Grenoble, March 17, 2009

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MODELLING AND SIMULATION OF LIQUID-VAPOR PHASE TRANSITION

A CONTRIBUTION TO THE STUDY OF THE BOILING CRISIS

[1]Équipe de Sismologie – IPGP

[2]DMA – ENS

[3]CMAP – École Polytechnique

[4]DEN/DANS/DM2S/SFME/LETR – CEA

Outline

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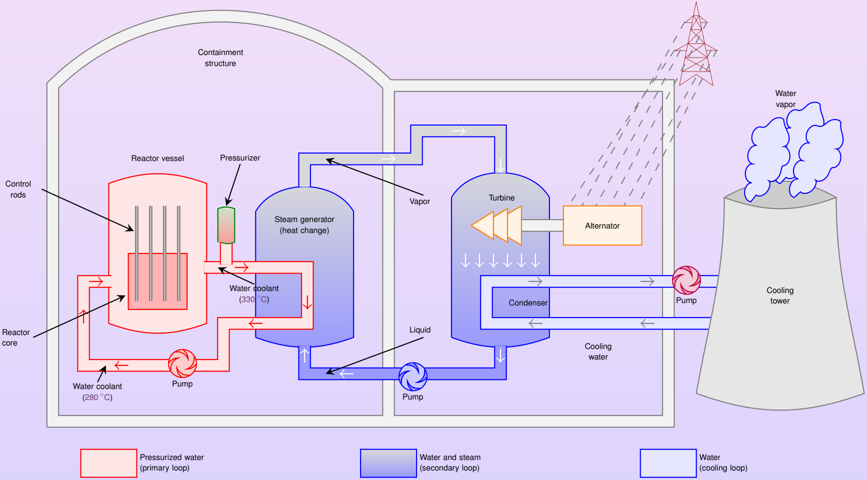
Pressurized Water Reactor



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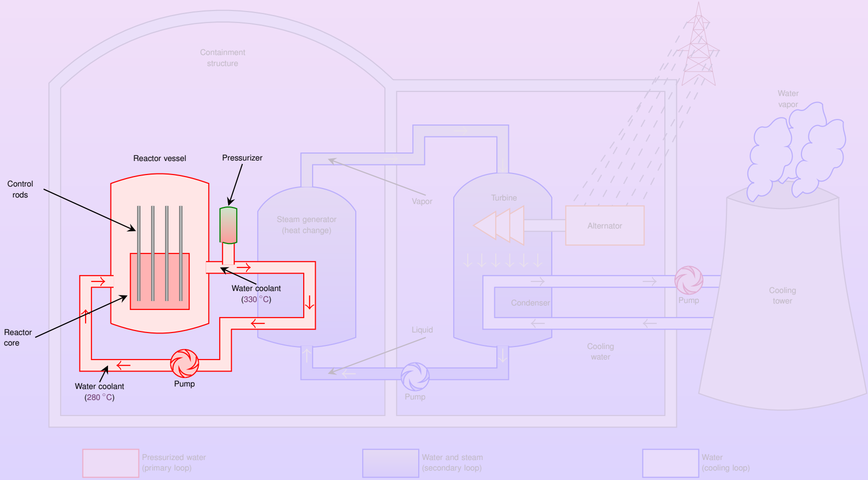

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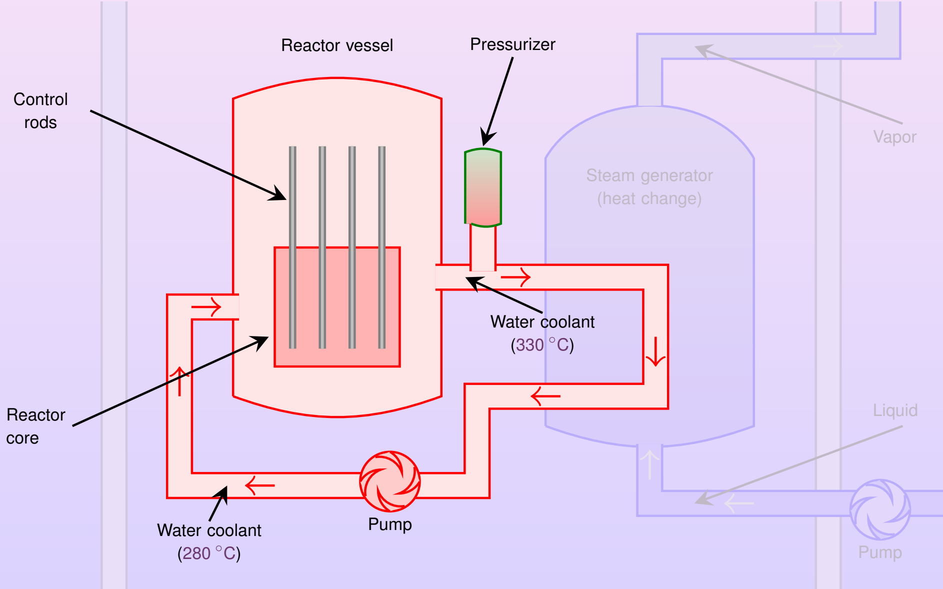
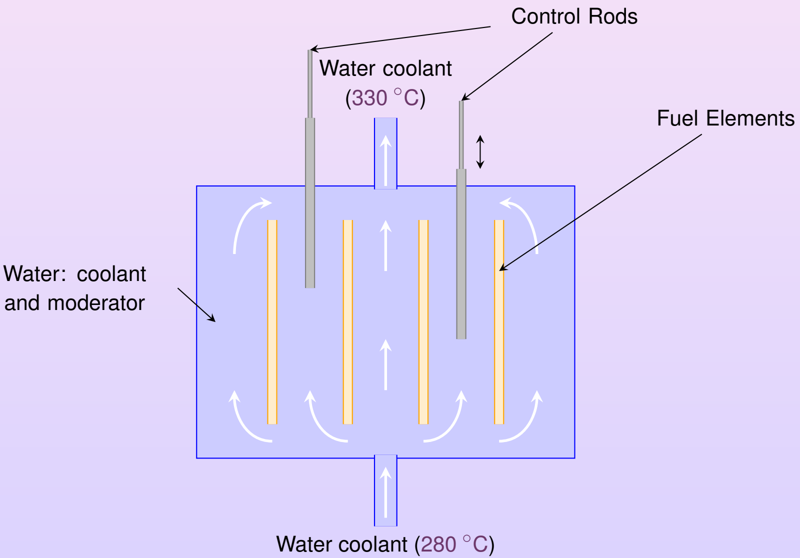


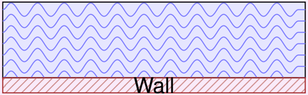
Core of a Pressurized Water Reactor



Boiling Crisis

Phenomenon

Liquid phase heated by a wall at a fixed temperature T^{wall}.
When T^{wall} increases, we switch from a **Nucleate Boiling** to a **Film Boiling**.



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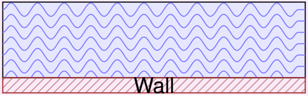

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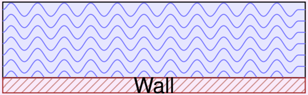

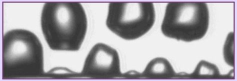

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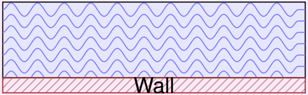

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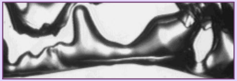

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OMEGA - CEA Grenoble

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Outline

“Ingredients” of the Model

- ✔ Simulating all bubbles,
- System of PDEs for the fluid flow (monophasic or diphasic),
- Phase transition (pressure and/or temperature variations),
- Heat Diffusion,
- Surface Tension,
- Gravity.

