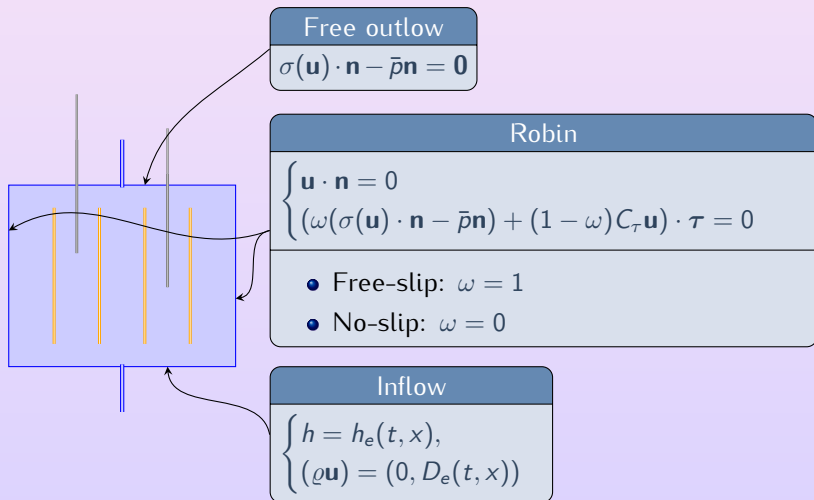
- **Unknowns**

- **Given quantities**

- **Equation Of State**

- $(t, y) \mapsto \mu$ viscosity et $\eta = -\frac{2}{3}\nu$,
- $h \mapsto \varrho$ density (stiffened gas or tabulated),
- $h \mapsto \beta \stackrel{\text{def}}{=} -\frac{p_0}{\varrho^2(h)} \varrho'(h)$ compressibility coefficient.

BOUNDARY CONDITIONS



Section 5

2D-MODEL

- Governing equations
- **Analytical solutions**
- Numerical scheme
- Numerical tests

VERTICAL FLOW (*i.e.* $\mathbf{u} = (0, v)$)

- Let v_e , D_e and Φ constant. With free-slip conditions

$$(u, v, h, \bar{p})(t, x, y) = (0, v_{1D}(t, y), h_{1D}(t, y), \bar{p}_{1D}(t, y))$$

- With no-slip conditions an asymptotic solution with $u^\infty(x, y) = 0$ does not exist.

Section 5

2D-MODEL

- Governing equations
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FREEFEM++ (1)

- Let ξ^n the foot at time t^n of the characteristic issuing from \mathbf{x} at time t^{n+1} , then

$$\partial_t \star + (\mathbf{u} \cdot \nabla) \star \approx \frac{\star^{n+1}(\mathbf{x}) - \star^n(\xi^n)}{\Delta t}, \quad \star = \mathbf{u} \text{ or } h$$

- Weak formulation of a semi-implicit temporal discretization: at time t^{n+1} find $(\mathbf{u}^{n+1}, \bar{p}^{n+1}, h^{n+1}) \in \mathcal{U} \times \mathcal{P} \times \mathcal{H}$ such that ...

FREEFEM++ (2)

- $\forall \mathbf{u}_{\text{test}} \in \mathcal{U}$

$$\begin{aligned} & \frac{1}{\Delta t} \int_{\Omega} \varrho(h^n)(\mathbf{u}^{n+1} - \mathbf{u}^n(\xi^n)) \cdot \mathbf{u}_{\text{test}} \\ & + \int_{\Omega} \mu(h^n)((\nabla \mathbf{u}^{n+1} + (\nabla \mathbf{u}^{n+1})^T) : \nabla(\mathbf{u}_{\text{test}})) \\ & + \int_{\Omega} \eta(h^n) \operatorname{div}(\mathbf{u}^{n+1}) \operatorname{div}(\mathbf{u}_{\text{test}}) - \int_{\Omega} \bar{p}^{n+1} \operatorname{div}(\mathbf{u}_{\text{test}}) = \int_{\Omega} \varrho(h^n) \mathbf{g} \cdot \mathbf{u}_{\text{test}} \end{aligned}$$

