Superbesse, 23/05/2012

A diphasic low Mach model with phase change

The L(ow) M(ach) N(uclear) C(ore) Model

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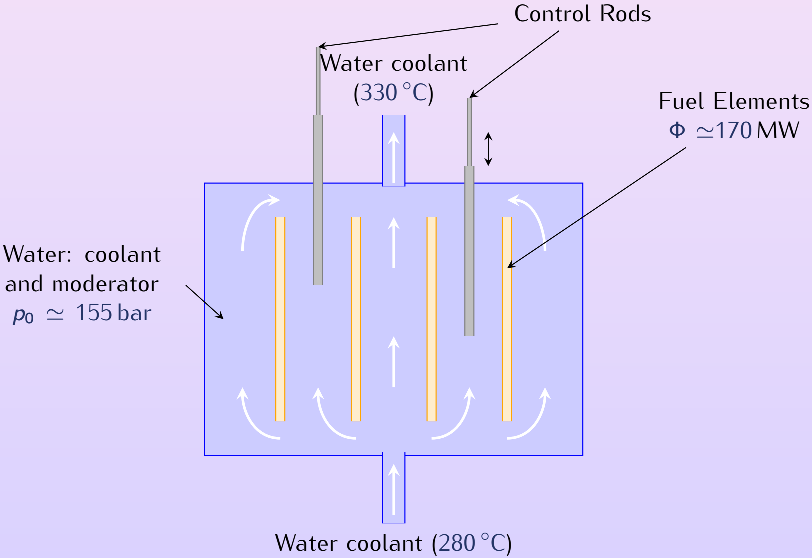
Outline

1. Introduction
2. The monophasic LMNC model with Stiffened Gas EOS
3. The diphasic LMNC model with Stiffened Gas EOS & Phase Transition
4. Numerical schemes
5. Numerical examples
6. Conclusion & Perspectives

Outline

Scheme of the core of nuclear reactors whose coolant is water



GOVERNING EQUATIONS: THE 1D-LMNC MODEL[1]

$$
\begin{cases}
\partial_y v = \dfrac{\beta}{p_0}\Phi, \\
\partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho}, \\
\partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\varrho g.
\end{cases}
$$

- Unknowns
- Given quantities
- Boundary Conditions
- Initial Conditions
- Equation Of State

[1] S. Dellacherie, *On A Low Mach Nuclear Core Model*, ESAIM: Proc., 35 (2012)

Governing equations: the 1D-LMNC model[1]

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi, \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho}, \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\varrho g. \end{cases}$$

- Unknowns
  - $(t, y) \mapsto v$ velocity,
  - $(t, y) \mapsto h$ enthalpy,
  - $(t, y) \mapsto \bar{p}$ dynamical pressure;
- Given quantities
- Boundary Conditions
- Initial Conditions
- Equation Of State

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Governing equations: the 1D-LMNC model[1]

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi, \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho}, \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\varrho g. \end{cases}$$

- Unknowns
- Given quantities
  - $p_0 > 0$ thermodynamical pressure (constant),
  - $(t, y) \mapsto \Phi \geq 0$ power density,
  - $(t, y) \mapsto \mu$ viscosity,
  - g gravity.
- Boundary Conditions
- Initial Conditions
- Equation Of State

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Governing equations: the 1D-LMNC model[1]

$$
\begin{cases}
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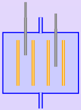
- Unknowns
- Given quantities
- Boundary Conditions



  - $\bar{p}(t, y = L) = p_0$ dynamical pressure on the top
  - $(\varrho v)(t, y = 0) = D_e(t)$ entrance flow rate
  - $h(t, y = 0) = h_e(t)$ entrance enthalpy
- Initial Conditions
- Equation Of State

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Governing equations: the 1D-LMNC model[1]

$$
\begin{cases}
\partial_y v = \dfrac{\beta}{p_0}\Phi, \\
\partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho}, \\
\partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\varrho g.
\end{cases}
$$

- Unknowns
- Given quantities
- Boundary Conditions
- Initial Conditions
  - $h(t=0,y) = h_0(y)$,
  - $v(t=0,y) = v_0(y) = v_e(0) + \int_0^y \beta(h_0(z))\Phi(0,z)\,\mathrm{d}z/p_0$,
  - $\bar{p}(t=0,y) = p_0$.
- Equation Of State

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Governing equations: the 1D-LMNC model[1]

$$\begin{cases} \partial_y v = \dfrac{\beta}{p_0}\Phi, \\ \partial_t h + v\partial_y h = \dfrac{\Phi}{\varrho}, \\ \partial_t(\varrho v) + \partial_y(\varrho v^2 + \bar{p}) - \partial_y(\mu \partial_y v) = -\varrho g. \end{cases}$$

- Unknowns
- Given quantities
- Boundary Conditions
- Initial Conditions
- Equation Of State
  - $(h, p = p_0) \mapsto \varrho$
  - $\beta \overset{\text{def}}{=} -\dfrac{p_0}{\varrho^2} \left.\dfrac{\partial \varrho}{\partial h}\right|_p$ compressibility coefficient.

[1] S. Dellacherie, *On A Low Mach Nuclear Core Model*, ESAIM: Proc., 35 (2012)

Outline

Monophasic Stiffened Gas EOS

$$\varrho(h, p_0) = \frac{\gamma}{\gamma - 1} \frac{p_0 + \pi}{h - q}$$

where

- $\gamma > 1$ adiabatic coefficient,
- π reference pressure,
- q binding energy.

$$\Downarrow$$

$$\beta = \frac{\gamma - 1}{\gamma} \frac{p_0}{p_0 + \pi} \qquad \text{constant}$$

$$\Downarrow$$

$$\varrho(h, p_0) = \frac{p_0/\beta}{h - q}$$

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Monophasic Stiffened Gas EOS

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$$\beta = \frac{\gamma - 1}{\gamma} \frac{p_0}{p_0 + \pi} \qquad \text{constant}$$

$$\Downarrow$$

$$\boxed{\varrho(h, p_0) = \frac{p_0/\beta}{h - q}}$$

Exact solutions

1. Velocity
2. Enthalpy
3. Dynamic pressure

Exact solutions

1. **Velocity**
   Direct integration of $\partial_y v = \frac{\beta}{p_0}\Phi$.

$$v(t,y) = v_e(t) + \frac{\beta}{p_0}\Psi(t,y), \qquad \Psi(t,y) \stackrel{\text{def}}{=} \int_0^y \Phi(t,z)\,\mathrm{d}z$$

2. **Enthalpy**
3. **Dynamic pressure**

Exact solutions

1. Velocity
2. Enthalpy

   Method of characteristics on $\partial_t h + v \partial_y h = \frac{\beta\Phi}{p_0}(h-q)$.

   Example: if Φ and v_e are constant, then

$$h(t,y) = \begin{cases} q + (h_0(\xi(t,y)) - q)\, e^{\frac{\beta\Phi}{p_0} t} & \text{if } \xi(t,y) \geq 0, \\ h_e(t^*(t,y)) + \frac{\Phi}{D_e(t^*(t,y))} y & \text{if } \xi(t,y) < 0. \end{cases}$$

   where

$$\xi(t,y) = \left(y + \frac{p_0}{\beta\Phi} v_e \right) e^{-\frac{\beta\Phi}{p_0} t} - \frac{p_0}{\beta\Phi} v_e,$$

$$t^*(t,y) = -\frac{p_0}{\beta\Phi} \ln\left(1 + \frac{\beta\Phi}{p_0 v_e} \xi(t,y) \right), \quad \text{for } \xi(t,y) < 0.$$

3. Dynamic pressure

Exact solutions

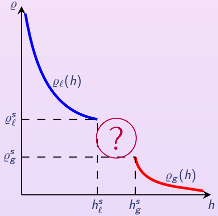
1. Velocity
2. Enthalpy
3. Dynamic pressure

   Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_t(\varrho v) - \partial_y(\varrho v^2) - \varrho g$.
   Example: if Φ and v_e are constant, then

$$
\begin{aligned}
\bar{p}(t,y) = p_0 &+ \frac{\beta\Phi}{p_0}(\mu(y) - \mu(L)) \\
&+ \frac{p_0\left(g + \frac{\beta\Phi}{p_0} v_e\right)}{\beta} \int_y^L \frac{1}{h(t,z) - q} \, \mathrm{d}z \\
&+ \beta\Phi^2 \int_y^L \frac{z}{h(t,z) - q} \, \mathrm{d}z
\end{aligned}
$$

Outline

Diphasic Stiffened Gas EOS with Phase Transition



- Liquid $\kappa = \ell$ and vapor $\kappa = g$ are characterized by their thermodynamical properties

$$(h, p_0) \mapsto \varrho_\kappa$$

- The two-phase mixture is constructed according to the second principle of thermodynamics: when phases coexist, they have the same pressure p_0, the same temperature and their chemical potentials are equal.