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Euler System

$$\begin{cases}\partial_t \rho + \mathrm{div}(\rho \mathbf{u}) = 0,\\ \partial_t(\rho \mathbf{u}) + \mathrm{div}(\rho \mathbf{u}\otimes\mathbf{u} + P\,\mathbb{I}) = \mathfrak{V}_{\mathrm{vf}} - \mathfrak{S}_{\mathrm{sf}},\\ \partial_t\left(\rho\left(\frac{|\mathbf{u}|^2}{2}+\varepsilon\right)\right) + \mathrm{div}\left(\rho\left(\frac{|\mathbf{u}|^2}{2}+\varepsilon\right)\mathbf{u} + P\,\mathbf{u}\right) = (\mathfrak{V}_{\mathrm{vf}} - \mathfrak{S}_{\mathrm{sf}})\cdot\mathbf{u} - \mathrm{div}(q).\end{cases}$$

- $(\mathbf{x},t)\mapsto\rho$ specific density,
- $(\mathbf{x},t)\mapsto\varepsilon$ specific internal energy,
- $(\mathbf{x},t)\mapsto\mathbf{u}$ velocity;
- $(\rho,\varepsilon)\mapsto\mathfrak{V}_{\mathrm{vf}}$ body forces,
- $(\rho,\varepsilon)\mapsto\mathfrak{S}_{\mathrm{sf}}$ surface forces,
- $(\rho,\varepsilon)\mapsto\mathrm{div}(q)$ heat transfer.

$(\rho,\varepsilon)\mapsto P$ pressure law.

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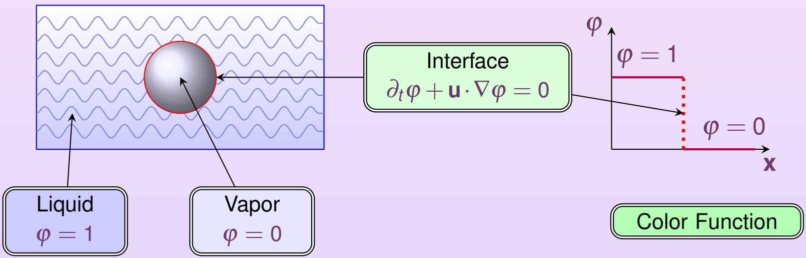
Liquid-Vapor Interface



$$(\rho,\varepsilon) \mapsto P = \begin{cases} P^{\text{liq}} & \text{if } \varphi = 1; \\ P^{\text{vap}} & \text{if } \varphi = 0. \end{cases}$$

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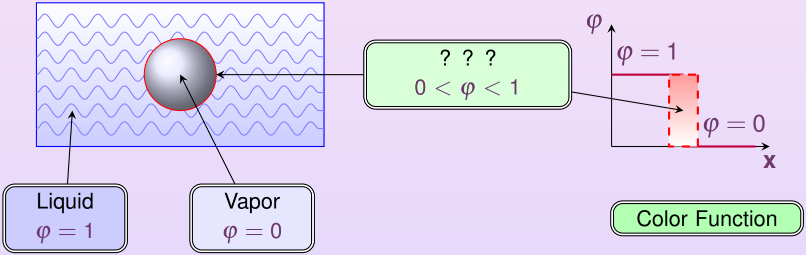

$$(\rho, \varepsilon) \mapsto P = \begin{cases} P^{\text{liq}} & \text{if } \varphi = 1; \\ ??? & \text{if } 0 < \varphi < 1; \\ P^{\text{vap}} & \text{if } \varphi = 0. \end{cases}$$

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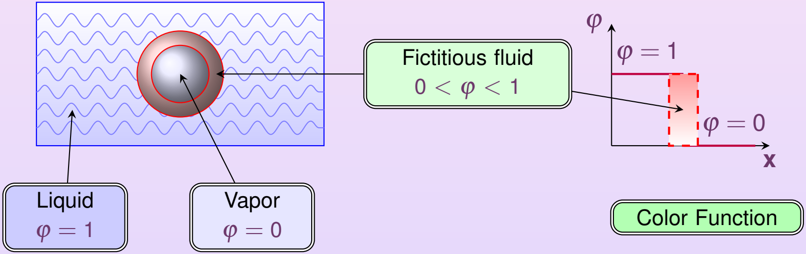

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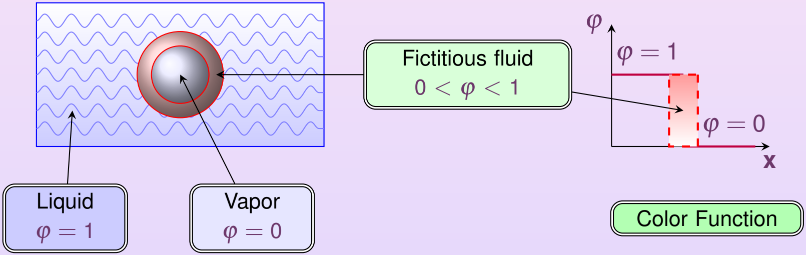

Goal: define a global pressure law such that

- $(\rho, \varepsilon, \mathbf{u}, P)$ are continuous (3 zones)
- the interface position and the phase change are implicit (i.e. $\not\varphi$)
- coherence with classical thermodynamics [H. CALLEN]

EOS of each PHASE $\alpha = 1, 2$

τ_α specific volume
ε_α specific internal energy

$$\Rightarrow \quad \mathbf{w}_\alpha \stackrel{\text{def}}{=} (\tau_\alpha, \varepsilon_\alpha);$$

$\mathbf{w}_\alpha \mapsto s_\alpha$ specific entropy (Hessian matrix neg. def.);

$$\begin{cases} T_\alpha \stackrel{\text{def}}{=} \left(\left.\dfrac{\partial s_\alpha}{\partial \varepsilon_\alpha}\right|_{\tau_\alpha} \right)^{-1} > 0 & \text{temperature,} \\ P_\alpha \stackrel{\text{def}}{=} T_\alpha \left.\dfrac{\partial s_\alpha}{\partial \tau_\alpha}\right|_{\varepsilon_\alpha} > 0 & \text{pressure,} \\ g_\alpha \stackrel{\text{def}}{=} \varepsilon_\alpha + P_\alpha \tau_\alpha - T_\alpha s_\alpha & \text{free enthalpy (Gibbs potential).} \end{cases}$$

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- $\mathbf{w} \stackrel{\text{def}}{=} y\mathbf{w}_1 + (1-y)\mathbf{w}_2$;
- y mass fraction;
- z volume fraction s.t. $y\tau_1 = z\tau$;
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ENTROPY WITHOUT PHASE CHANGE

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EOS with Phase Change

ENTROPY WITHOUT PH.CH.

$(\mathbf{w}, z, y, \psi) \mapsto \sigma$

→

ENTROPY AT EQUILIBRIUM

$\mathbf{w} \mapsto s^{\text{eq}}$

DEFINITION [H. CALLEN, PH. HELLUY ...]

Optimization Problem:

$$s^{\text{eq}}(\mathbf{w}) \stackrel{\text{def}}{=} \max_{z,y,\psi \in [0,1]^3} \sigma(\mathbf{w}, z, y, \psi)$$

Optimality Condition:
$$\begin{cases} T_1(z,y,\psi) = T_2(z,y,\psi) \\ P_1(z,y,\psi) = P_2(z,y,\psi) \\ g_1(z,y,\psi) = g_2(z,y,\psi) \\ z,y,\psi \in]0,1[^3 \end{cases}$$

Solution: (z^*, y^*, ψ^*)

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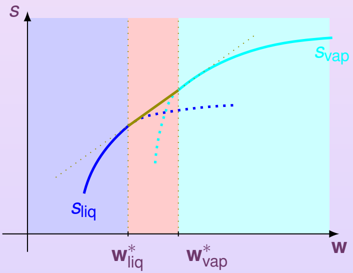

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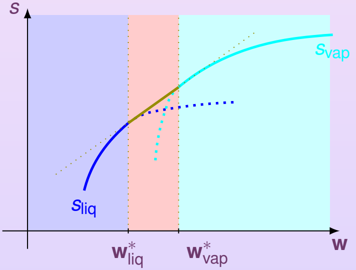

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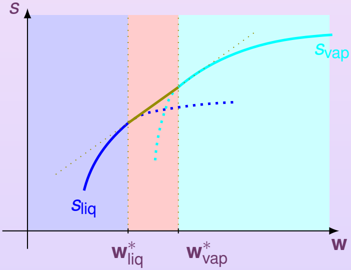