- $\forall p_{\text{test}} \in \mathcal{P}$

$$\int_{\Omega} \operatorname{div}(\mathbf{u}^{n+1}) p_{\text{test}} - \varepsilon \int_{\Omega} \bar{p}^{n+1} p_{\text{test}} = \int_{\Omega} \frac{\beta(h^n) \Phi(t^{n+1})}{p_0} p_{\text{test}}$$

- $\forall h_{\text{test}} \in \mathcal{H}$

$$\frac{1}{\Delta t} \int_{\Omega} (h^{n+1} - h^n(\xi^n)) h_{\text{test}} = \int_{\Omega} \frac{\Phi(t^{n+1})}{\varrho(h^n)} h_{\text{test}}$$

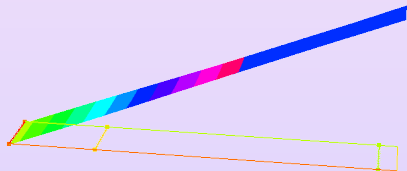
Section 5

2D-MODEL

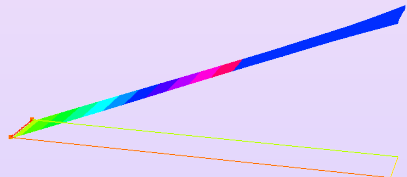
- Governing equations
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SG vs TAB

Enthalpy



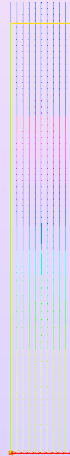
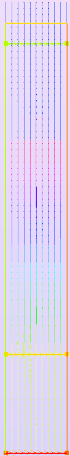
$$h \in [1.19 \times 10^6; 3.16367 \times 10^6]$$



$$h \in [1.19 \times 10^6; 3.08409 \times 10^6]$$

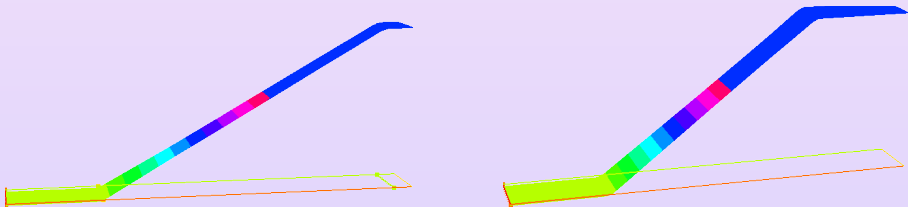
SG vs TAB

u



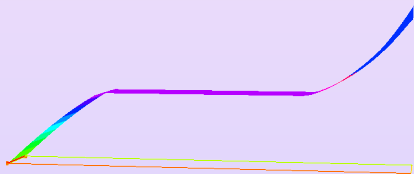
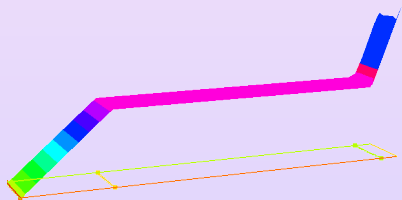
SG vs TAB

Mass fraction



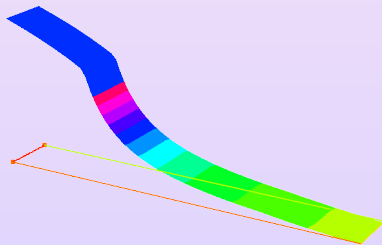
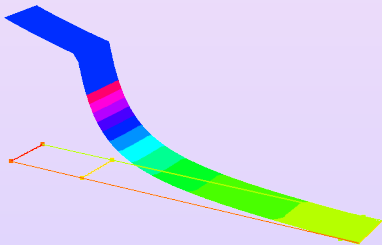
SG vs TAB