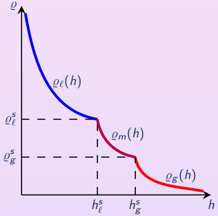
$$g_\ell(p_0, T) = g_g(p_0, T) \qquad \Longrightarrow \qquad T = T^s(p_0).$$

- We define values at saturation:

$$h_\kappa^s \stackrel{\text{def}}{=} h_\kappa(p_0, T^s(p_0)), \qquad \varrho_\kappa^s \stackrel{\text{def}}{=} \varrho_\kappa(p_0, T^s(p_0)) = \varrho_\kappa(h_\kappa^s, p_0).$$

Diphasic Stiffened Gas EOS with Phase Transition

$$\varrho(h, p_0) = \frac{p_0/\beta(h, p_0)}{h - q(h, p_0)}$$



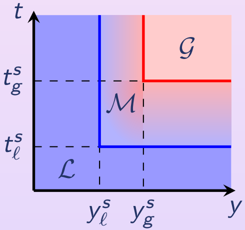
where

$$\beta(h, p_0) = \begin{cases} \beta_\ell, & \text{if } h \le h_\ell^s(p_0), \\ \beta_m \stackrel{\text{def}}{=} -p_0 \dfrac{\varrho_g^s - \varrho_\ell^s}{\varrho_g^s \varrho_\ell^s (h_g^s - h_\ell^s)} & \text{if } h_\ell^s(p_0) < h < h_g^s(p_0), \\ \beta_g, & \text{if } h \ge h_g^s(p_0), \end{cases}$$

$$q(h, p_0) = \begin{cases} q_\ell, & \text{if } h \le h_\ell^s(p_0), \\ q_m \stackrel{\text{def}}{=} \dfrac{\varrho_g^s h_g^s - \varrho_\ell^s h_\ell^s}{\varrho_g^s - \varrho_\ell^s} & \text{if } h_\ell^s(p_0) < h < h_g^s(p_0), \\ q_g, & \text{if } h \ge h_g^s(p_0), \end{cases}$$

Exact solutions

Let Φ, v_e, h_e and h_0 be constant and suppose liquid phase in entrance.

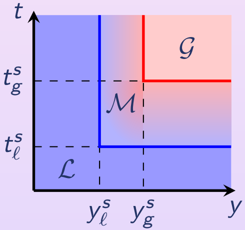


$$y_\ell^s = \frac{D_e}{\Phi}(h_\ell^s - h_e)$$
$$y_g^s = \frac{D_e}{\Phi}(h_g^s - h_e)$$
$$t_\ell^s = \frac{p_0}{\beta_\ell \Phi} \ln\left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell}\right)$$
$$t_g^s = t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln\left(\frac{h_g^s - q_m}{h_\ell^s - q_m}\right)$$

1. Velocity
2. Enthalpy

Exact solutions

Let Φ, v_e, h_e and h_0 be constant and suppose liquid phase in entrance.



$$
\begin{aligned}
y_\ell^s &= \frac{D_e}{\Phi}(h_\ell^s - h_e)\\
y_g^s &= \frac{D_e}{\Phi}(h_g^s - h_e)\\
t_\ell^s &= \frac{p_0}{\beta_\ell \Phi} \ln\left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell}\right)\\
t_g^s &= t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln\left(\frac{h_g^s - q_m}{h_\ell^s - q_m}\right)
\end{aligned}
$$

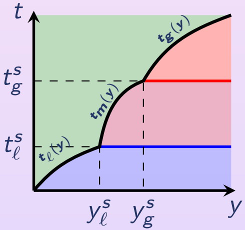
1. **Velocity**: direct integration of $\partial_y v = \frac{\beta(h,p_0)}{p_0}\Phi$.

$$
v(t,y) = \begin{cases}
v_e + \frac{\beta_\ell \Phi}{p_0} y & \text{if } (t,y) \in \mathcal{L},\\
v_e + \frac{\beta_\ell \Phi}{p_0} y_\ell^s + \frac{\beta_m \Phi}{p_0}(y - y_\ell^s) & \text{if } (t,y) \in \mathcal{M},\\
v_e + \frac{\beta_\ell \Phi}{p_0} y_\ell^s + \frac{\beta_m \Phi}{p_0}(y_g^s - y_\ell^s) + \frac{\beta_g \Phi}{p_0}(y - y_g^s) & \text{if } (t,y) \in \mathcal{G},
\end{cases}
$$

2. **Enthalpy**

Exact solutions

Let Φ, v_e, h_e and h_0 be constant and suppose liquid phase in entrance.



$$
\begin{aligned}
y_\ell^s &= \frac{D_e}{\Phi}(h_\ell^s - h_e)\\
y_g^s &= \frac{D_e}{\Phi}(h_g^s - h_e)\\
t_\ell^s &= \frac{p_0}{\beta_\ell \Phi} \ln\left(\frac{h_\ell^s - q_\ell}{h_0 - q_\ell}\right)\\
t_g^s &= t_\ell^s + \frac{p_0}{\beta_m \Phi} \ln\left(\frac{h_g^s - q_m}{h_\ell^s - q_m}\right)
\end{aligned}
$$

1. Velocity
2. Enthalpy: method of characteristics on $\partial_t h + v \partial_y h = \frac{\beta(h,p_0)\Phi}{p_0}(h - q(h,p_0))$.

$$
h(t,y) = \begin{cases}
q_\ell + (h_0 - q_\ell) e^{\frac{\beta_\ell \Phi}{p_0} t} & \text{if } (t,y) \in \mathcal{L} \text{ and } t < t_\ell(y),\\
q_m + (h_\ell^s - q_m) e^{\frac{\beta_m \Phi}{p_0}(t - t_\ell^s)} & \text{if } (t,y) \in \mathcal{M} \text{ and } t < t_m(y),\\
q_g + (h_g^s - q_g) e^{\frac{\beta_g \Phi}{p_0}(t - t_g^s)} & \text{if } (t,y) \in \mathcal{G} \text{ and } t < t_g(y),\\
h_e + \frac{\Phi}{D_e} y & \text{otherwise.}
\end{cases}
$$

Asymptotic solutions

$$(h_e^\infty, D_e^\infty > 0, \Phi^\infty(y)) \stackrel{\text{def}}{=} \lim_{t\to+\infty} (h_e(t), D_e(t), \Phi(t,y))$$

1. **Enthalpy**
   Using $\partial_y(\varrho^\infty v^\infty) = 0$ we have $\partial_y h^\infty = \frac{\Phi^\infty}{D_e^\infty}$.

$$h^\infty(y) = h_e^\infty + \frac{\Psi(y)}{D_e^\infty}, \qquad \Psi(y) \stackrel{\text{def}}{=} \int_0^y \Phi^\infty(z)\, dz$$

2. **Velocity**

$$v^\infty(y) = \frac{D_e^\infty}{\varrho(h^\infty(y))}$$

3. **Dynamic pressure**
   Direct integration of $\partial_y \bar{p} = \partial_y(\mu \partial_y v) - \partial_y(\varrho v^2) - \varrho g$.

Outline

MOC SCHEME

1. Enthalpy
2. Velocity

MOC SCHEME

1. Enthalpy

$$\partial_t h(t^{n+1}, y_i) + v(t^{n+1}, y_i)\partial_y h(t^{n+1}, y_i) = \frac{\beta(h(t^{n+1}, y_i))\Phi(t^{n+1}, y_i)}{p_0}(h(t^{n+1}, y_i) - q(h(t^{n+1}, y_i)))$$

$$\rightsquigarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau) = \frac{\beta(\tilde{h}_i^{n+1}(\tau))\Phi(\tau, \chi(\tau; t^{n+1}, y_i))}{p_0}\left(\tilde{h}_i^{n+1}(\tau) - q(\tilde{h}_i^{n+1}(\tau))\right)$$

where $\tau \mapsto \tilde{h}_i^{n+1}(\tau) \stackrel{\text{def}}{=} h(\tau, \chi(\tau; t^{n+1}, y_i))$ and χ is the solution of

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}\tau}\chi(\tau; t^{n+1}, y_i) = v\left(\tau, \chi(\tau; t^{n+1}, y_i)\right), & \tau \le t^{n+1}, \\ \chi(t^{n+1}; t^{n+1}, y_i) = y_i. \end{cases}$$