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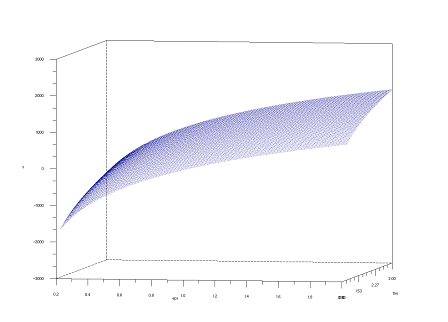
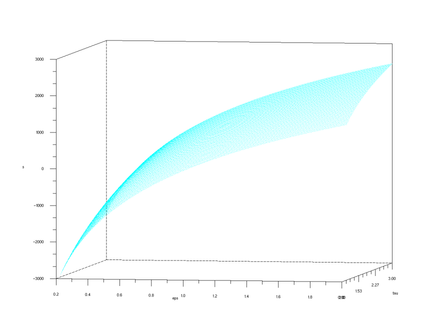
Concave Hull with two Perfect Gases

$(\tau, \varepsilon) \mapsto s_{\text{liq}}$



$(\tau, \varepsilon) \mapsto s_{\text{vap}}$



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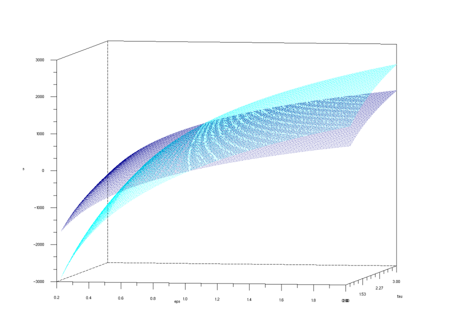

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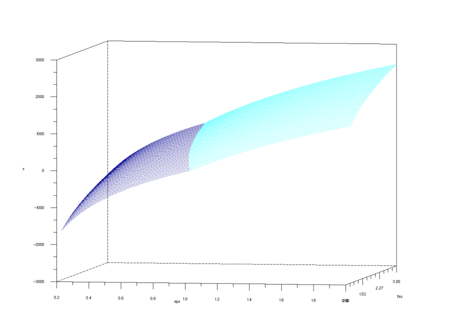

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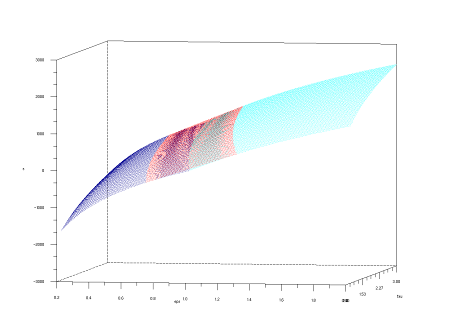

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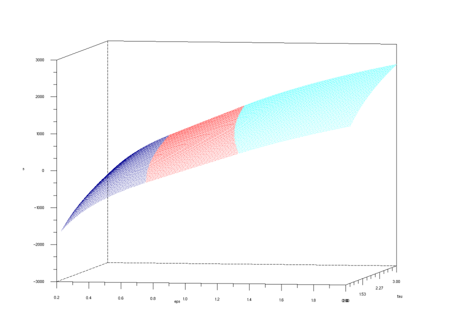


From $\mathbf{w} \mapsto s^{\text{eq}}$ to $\mathbf{w} \mapsto P^{\text{eq}}$

For all $\widetilde{\mathbf{w}}$ fixed, we seek $(\mathbf{w}^*_{\text{liq}}, \mathbf{w}^*_{\text{vap}}, y^*)$ as the solution of the system

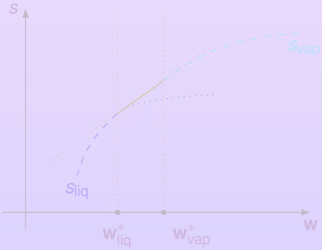
$$\begin{cases} P_{\text{liq}}(\mathbf{w}_{\text{liq}}) = P_{\text{vap}}(\mathbf{w}_{\text{vap}}) \\ T_{\text{liq}}(\mathbf{w}_{\text{liq}}) = T_{\text{vap}}(\mathbf{w}_{\text{vap}}) \\ g_{\text{liq}}(\mathbf{w}_{\text{liq}}) = g_{\text{vap}}(\mathbf{w}_{\text{vap}}) \\ \widetilde{\mathbf{w}} = y\mathbf{w}_{\text{liq}} + (1-y)\mathbf{w}_{\text{vap}} \end{cases}$$

1. if $y^* \in]0,1[$ then $\widetilde{\mathbf{w}}$ is an **equilibrium mixture state**

$$s^{\text{eq}}(\widetilde{\mathbf{w}}) = y^* s_{\text{liq}}(\mathbf{w}^*_{\text{liq}}) + (1-y^*) s_{\text{vap}}(\mathbf{w}^*_{\text{vap}}),$$
$$P^{\text{eq}}(\widetilde{\mathbf{w}}) = P_{\text{liq}}(\mathbf{w}^*_{\text{liq}}) = P_{\text{vap}}(\mathbf{w}^*_{\text{vap}});$$

2. if the system has no solution or $y^* \notin]0,1[$ then $\widetilde{\mathbf{w}}$ is a **monophasic pure state**

$$s^{\text{eq}}(\widetilde{\mathbf{w}}) = \max\{s_{\text{liq}}(\widetilde{\mathbf{w}}), s_{\text{vap}}(\widetilde{\mathbf{w}})\},$$
$$P^{\text{eq}}(\widetilde{\mathbf{w}}) = P_{\alpha}(\widetilde{\mathbf{w}}).$$



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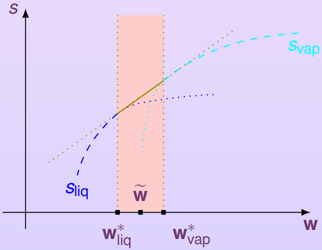

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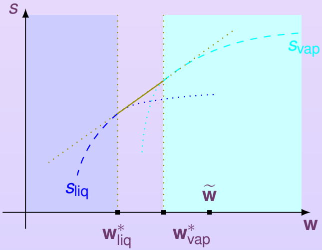

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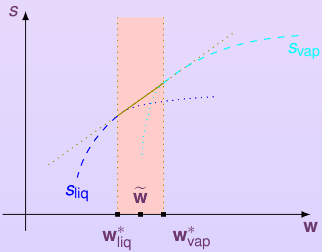

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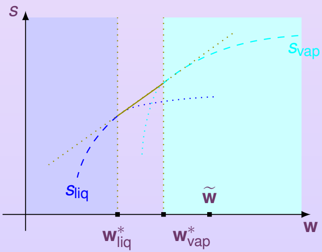

Summary of the Model

$\mathbf{w} \mapsto s^{\text{eq}}$

$$\begin{cases} g_1(\mathbf{w}_1) = g_2(\mathbf{w}_2) \\ P_1(\mathbf{w}_1) = P_2(\mathbf{w}_2) \\ T_1(\mathbf{w}_1) = T_2(\mathbf{w}_2) \\ \mathbf{w} = y\mathbf{w}_1 + (1-y)\mathbf{w}_2 \end{cases}$$

Euler System

$\mathbf{w} \mapsto P^{\text{eq}}$

Phase Change Equation

$$\frac{\tau - \tau_{\text{vap}}^{\text{sat}}}{\tau_{\text{liq}}^{\text{sat}} - \tau_{\text{vap}}^{\text{sat}}} = \frac{\varepsilon - \varepsilon_{\text{vap}}^{\text{sat}}}{\varepsilon_{\text{liq}}^{\text{sat}} - \varepsilon_{\text{vap}}^{\text{sat}}}$$

Summary of the Model

$\mathbf{w} \longmapsto s^{\mathrm{eq}}$

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Euler System

$\mathbf{w} \longmapsto P^{\mathrm{eq}}$

Phase Change Equation

$$\frac{\tau - \tau^{\mathrm{sat}}_{\mathrm{vap}}}{\tau^{\mathrm{sat}}_{\mathrm{liq}} - \tau^{\mathrm{sat}}_{\mathrm{vap}}} = \frac{\varepsilon - \varepsilon^{\mathrm{sat}}_{\mathrm{vap}}}{\varepsilon^{\mathrm{sat}}_{\mathrm{liq}} - \varepsilon^{\mathrm{sat}}_{\mathrm{vap}}}$$