Temperature



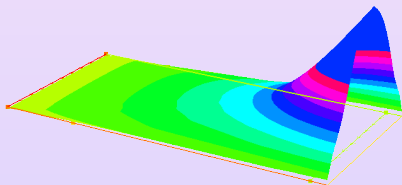
SG vs TAB

Density

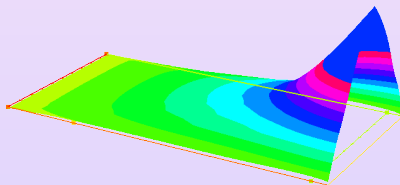


FREE-SLIP VS NO-SLIP

Enthalpy



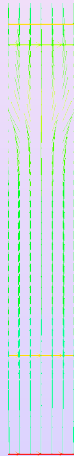
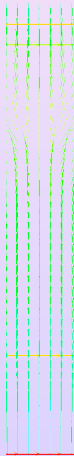
$$h \in [1.19 \times 10^6; 2.73224 \times 10^6]$$



$$h \in [1.19 \times 10^6; 2.54928 \times 10^6]$$

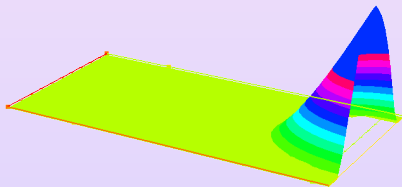
FREE-SLIP VS NO-SLIP

u

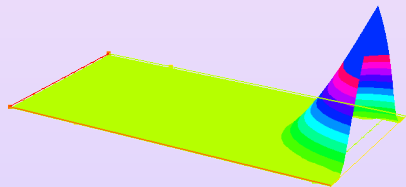


FREE-SLIP VS NO-SLIP

Mass fraction



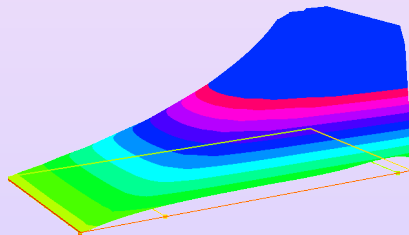
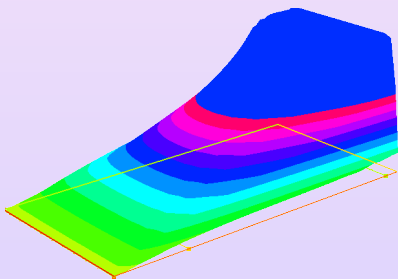
[0; 0.803611]



[0; 0.670913]

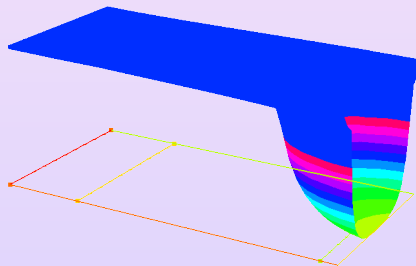
FREE-SLIP VS NO-SLIP

Temperature

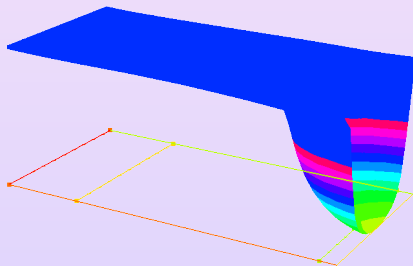


FREE-SLIP VS NO-SLIP

Density



$$\varrho \in [64.6185; 749.97]$$



$$\varrho \in [75.8681; 749.97]$$

Section 6

CONCLUSION & PERSPECTIVES

SUMMARY & PERSPECTIVES

- Model

- ✓ mono/diphasic low Mach model with phase transition (stiffened gas & tabulated EoS),

- Theoretical study

- ✓ unsteady exact solutions on some cases (1D-SG-diphasic),
steady exact solutions on all 1D cases (also with tabulated EOS),

- Numerical Method

- ✓ preliminary results: 1D & 2D

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 - ✗ Heat diffusion (see E. Najir),
 $t \mapsto p_0(t)$,
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- Numerical Method

- ✓ preliminary results: 1D & 2D
- ✗ quantitative simulations: comparison with compressible model and experimental data,
 2D (C. Calgaro, E. Creusé, T. Goudon).