2. Velocity

MOC scheme

1. Enthalpy

$$\partial_t h(t^{n+1},y_i)+v(t^{n+1},y_i)\partial_y h(t^{n+1},y_i) = \frac{\beta(h(t^{n+1},y_i))\Phi(t^{n+1},y_i)}{p_0}(h(t^{n+1},y_i)-q(h(t^{n+1},y_i)))$$

$$\Downarrow$$

$$\int_{t^n}^{t^{n+1}}\frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau)\,\mathrm{d}\tau = \int_{t^n}^{t^{n+1}}\frac{\beta(\tilde{h}_i^{n+1}(\tau))\Phi\left(\tau,\chi(\tau;t^{n+1},y_i)\right)}{p_0}\left(\tilde{h}_i^{n+1}(\tau)-q(\tilde{h}_i^{n+1}(\tau))\right)\,\mathrm{d}\tau$$

where $\tau\mapsto\tilde{h}_i^{n+1}(\tau)\stackrel{\text{def}}{=}h\left(\tau,\chi(\tau;t^{n+1},y_i)\right)$ and χ is the solution of

$$\begin{cases}\frac{\mathrm{d}}{\mathrm{d}\tau}\chi(\tau;t^{n+1},y_i)=v\left(\tau,\chi(\tau;t^{n+1},y_i)\right), & \tau\le t^{n+1},\\ \chi(t^{n+1};t^{n+1},y_i)=y_i.\end{cases}$$

2. Velocity

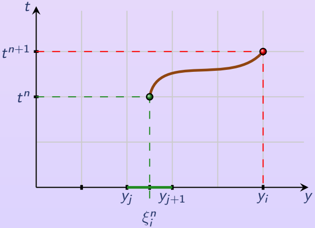
MOC scheme

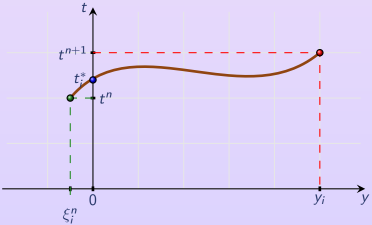
1. **Enthalpy**: let $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.
   - If $\xi_i^n > 0$, let $\hat{h}_i^n \approx \tilde{h}_i^{n+1}(t^n)$ (at order 1 or 2) and then

$$h_i^{n+1} = \hat{h}_i^n + \Delta t \frac{\beta(\hat{h}_i^n)\Phi(t^n, \xi_i^n)}{p_0}(\hat{h}_i^n - q(\hat{h}_i^n))$$

   - If $\xi_i^n \le 0$, let $t_i^* = t^{n+1} - y_i/v_i^n \approx \tau$ such that $\chi(\tau; t^{n+1}, y_i) = 0$ and then

$$h_i^{n+1} = h_e(t_i^*) + (t^{n+1} - t_i^*)\frac{\beta(h_e(t_i^*))\Phi(t^*, 0)}{p_0}(h_e(t_i^*) - q(h_e(t_i^*)))$$





2. **Velocity**

MOC scheme

1. Enthalpy
2. Velocity

$$
\begin{aligned}
v_i^{n+1} &= v_{i-1}^{n+1} + \frac{1}{p_0}\int_{y_{i-1}}^{y_i} \beta(h(t^{n+1},z))\Phi(t^{n+1},z)\,\mathrm{d}z \\
&\approx v_{i-1}^{n+1} + \frac{\Delta y}{p_0}\beta(h_{i-1}^{n+1})\Phi(t^{n+1},y_{i-1}).
\end{aligned}
$$

β is discontinuous at phase change points, so that if $h^s_\kappa \in (h_{i-1}^{n+1}, h_i^{n+1})$, let $y^* = y_{i-1} + \Delta y \frac{h^s_\kappa - h_{i-1}^{n+1}}{h_i^{n+1} - h_{i-1}^{n+1}}$ and then

$$
\begin{aligned}
&\int_{y_{i-1}}^{y_i} \beta(h(t^{n+1},z))\Phi(t^{n+1},z)\,\mathrm{d}z \\
&\approx (y^* - y_{i-1})\beta(h_{i-1}^{n+1})\Phi(t^{n+1},y_{i-1})\,\mathrm{d}y + (y_i - y^*)\beta(h_i^{n+1})\Phi(t^{n+1},y_i)\,\mathrm{d}y
\end{aligned}
$$

INTMOC scheme

1. Enthalpy

$$\frac{\frac{\mathrm{d}}{\mathrm{d}\tau}\tilde{h}_i^{n+1}(\tau)}{\beta(\tilde{h}_i^{n+1}(\tau))\left(\tilde{h}_i^{n+1}(\tau)-q(\tilde{h}_i^{n+1}(\tau))\right)}=\frac{\Phi\big(\tau,\chi(\tau;t^{n+1},y_i)\big)}{p_0}$$

$$\rightsquigarrow$$

$$\int_{\tilde{h}_i^{n+1}(t^n)}^{\tilde{h}_i^{n+1}(t^{n+1})}\frac{1}{\beta(h)(h-q(h))}\,\mathrm{d}h=\frac{1}{p_0}\int_{t^n}^{t^{n+1}}\Phi\big(\tau,\chi(\tau;t^{n+1},y_i)\big)\,\mathrm{d}\tau$$

so that

$$\tilde{h}_i^{n+1}(t^{n+1})=R^{-1}\left(R(\tilde{h}_i^{n+1}(t^n))+\frac{1}{p_0}\int_{t^n}^{t^{n+1}}\Phi\big(\tau,\chi(\tau;t^{n+1},y_i)\big)d\tau\right)$$

where

$$R(h)\stackrel{\text{def}}{=}\int_0^{\tilde{h}}\frac{1}{\beta(h)(h-q(h))}\,\mathrm{d}h$$

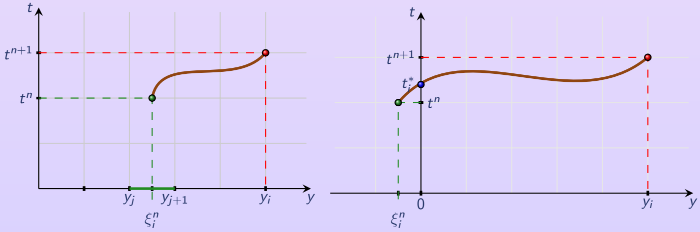
INTMOC scheme

1. **Enthalpy** : let $\xi_i^n \approx \chi(t^n; t^{n+1}, y_i)$.
   - If $\xi_i^n > 0$, let $\hat{h}_i^n \approx \tilde{h}_i^{n+1}(t^n)$ (at order 1 or 2) and then

$$h_i^{n+1} = R^{-1}\left(R(\hat{h}_i^n) + \frac{\Delta t}{p_0}\frac{\Phi(t^n,\xi_i^n) + \Phi(t^{n+1},y_j)}{2} \right)$$

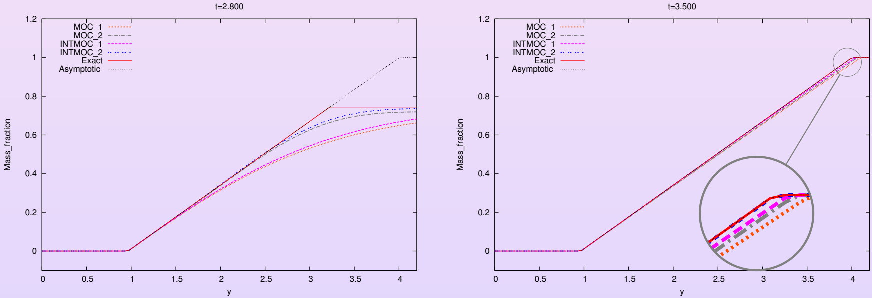
   - If $\xi_i^n \le 0$, let $t_i^* = t^{n+1} - y_i/v_i^n \approx \tau$ such that $\chi(\tau; t^{n+1}, y_i) = 0$ and then

$$h_i^{n+1} = R^{-1}\left(R(h_e(t_i^*)) + \frac{t^{n+1}-t_i^*}{p_0}\frac{\Phi(t_i^*,0) + \Phi(t^{n+1},y_i)}{2} \right)$$



Outline

MOC vs. INTMOC

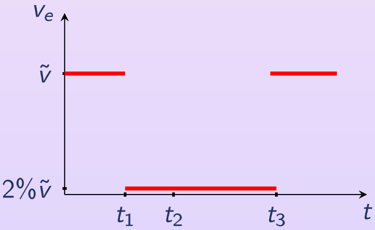


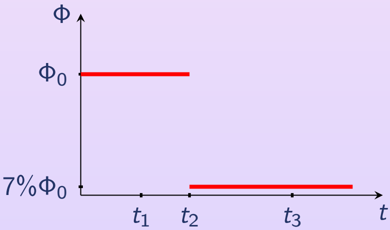
- Initially the domain is filled with liquid phase
- At $t = 1.769\,\mathrm{s}$ mixture appears for $y > y_\ell^s \simeq 0.964\,\mathrm{m}$
- At $t = 2.929\,\mathrm{s}$ pure vapor phase appears for $y > y_g^s \simeq 4.002\,\mathrm{m}$
- The asymptotic state is reached at $t = 2.957\,\mathrm{s}$