Summary of the Model

$$\begin{cases} g_1(\mathbf{w}_1) = g_2(\mathbf{w}_2) \\ P_1(\mathbf{w}_1) = P_2(\mathbf{w}_2) \\ T_1(\mathbf{w}_1) = T_2(\mathbf{w}_2) \\ \mathbf{w} = y\mathbf{w}_1 + (1-y)\mathbf{w}_2 \end{cases}$$

$$\mathbf{w} \longmapsto s^{\mathrm{eq}}$$

Euler System

$$\mathbf{w} \longmapsto P^{\mathrm{eq}}$$

Phase Change Equation

$$\frac{\tau - \tau_{\mathrm{vap}}^{\mathrm{sat}}}{\tau_{\mathrm{liq}}^{\mathrm{sat}} - \tau_{\mathrm{vap}}^{\mathrm{sat}}} = \frac{\varepsilon - \varepsilon_{\mathrm{vap}}^{\mathrm{sat}}}{\varepsilon_{\mathrm{liq}}^{\mathrm{sat}} - \varepsilon_{\mathrm{vap}}^{\mathrm{sat}}}$$

Summary of the Model

$$\mathbf{w} \mapsto s^{\text{eq}}$$

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Euler System

$$\mathbf{w} \mapsto P^{\text{eq}}$$

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$$\frac{\tau-\tau_{\text{vap}}^{\text{sat}}}{\tau_{\text{liq}}^{\text{sat}}-\tau_{\text{vap}}^{\text{sat}}}=\frac{\varepsilon-\varepsilon_{\text{vap}}^{\text{sat}}}{\varepsilon_{\text{liq}}^{\text{sat}}-\varepsilon_{\text{vap}}^{\text{sat}}}$$

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Euler System

$\mathbf{w} \mapsto P^{\text{eq}}$

Phase Change Equation

$$\frac{\tau - \tau_{\text{vap}}^{\text{sat}}}{\tau_{\text{liq}}^{\text{sat}} - \tau_{\text{vap}}^{\text{sat}}} = \frac{\varepsilon - \varepsilon_{\text{vap}}^{\text{sat}}}{\varepsilon_{\text{liq}}^{\text{sat}} - \varepsilon_{\text{vap}}^{\text{sat}}}$$

Outline

Analytical EOS

(τ,ε) fixed

$(\tau_1,\varepsilon_1,\tau_2,\varepsilon_2,y)$ SOLUTION OF

$$\begin{cases} g_1(\tau_1,\varepsilon_1)=g_2(\tau_2,\varepsilon_2)\\ P_1(\tau_1,\varepsilon_1)=P_2(\tau_2,\varepsilon_2)\\ T_1(\tau_1,\varepsilon_1)=T_2(\tau_2,\varepsilon_2)\\ \tau=y\tau_1+(1-y)\tau_2\\ \varepsilon=y\varepsilon_1+(1-y)\varepsilon_2 \end{cases}$$

(P,T) SOLUTION OF

$$\begin{cases} g_1(P,T)=g_2(P,T)\\ \dfrac{\tau-\tau_2(P,T)}{\tau_1(P,T)-\tau_2(P,T)}=\dfrac{\varepsilon-\varepsilon_2(P,T)}{\varepsilon_1(P,T)-\varepsilon_2(P,T)} \end{cases}$$

$$T\mapsto P=P^{\text{sat}}(T)\approx P^{\text{sat}}(T)$$

T SOLUTION OF

$$\frac{\tau-\tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T)-\tau_2^{\text{sat}}(T)}=\frac{\varepsilon-\varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T)-\varepsilon_2^{\text{sat}}(T)}\quad\text{where}\quad\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T)\stackrel{\text{def}}{=}\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha(P^{\text{sat}}(T),T)$$

Analytical EOS

(τ, ε) fixed

$(\tau_1, \varepsilon_1, \tau_2, \varepsilon_2, y)$ SOLUTION OF

$$\begin{cases} g_1(\tau_1, \varepsilon_1) = g_2(\tau_2, \varepsilon_2) \\ P_1(\tau_1, \varepsilon_1) = P_2(\tau_2, \varepsilon_2) \\ T_1(\tau_1, \varepsilon_1) = T_2(\tau_2, \varepsilon_2) \\ \tau = y\tau_1 + (1-y)\tau_2 \\ \varepsilon = y\varepsilon_1 + (1-y)\varepsilon_2 \end{cases}$$

(P, T) SOLUTION OF

$$\begin{cases} g_1(P,T) = g_2(P,T) \\ \dfrac{\tau - \tau_2(P,T)}{\tau_1(P,T) - \tau_2(P,T)} = \dfrac{\varepsilon - \varepsilon_2(P,T)}{\varepsilon_1(P,T) - \varepsilon_2(P,T)} \end{cases}$$

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Analytical EOS

(τ, ε) fixed

$(\tau_1, \varepsilon_1, \tau_2, \varepsilon_2, y)$ **SOLUTION OF**

$$\begin{cases} g_1(\tau_1,\varepsilon_1) = g_2(\tau_2,\varepsilon_2) \\ P_1(\tau_1,\varepsilon_1) = P_2(\tau_2,\varepsilon_2) \\ T_1(\tau_1,\varepsilon_1) = T_2(\tau_2,\varepsilon_2) \\ \tau = y\tau_1 + (1-y)\tau_2 \\ \varepsilon = y\varepsilon_1 + (1-y)\varepsilon_2 \end{cases}$$

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$$\begin{cases} g_1(P,T) = g_2(P,T) \\ \dfrac{\tau - \tau_2(P,T)}{\tau_1(P,T) - \tau_2(P,T)} = \dfrac{\varepsilon - \varepsilon_2(P,T)}{\varepsilon_1(P,T) - \varepsilon_2(P,T)} \end{cases}$$

$$T \mapsto P = P^{\text{sat}}(T) \approx P^{\text{sat}}(T)$$

T **SOLUTION OF**

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{where} \quad \begin{pmatrix} \tau \\ \varepsilon \end{pmatrix}_\alpha^{\text{sat}}(T) \stackrel{\text{def}}{=} \begin{pmatrix} \tau \\ \varepsilon \end{pmatrix}_\alpha (P^{\text{sat}}(T), T)$$

Analytical EOS

(τ, ε) fixed

$(\tau_1, \varepsilon_1, \tau_2, \varepsilon_2, y)$ SOLUTION OF

$$\begin{cases} g_1(\tau_1,\varepsilon_1) = g_2(\tau_2,\varepsilon_2) \\ P_1(\tau_1,\varepsilon_1) = P_2(\tau_2,\varepsilon_2) \\ T_1(\tau_1,\varepsilon_1) = T_2(\tau_2,\varepsilon_2) \\ \tau = y\tau_1 + (1-y)\tau_2 \\ \varepsilon = y\varepsilon_1 + (1-y)\varepsilon_2 \end{cases}$$

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$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{where} \quad \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \stackrel{\text{def}}{=} \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha (P^{\text{sat}}(T), T)$$

Analytical EOS

(τ, ε) fixed

$(\tau_1, \varepsilon_1, \tau_2, \varepsilon_2, y)$ SOLUTION OF

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(P, T) SOLUTION OF

$$\begin{cases} g_1(P,T) = g_2(P,T) \\ \dfrac{\tau - \tau_2(P,T)}{\tau_1(P,T) - \tau_2(P,T)} = \dfrac{\varepsilon - \varepsilon_2(P,T)}{\varepsilon_1(P,T) - \varepsilon_2(P,T)} \end{cases}$$

least square approximation

$$T \mapsto P = \widehat{P}^{\text{sat}}(T) \approx P^{\text{sat}}(T)$$

T SOLUTION OF

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{where} \quad \begin{pmatrix}\tau \\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \stackrel{\text{def}}{=} \begin{pmatrix}\tau \\ \varepsilon\end{pmatrix}_\alpha (\widehat{P}^{\text{sat}}(T), T)$$

Tabulated EOS

		Volume (m^3/kg)		Internal Energy (kJ/kg)	
T (K)	P^{sat} (MPa)	τ_{liq}^{sat}	τ_{vap}^{sat}	ε_{liq}^{sat}	ε_{vap}^{sat}
275	0,00069845	0,0010001	181,60	7,7590	2377,5
278	0,00086349	0,0010001	148,48	20,388	2381,6
281	0,0010621	0,0010002	122,01	32,996	2385,7
284	0,0012999	0,0010004	100,74	45,586	2389,8
287	0,0015835	0,0010008	83,560	58,162	2393,9
290	0,0019200	0,0010012	69,625	70,727	2398,0
293	0,0023177	0,0010018	58,267	83,284	2402,1
296	0,0027856	0,0010025	48,966	95,835	2406,2
299	0,0033342	0,0010032	41,318	108,38	2410,3
302	0,0039745	0,0010041	35,002	120,92	2414,4
305	0,0047193	0,0010050	29,764	133,46	2418,4
308	0,0055825	0,0010060	25,403	146	2422,5
...	...	...	...	...	...