Loss of Flow

$$v_e(t) = \begin{cases} \tilde{v} & \text{if } 0 \le t < t_1, \\ 2\%\tilde{v} & \text{if } t_1 \le t < t_3, \\ \tilde{v} & \text{if } t \ge t_3, \end{cases}$$

$$\Phi(t) = \begin{cases} \Phi_0 & \text{if } 0 \le t < t_2, \\ 7\%\Phi_0 & \text{if } t \ge t_2. \end{cases}$$





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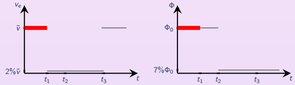
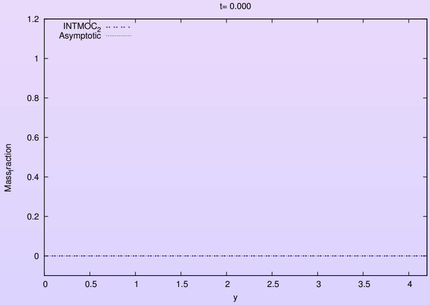
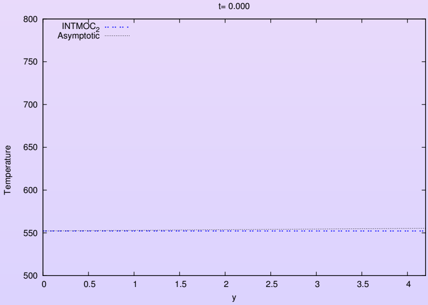


Initial data: $v_e(t) = \tilde v$ and $\Phi(t, y) = \Phi_0$.

Mass fraction

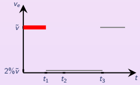


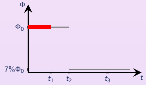
Temperature



Description | $[t_0 - t_1]$ | $[t_1 - t_2]$ | $[t_2 - t_3]$ | $t > t_3$ | Fin

Loss of Flow





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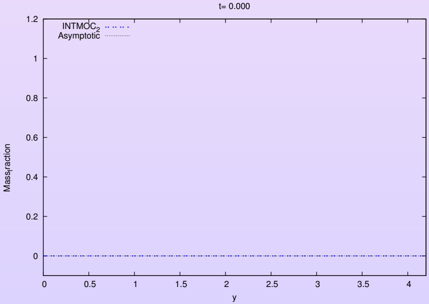

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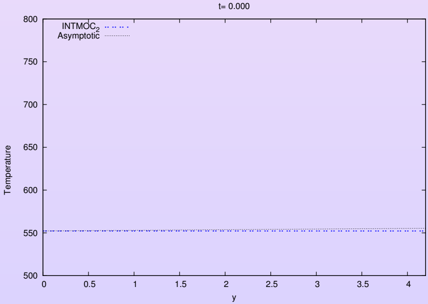

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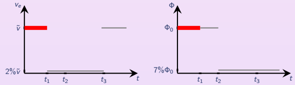

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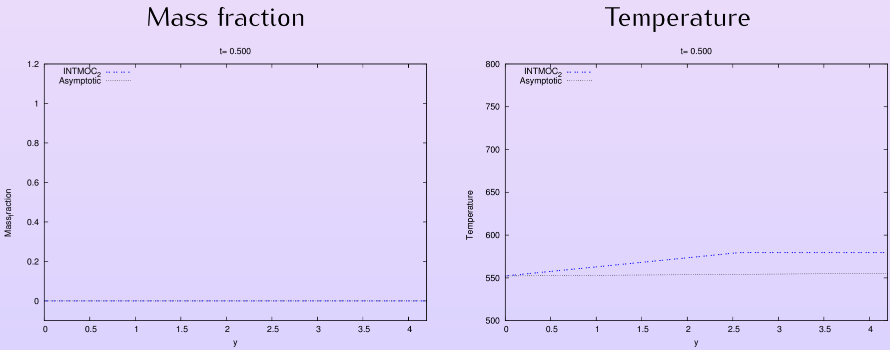

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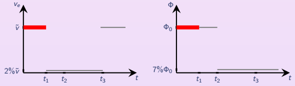

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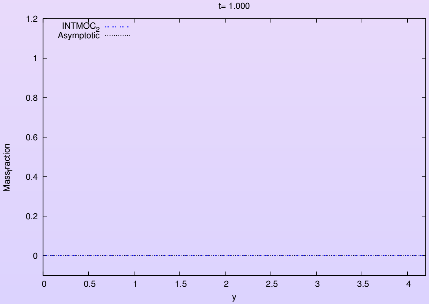

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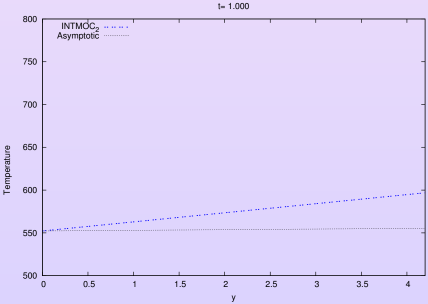

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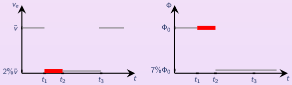

At t_1 most of the pumps stop $\Longrightarrow$ $v_e(t) \searrow$.

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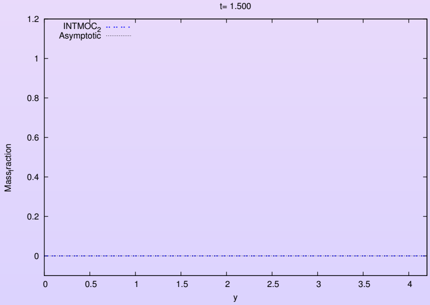

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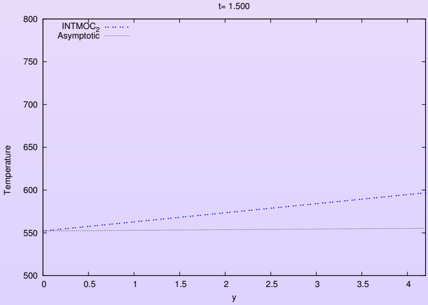

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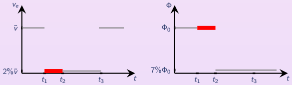

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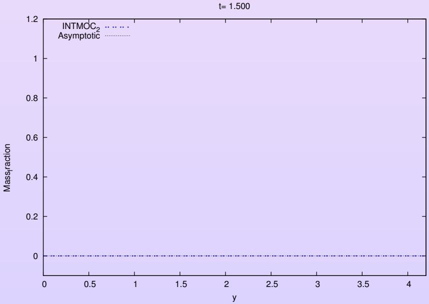

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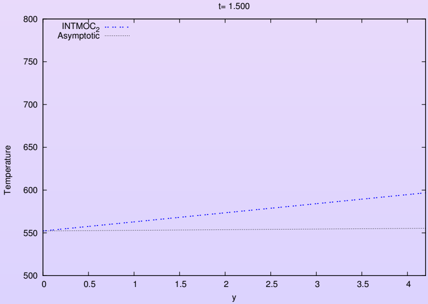

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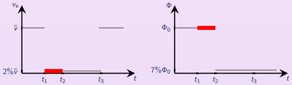

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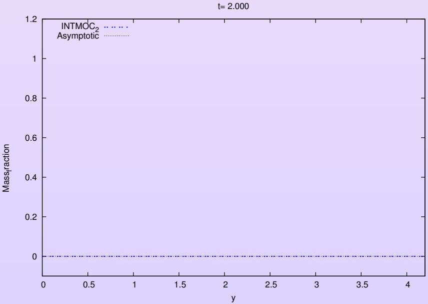

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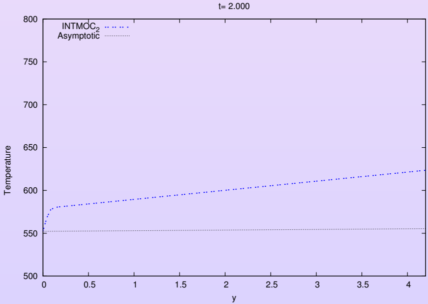

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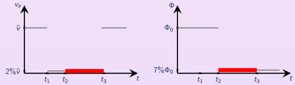

At t_2 the security system drops control rods into the core

$\Longrightarrow \Phi(t) \searrow 7\%\Phi_0$.

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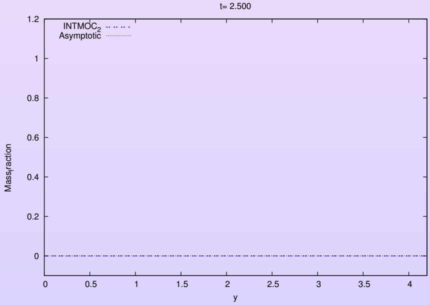

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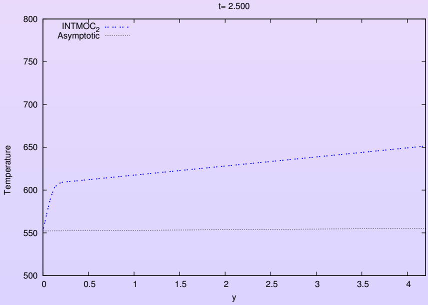

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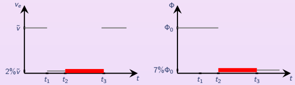

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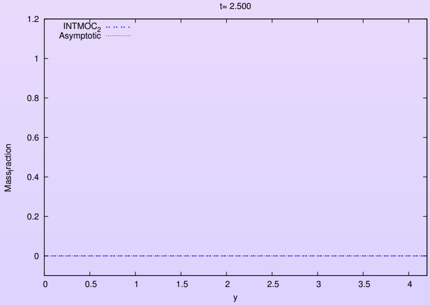

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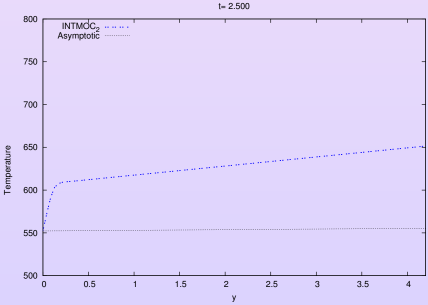

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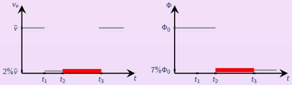

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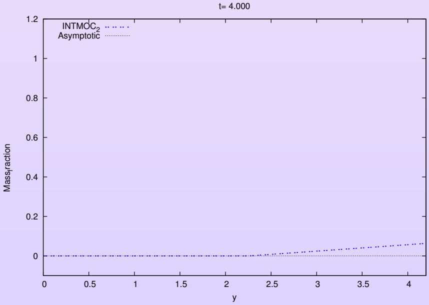

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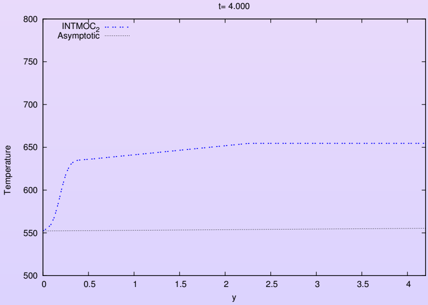

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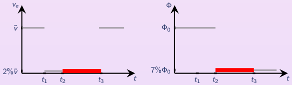

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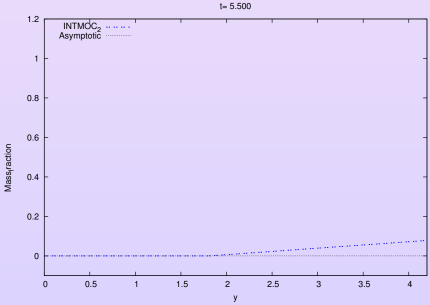

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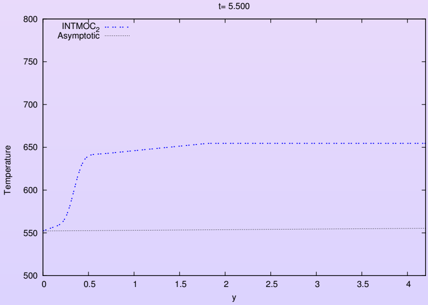

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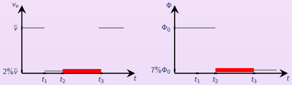

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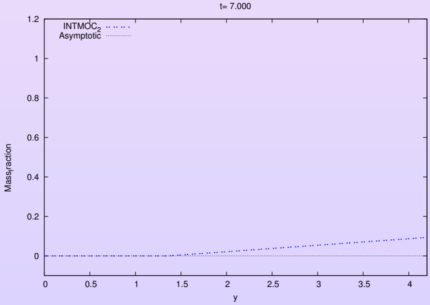

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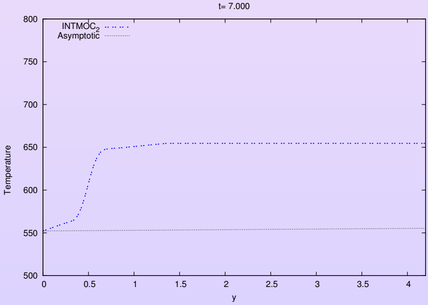

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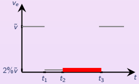

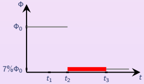

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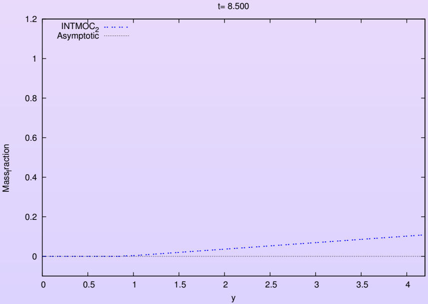

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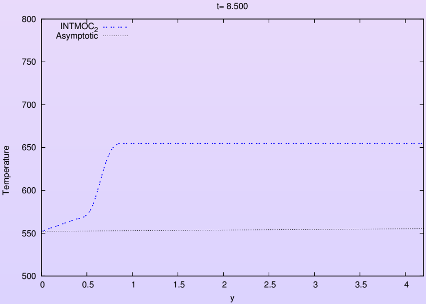

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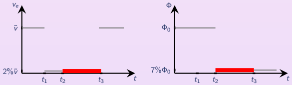

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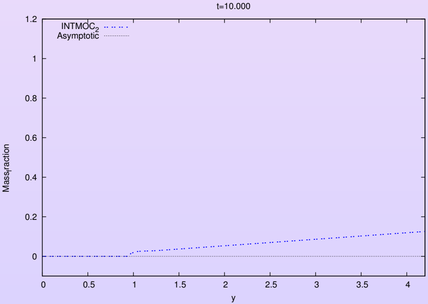

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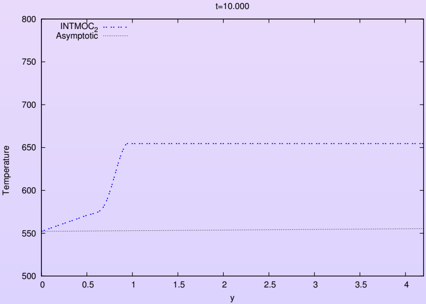

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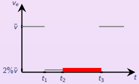

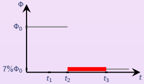

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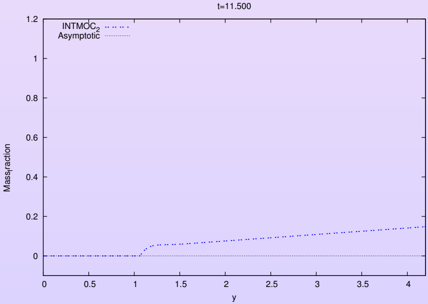

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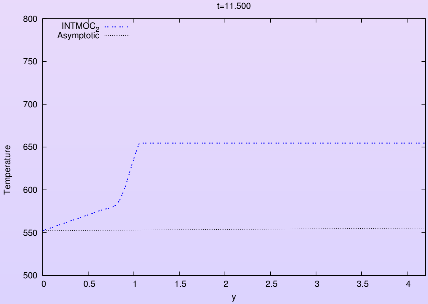

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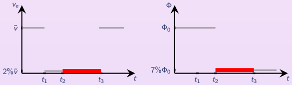

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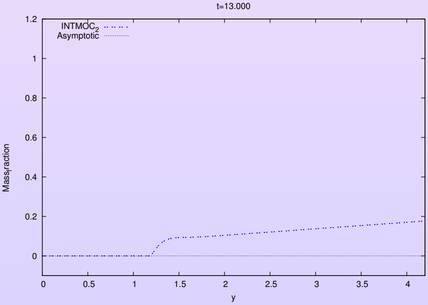

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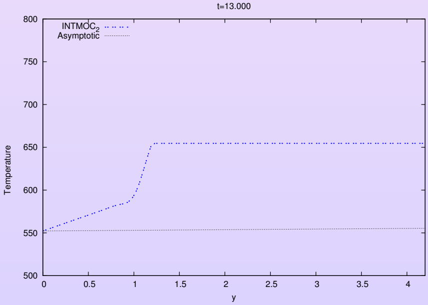

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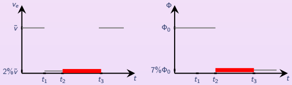