Source: `http://webbook.nist.gov/chemistry/fluid/`

Tabulated EOS

(τ, ε) fixed

T SOLUTION OF

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \quad \text{tabulated}$$

$\wr\wr$

$$\frac{\tau - \widehat{\tau}_2^{\text{sat}}(T)}{\widehat{\tau}_1^{\text{sat}}(T) - \widehat{\tau}_2^{\text{sat}}(T)} = \frac{\varepsilon - \widehat{\varepsilon}_2^{\text{sat}}(T)}{\widehat{\varepsilon}_1^{\text{sat}}(T) - \widehat{\varepsilon}_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix}\widehat{\tau}\\ \widehat{\varepsilon}\end{pmatrix}_\alpha^{\text{sat}}(T)$$

least square approximations

Tabulated EOS

(τ, ε) fixed

T SOLUTION OF

$$\frac{\tau-\tau_2^{\rm sat}(T)}{\tau_1^{\rm sat}(T)-\tau_2^{\rm sat}(T)}=\frac{\varepsilon-\varepsilon_2^{\rm sat}(T)}{\varepsilon_1^{\rm sat}(T)-\varepsilon_2^{\rm sat}(T)}$$ with $\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\rm sat}(T)$ tabulated

$\wr\wr$

$$\frac{\tau-\widehat{\tau}_2^{\rm sat}(T)}{\widehat{\tau}_1^{\rm sat}(T)-\widehat{\tau}_2^{\rm sat}(T)}=\frac{\varepsilon-\widehat{\varepsilon}_2^{\rm sat}(T)}{\widehat{\varepsilon}_1^{\rm sat}(T)-\widehat{\varepsilon}_2^{\rm sat}(T)}$$ with $\begin{pmatrix}\widehat{\tau}\\ \widehat{\varepsilon}\end{pmatrix}_\alpha^{\rm sat}(T)$

least square approximations

Tabulated EOS

(τ, ε) fixed

T SOLUTION OF

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix}\tau \\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \quad \text{tabulated}$$

$\wr\wr$ ← least square approximations

$$\frac{\tau - \widehat{\tau}_2^{\text{sat}}(T)}{\widehat{\tau}_1^{\text{sat}}(T) - \widehat{\tau}_2^{\text{sat}}(T)} = \frac{\varepsilon - \widehat{\varepsilon}_2^{\text{sat}}(T)}{\widehat{\varepsilon}_1^{\text{sat}}(T) - \widehat{\varepsilon}_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix}\widehat{\tau} \\ \widehat{\varepsilon}\end{pmatrix}_\alpha^{\text{sat}}(T)$$

Phase Change Equation: Summary

Phase Change Equation

$$\frac{\tau-\tau_{\text{vap}}^{\text{sat}}}{\tau_{\text{liq}}^{\text{sat}}-\tau_{\text{vap}}^{\text{sat}}}=\frac{\varepsilon-\varepsilon_{\text{vap}}^{\text{sat}}}{\varepsilon_{\text{liq}}^{\text{sat}}-\varepsilon_{\text{vap}}^{\text{sat}}}$$

with

$$T \mapsto \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_{\alpha}^{\text{sat}}(T)=\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_{\alpha}(T,P^{\text{sat}}(T))$$

or

$$P \mapsto \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_{\alpha}^{\text{sat}}(P)=\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_{\alpha}(T^{\text{sat}}(P),P)$$

Phase Change Equation: Summary

How to compute saturation functions τ_α^{sat} and $\varepsilon_\alpha^{\text{sat}}$

- Analytical EOS: we compute the saturation functions τ_α^{sat} and $\varepsilon_\alpha^{\text{sat}}$ by the Coexistence Curve:
  - Exact: $T \mapsto P^{\text{sat}}(T)$ or $P \mapsto T^{\text{sat}}(P)$

$$\begin{pmatrix}\tau\\\varepsilon\end{pmatrix}_\alpha^{\text{sat}}(P) = \begin{pmatrix}\tau\\\varepsilon\end{pmatrix}_\alpha(T^{\text{sat}}(P), P) \qquad \text{e.g. Simplified Stiffened Gases}$$

  - Approximated: $T \mapsto \widehat{P}^{\text{sat}}(T) \approx P^{\text{sat}}(T)$

$$\begin{pmatrix}\tau\\\varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \approx \begin{pmatrix}\tau\\\varepsilon\end{pmatrix}_\alpha(T, \widehat{P}^{\text{sat}}(T)) \qquad \text{e.g. General Stiffened Gases}$$

- Tabulated EOS: the saturation functions τ_α^{sat} and $\varepsilon_\alpha^{\text{sat}}$ are given by experiments and we set

$$\begin{pmatrix}\tau\\\varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T \text{ or } P) \approx \begin{pmatrix}\widehat{\tau}\\\widehat{\varepsilon}\end{pmatrix}_\alpha^{\text{sat}}(T \text{ or } P)$$

Outline

Dynamic Liquid-Vapor Phase Change

Euler System

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0, \\ \partial_t(\rho\mathbf{u}) + \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u} + P^{\text{eq}}\,\mathbb{I}) = 0 \\ \partial_t\left(\rho\left(\frac{|\mathbf{u}|^2}{2} + \varepsilon\right)\right) + \operatorname{div}\left(\rho\left(\frac{|\mathbf{u}|^2}{2}+\varepsilon\right)\mathbf{u} + P^{\text{eq}}\mathbf{u}\right) = 0 \end{cases} \quad \text{with} \quad P^{\text{eq}} \overset{\text{def}}{=} \frac{s_\tau^{\text{eq}}}{s_\varepsilon^{\text{eq}}}.$$

Properties

If $\tau_1^* \neq \tau_2^*$ and $\varepsilon_1^* \neq \varepsilon_2^*$ (first order phase transition) then

❶ $c(\mathbf{w}) > 0$, ❷ $s^{\text{eq}}_{\tau\varepsilon}(\mathbf{w}) > 0$

❶ Euler system: strict hyperbolicity ($\neq$ p-system),

❷ Riemann problem: multitude of entropy (Lax) solutions [R. Menikoff, B. J. Plohr], uniqueness of Liu solution.

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Outline

Relaxation Approach

$$\partial_t \mathbf{U} + \operatorname{div} \mathbf{F}(\mathbf{U}) = \mathbf{0}$$

Relaxation Approach

$$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \frac{1}{\mu}\mathbf{R}(\mathbf{V}) \quad \xrightarrow[\mu \to 0]{\text{Formally}} \quad \partial_t \mathbf{U} + \operatorname{div} \mathbf{F}(\mathbf{U}) = \mathbf{0}$$

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Equilibrium System

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho\mathbf{u}) = 0 \\ \partial_t(\rho\mathbf{u}) + \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u} + P^{\text{eq}}\mathbb{I}) = 0 \\ \partial_t(\rho e) + \operatorname{div}((\rho e + P^{\text{eq}})\mathbf{u}) = 0 \end{cases}$$

$$P^{\text{eq}}(\rho,\varepsilon) = \frac{s_\tau^{\text{eq}}}{s_\varepsilon^{\text{eq}}}, \quad e \stackrel{\text{def}}{=} \frac{|\mathbf{u}|^2}{2} + \varepsilon$$

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How to build the Augmented System

1. Lagrangian:
$$\mathscr{L}(\rho,\mathbf{u},\sigma,y,z,\psi) \stackrel{\text{def}}{=} \rho\left(\frac{|\mathbf{u}|^2}{2} - \varepsilon(\rho,\sigma,y,z,\psi)\right)$$

   Action:
$$\mathscr{A}(v) \stackrel{\text{def}}{=} \int_{t_1}^{t_2}\int_{\widehat{\Omega}(t;v)} \mathscr{L}(\widehat{\rho},\widehat{\rho\mathbf{u}},\widehat{s},\widehat{y},\widehat{z},\widehat{\psi})(\widehat{\mathbf{x}},t;v)\,\mathrm{d}\widehat{\mathbf{x}}\,\mathrm{d}t$$