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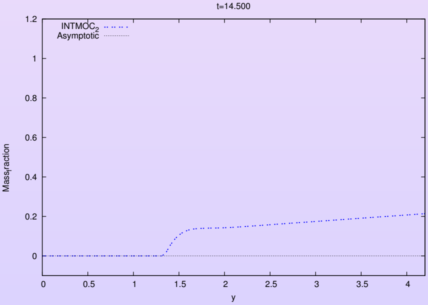

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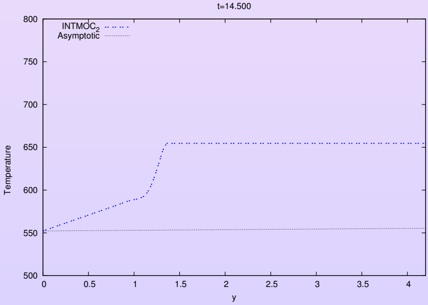

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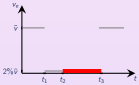

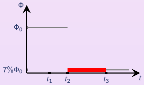

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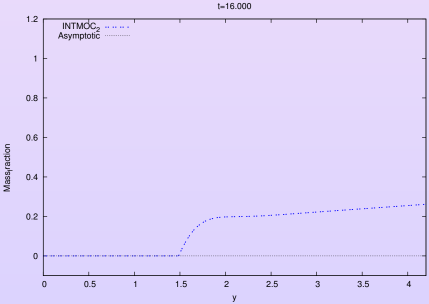

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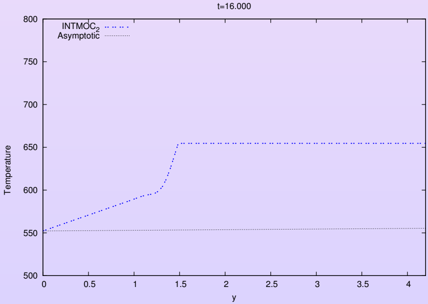

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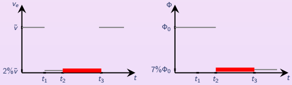
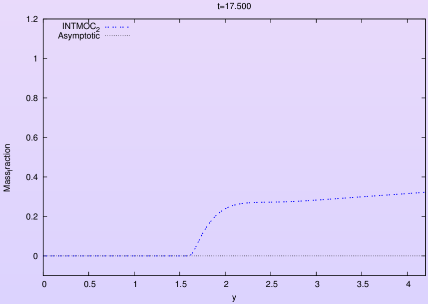
Loss of Flow

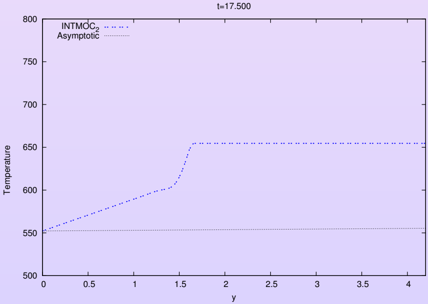


At t_2 the security system drops control rods into the core
$\implies \Phi(t) \searrow 7\%\Phi_0$.

Mass fraction



Temperature



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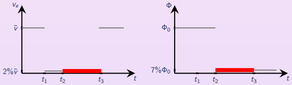

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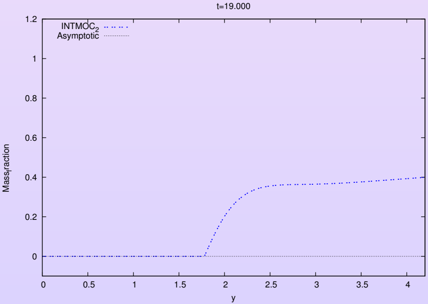
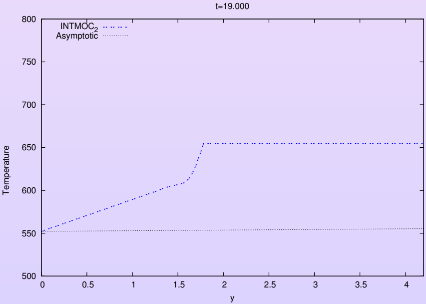
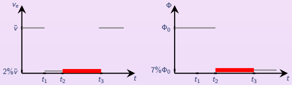


Temperature



◀ Description ▶ $[t_0 - t_1]$ ▶ $[t_1 - t_2]$ ⏩ $[t_2 - t_3]$ ⏩ $t > t_3$ ⏩ Fin

Loss of Flow



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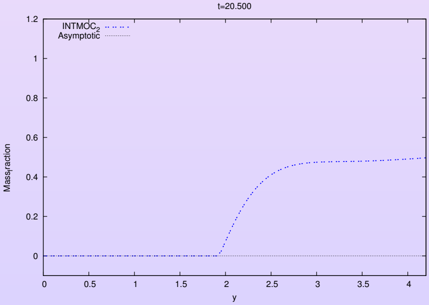
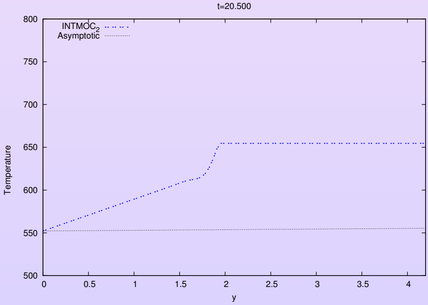


Temperature



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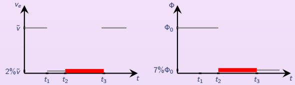

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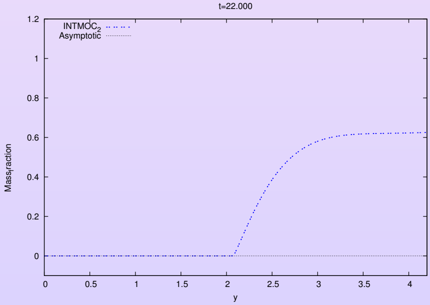
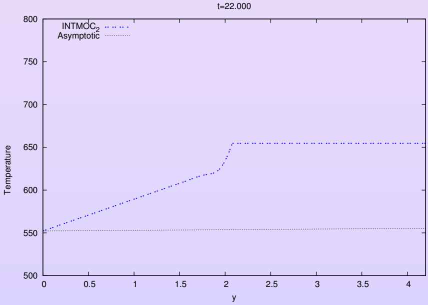


Temperature



◂ Description ▸ $[t_0 - t_1]$ ▸ $[t_1 - t_2]$ ▸▸ $[t_2 - t_3]$ ▸▸ $t > t_3$ ▸▸ Fin

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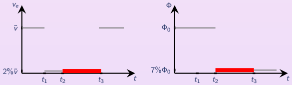

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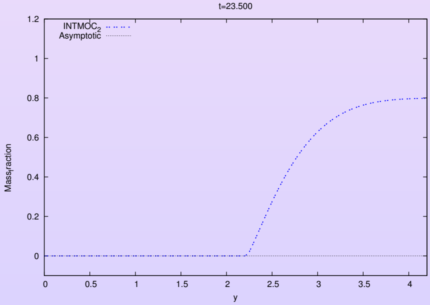
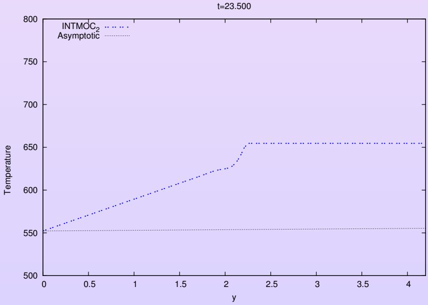


Temperature



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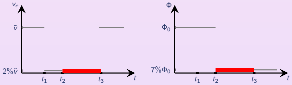

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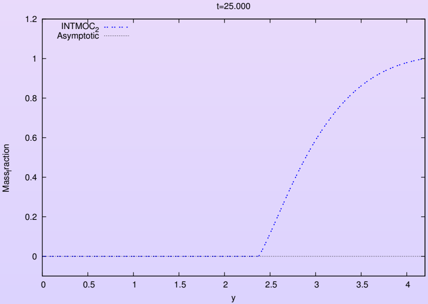

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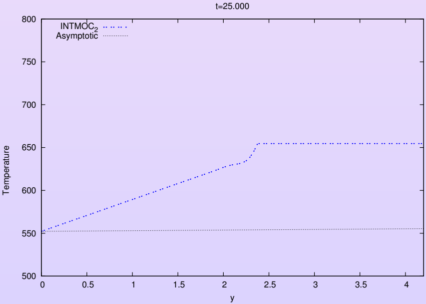

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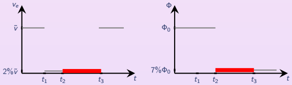

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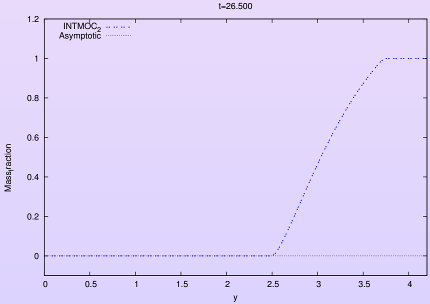
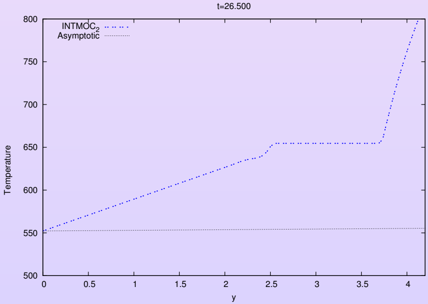


Temperature



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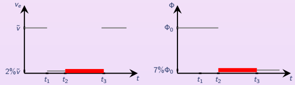

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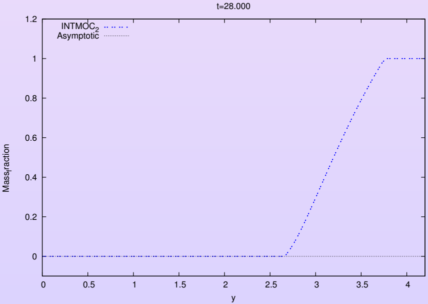
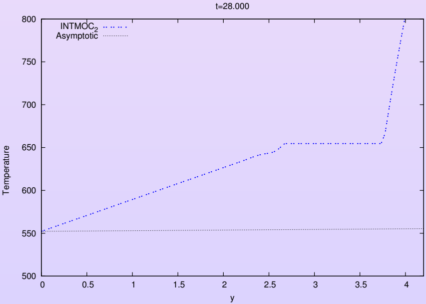


Temperature



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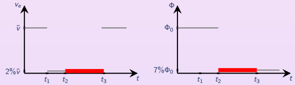

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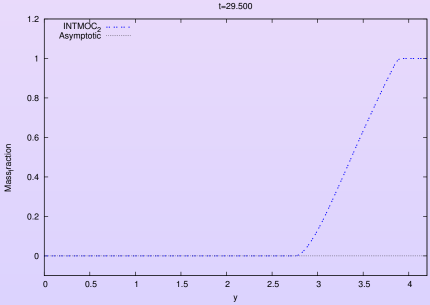

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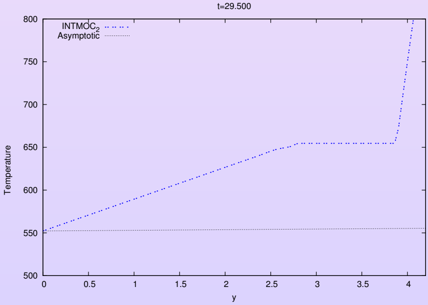

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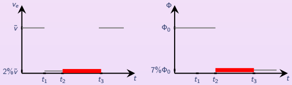

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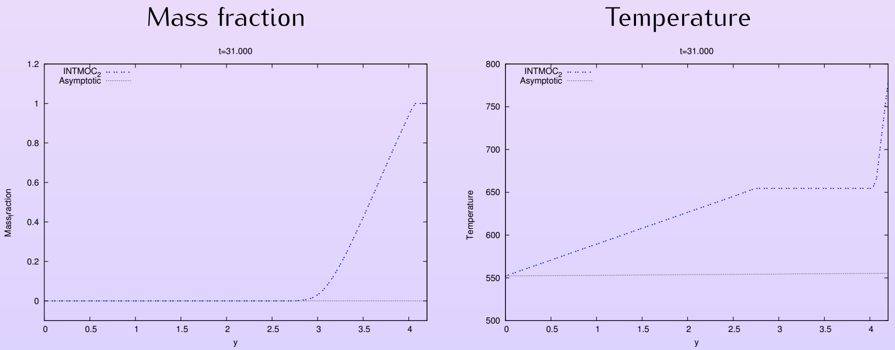

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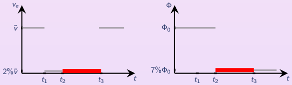

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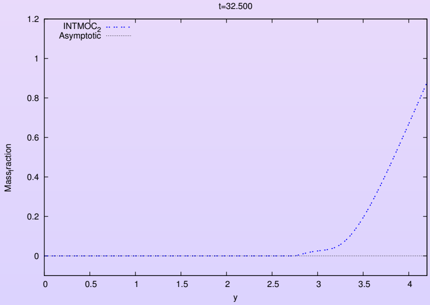
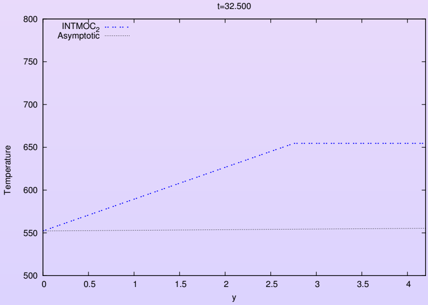


Temperature



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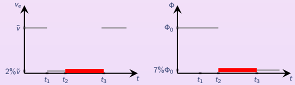

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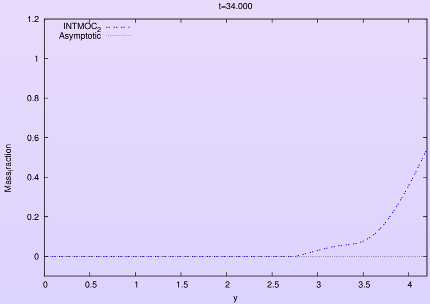

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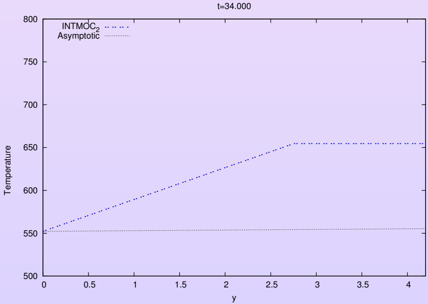

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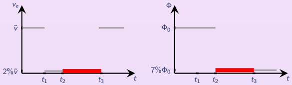

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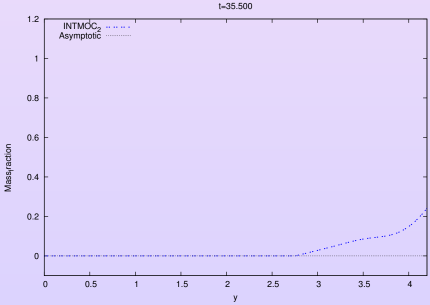

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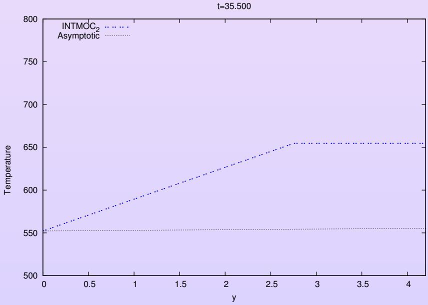

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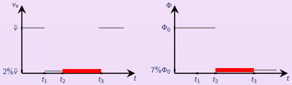

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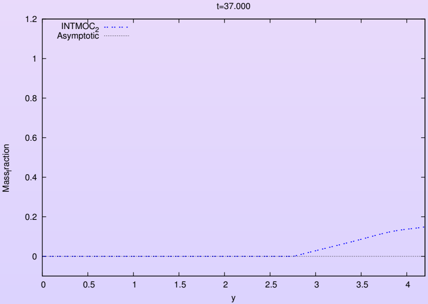

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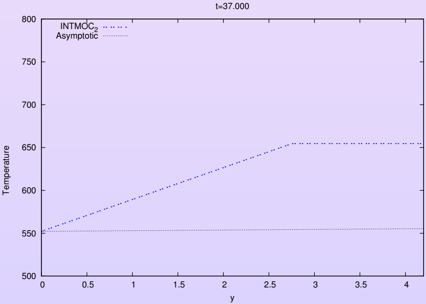

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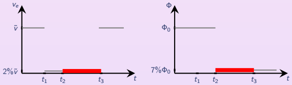

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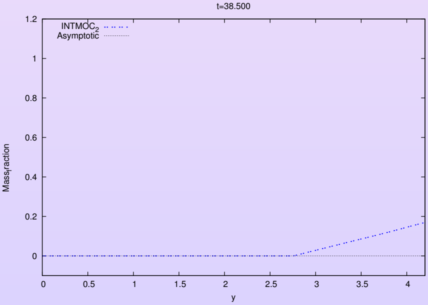

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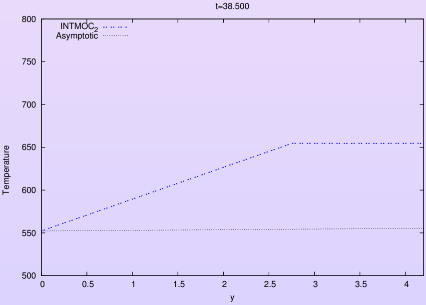

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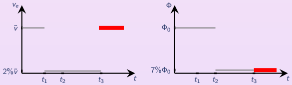

At t_3 the security pumps are turned on $\implies$ $v_e(t) \nearrow$ and the fluid comes back to the liquid phase.