   Minimization of the Action: $\frac{\mathrm{d}\mathscr{A}}{\mathrm{d}v}(v=0)=0$

2. Energy: $\varepsilon \stackrel{\text{def}}{=} \sum_\alpha y_\alpha \varepsilon_\alpha\left(\frac{z_\alpha}{y_\alpha}\frac{1}{\rho},\frac{\psi_\alpha}{y_\alpha}\sigma\right)$

3. Positive Entropy Production: $\mathrm{D}_t\sigma \geq 0$

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$$P(\rho,\varepsilon,z,y,\psi) = \frac{\sigma_\tau}{\sigma_\varepsilon}$$

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$$\begin{cases} \partial_t z + \mathbf{u} \cdot \mathbf{grad}\, z = \\ \partial_t y + \mathbf{u} \cdot \mathbf{grad}\, y = \\ \partial_t \psi + \mathbf{u} \cdot \mathbf{grad}\, \psi = \end{cases}$$

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NOTE: we can replace an EDP by an algebraic closure, for example

$$\cancel{\partial_t \psi + \mathbf{u}\cdot\mathbf{grad}\,\psi = \frac{1}{\mu_\psi}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\varepsilon} \rightsquigarrow T_1 = T_2.$$

Numerical Scheme

$$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{S}(\mathbf{V}) \quad + \frac{1}{\mu} \quad \mathbf{R}(\mathbf{V})$$

$\mathbf{V}_i^n \longrightarrow \mathbf{V}_i^{n+1}$

❶ $\mu_j \longrightarrow +\infty$

$\Downarrow$

$$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{S}(\mathbf{V})$$

Aug. System:	5-eq. iso-T
Num. Scheme:	op. splitting
$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{0}$	[G. ALLAIRE and all.]

$$\mathbf{S}(\mathbf{V}) = \mathbf{S}_{st}(\mathbf{V}) + \mathbf{S}_{heat}(\mathbf{V}) + \mathbf{S}_{g}(\mathbf{V})$$

$\partial_t \mathbf{V} = \mathbf{S}_{st}(\mathbf{V})$	[J. U. BRACKBILL and all.]
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$$\mathbf{R}(\mathbf{V}) = \mathbf{0}$$

update fractions (y, z, ψ) solving the Phase Change Equation

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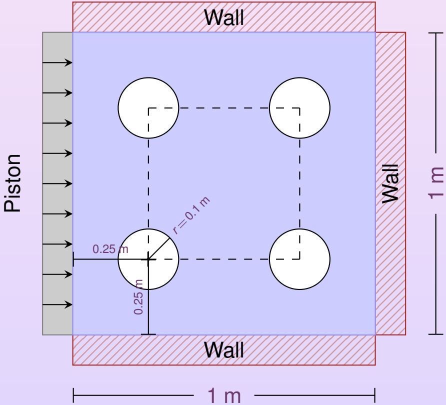
Outline

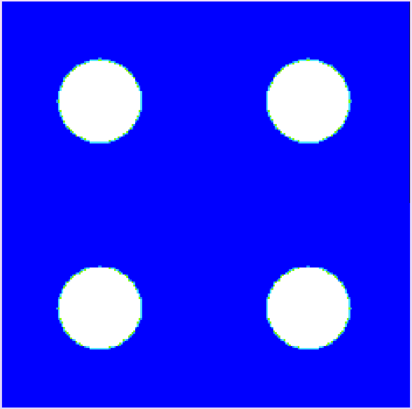
Compression of Vapor Bubbles



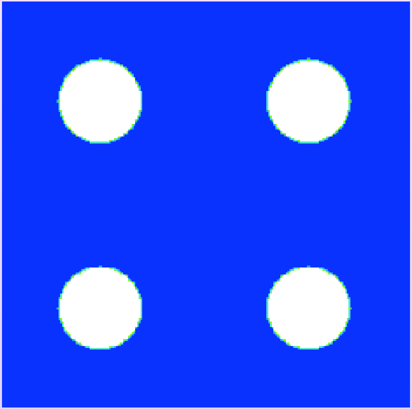
Compression of 4 Vapor Bubbles involving two Stiffened Gases for water and steam. The piston moves towards right at constant speed $u_p = 30$ m/s.

Compression of Vapor Bubbles

Mass Fraction y



Density ρ



◂ Geometry ▸ Play ▸▸ Skip

Compression of Vapor Bubbles

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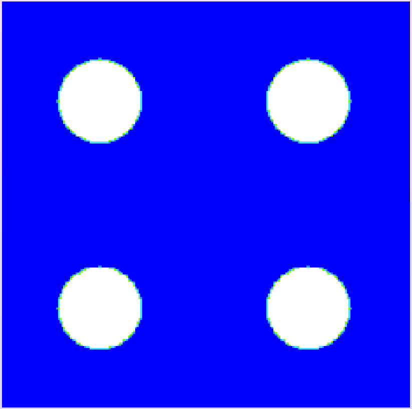

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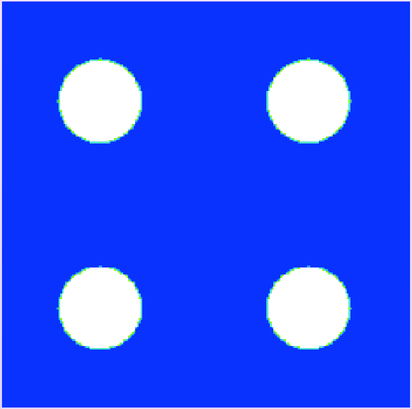

Geometry Play Skip

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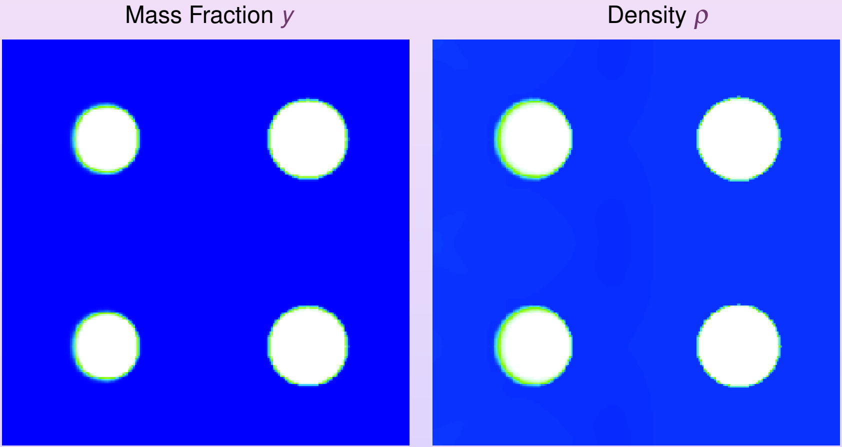

◀ Geometry ▶ Play ▶▶ Skip

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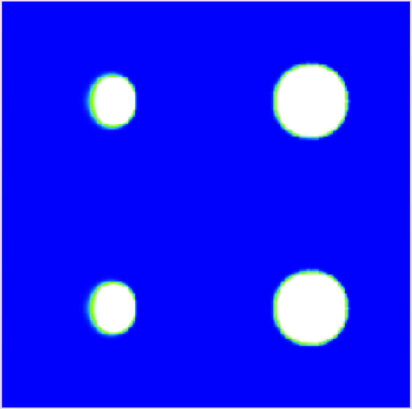

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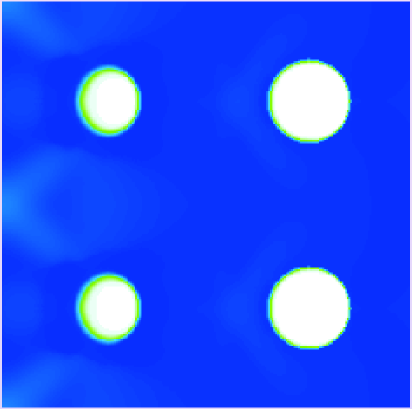