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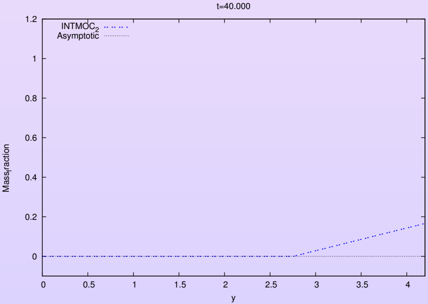

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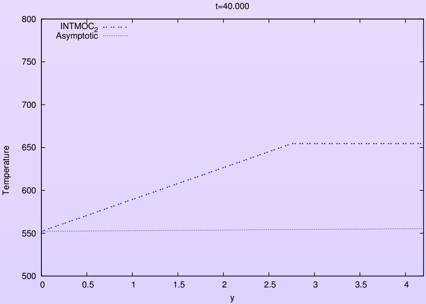

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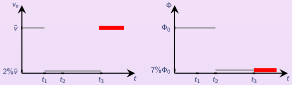

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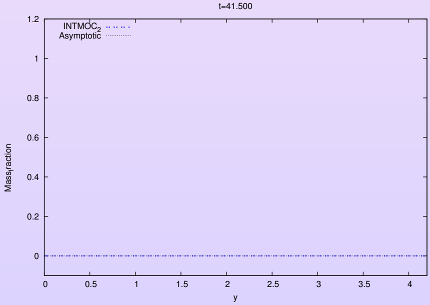

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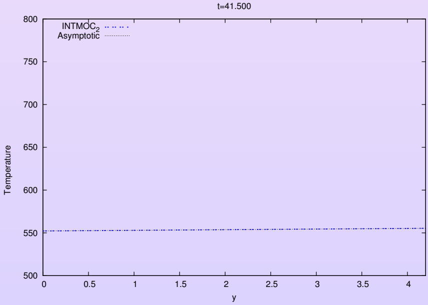

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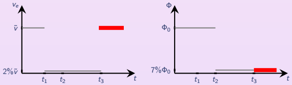

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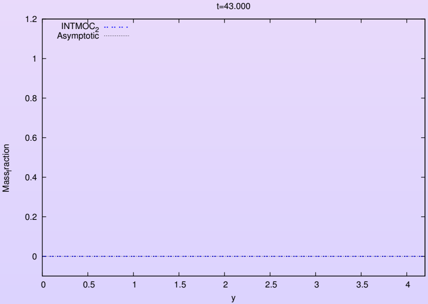
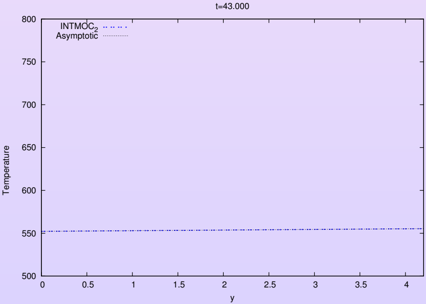


Temperature



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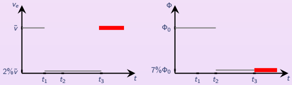

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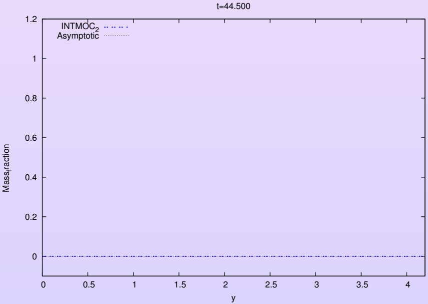

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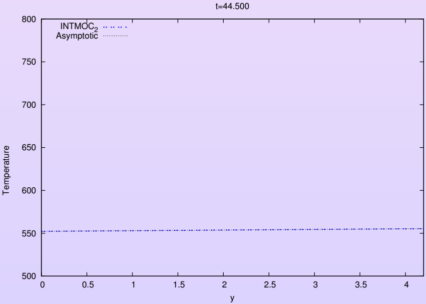

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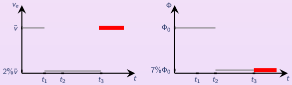

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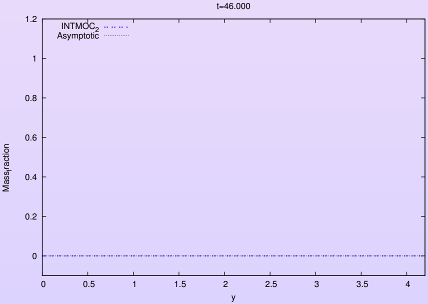

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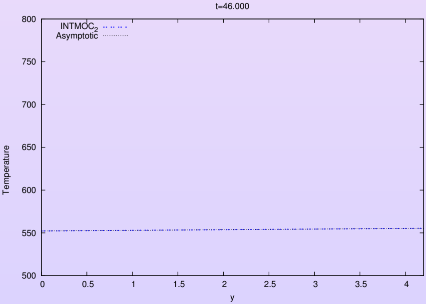

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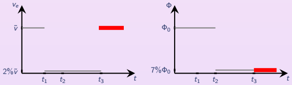

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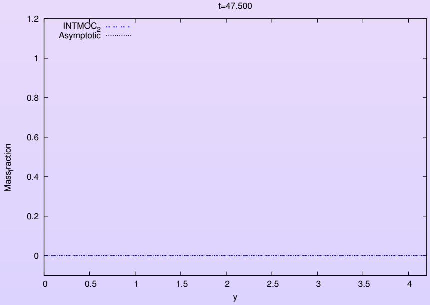
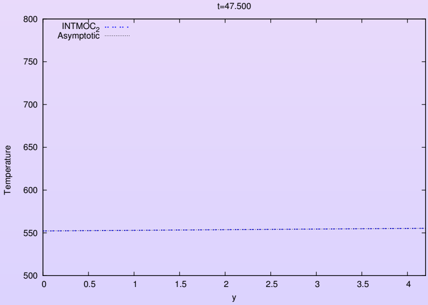
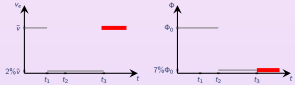


Temperature



Description | $[t_0 - t_1]$ | $[t_1 - t_2]$ | $[t_2 - t_3]$ | $t > t_3$ | Fin

Loss of Flow



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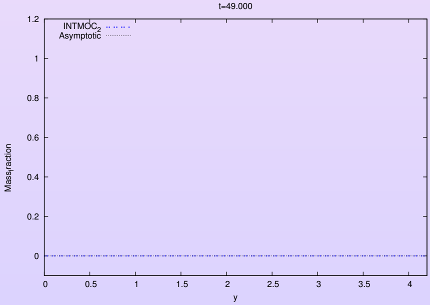

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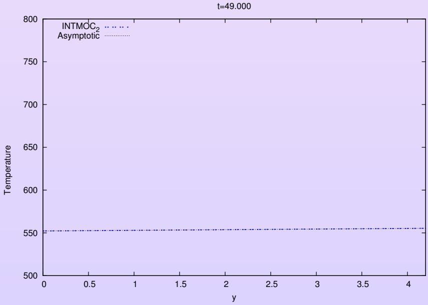

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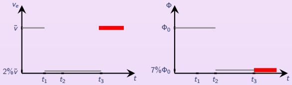

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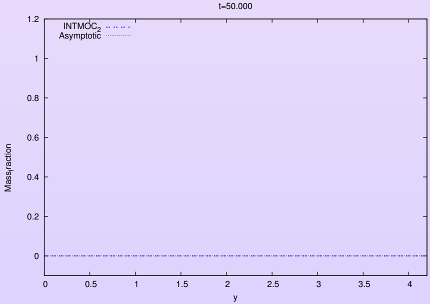

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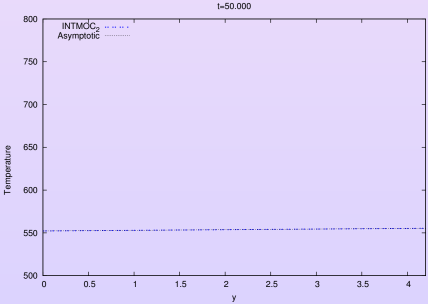

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Outline

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  - ✓ unsteady exact solutions on some cases (also with phase transition), steady exact solutions on all cases (also with tabulated EOS),
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