Geometry Play Skip

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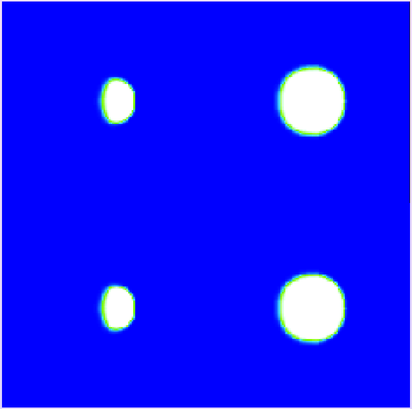

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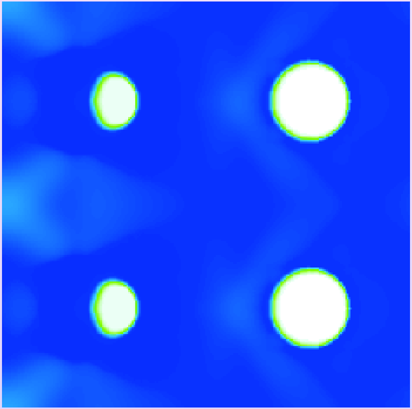
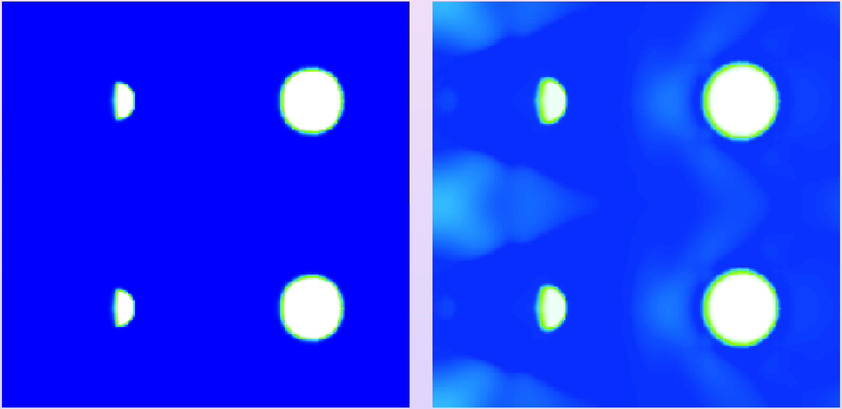


Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y Density ρ



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Compression of Vapor Bubbles

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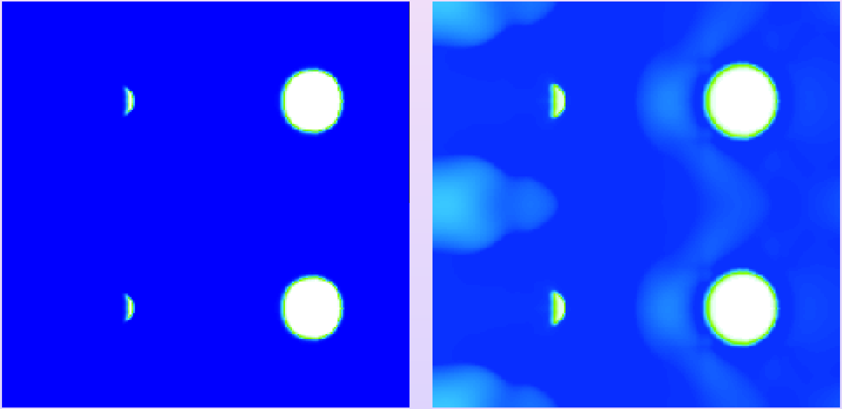

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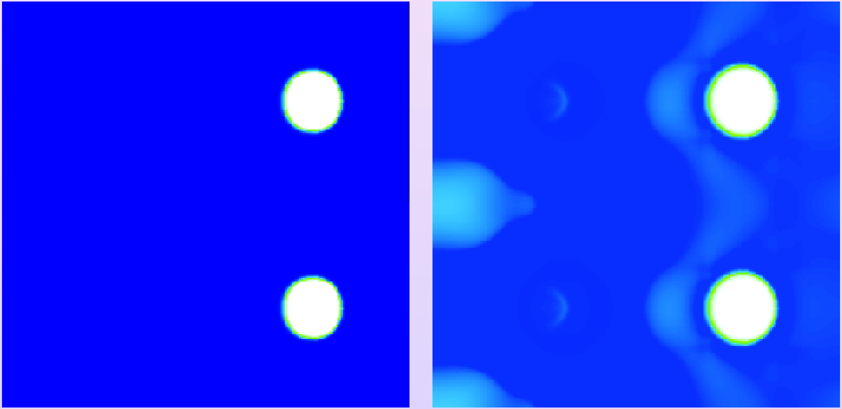

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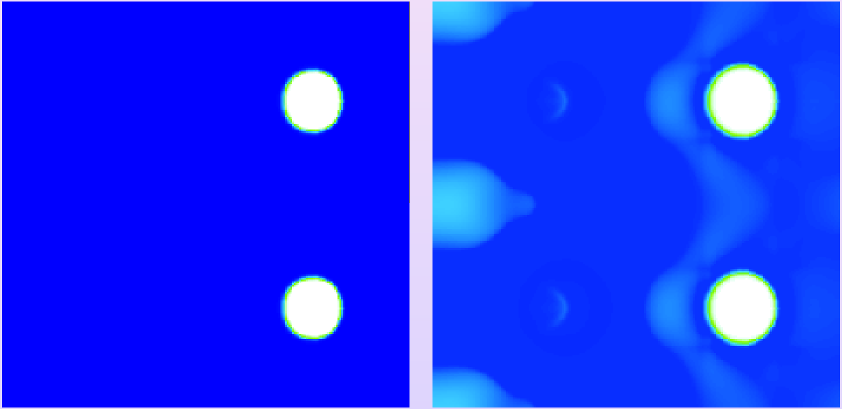

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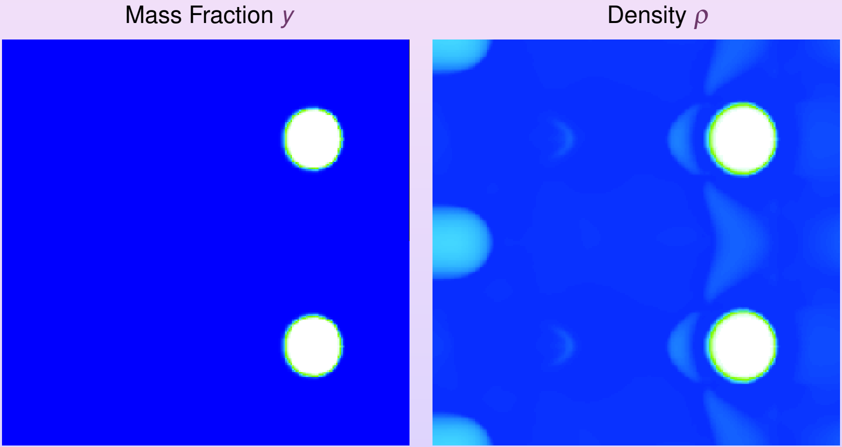

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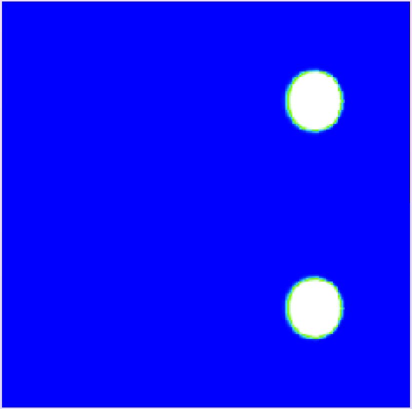
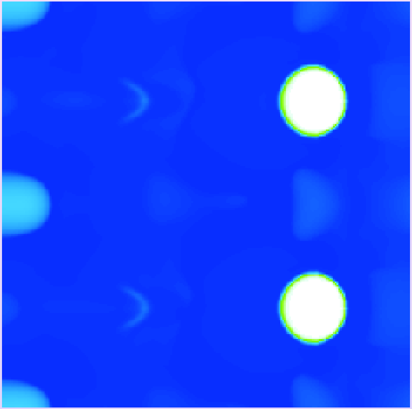


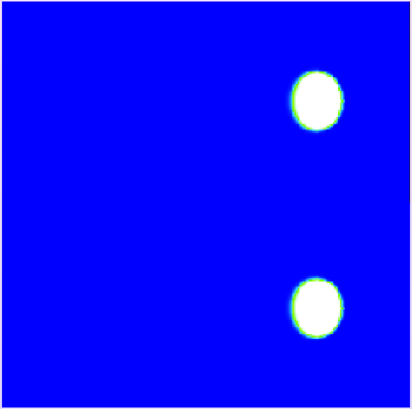
Density ρ



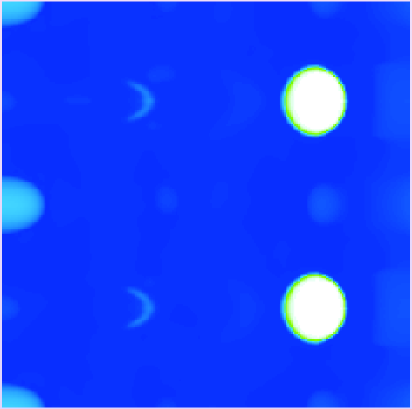
Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y



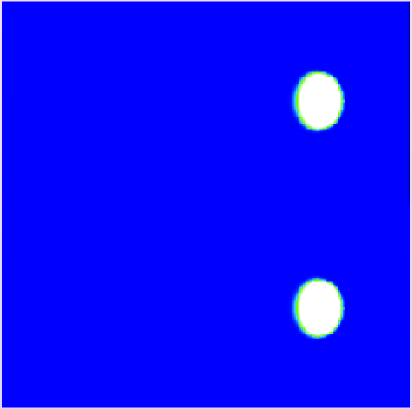
Density ρ



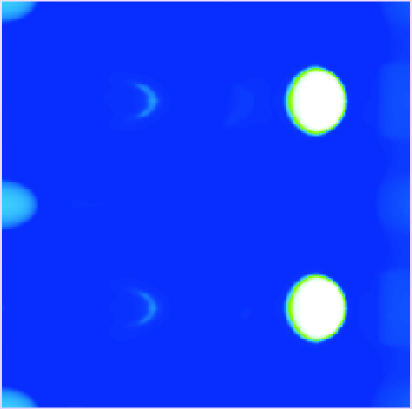
Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y

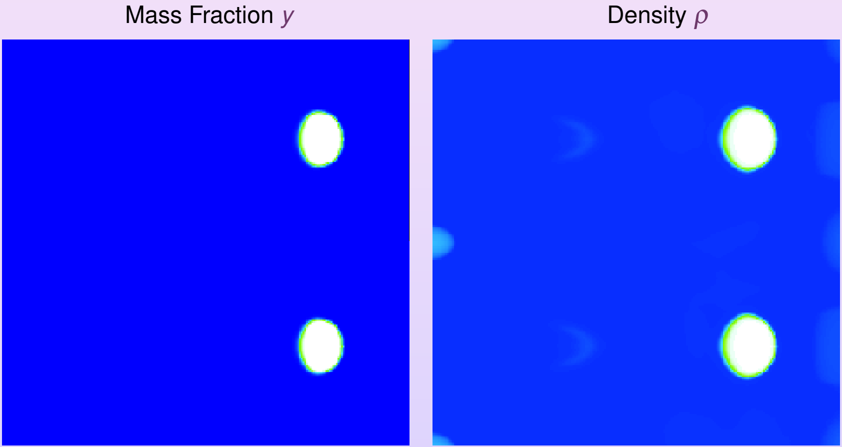


Density ρ



◀ Geometry ▶ Play ▶▶ Skip

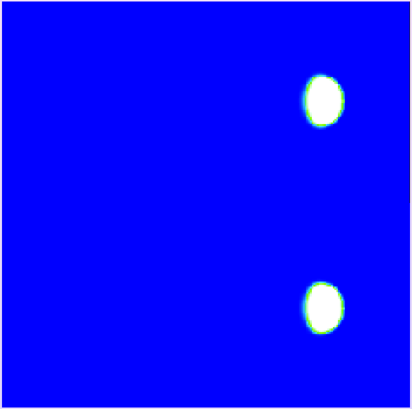
Compression of Vapor Bubbles



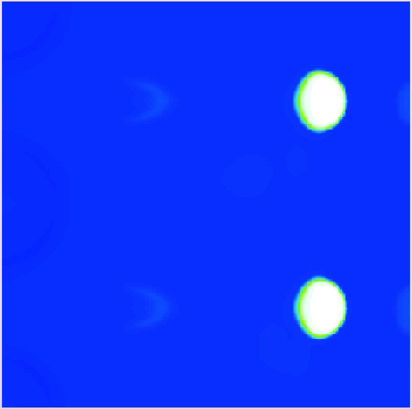
Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y



Density ρ

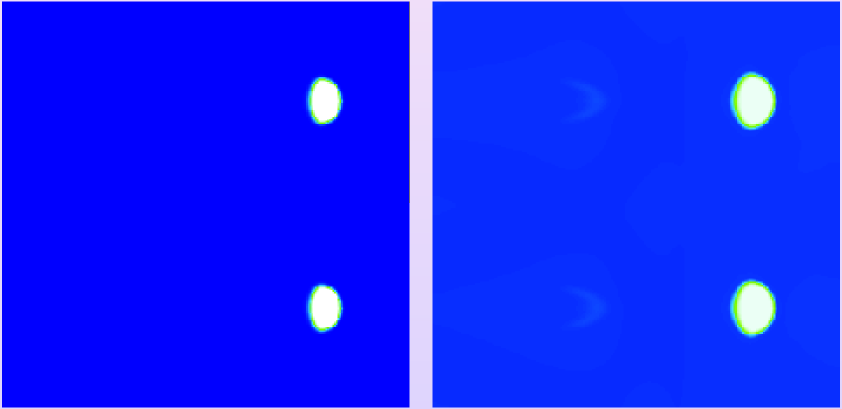


◂ Geometry ▸ Play ▸▸ Skip

Compression of Vapor Bubbles

Mass Fraction y

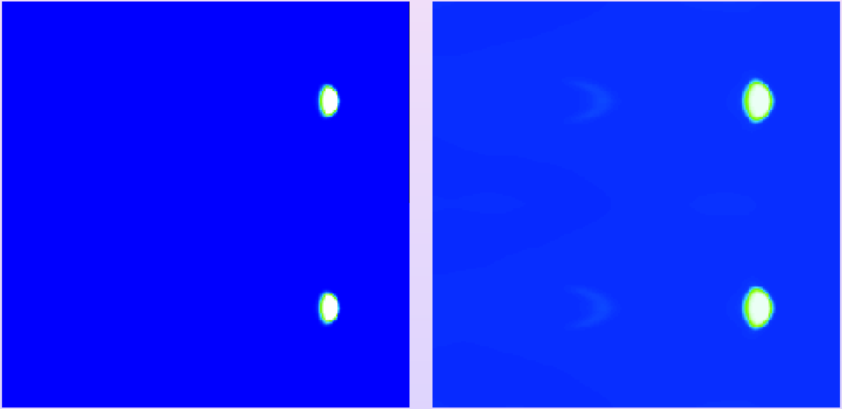
Density ρ



◀ Geometry ▶ Play ▶▶ Skip

Compression of Vapor Bubbles

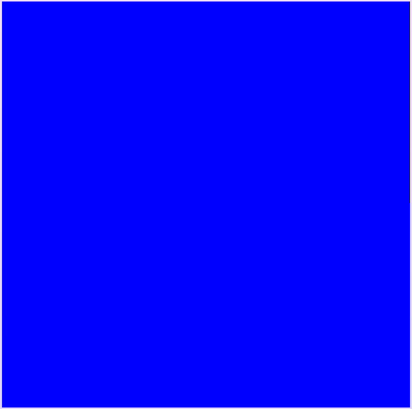
Mass Fraction y Density ρ



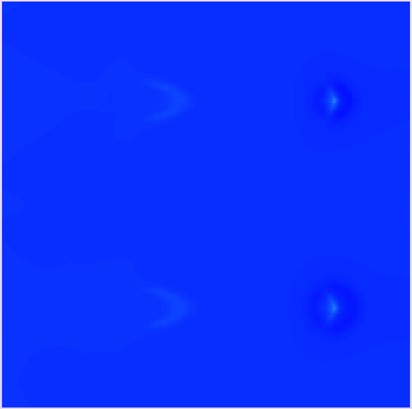
Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y



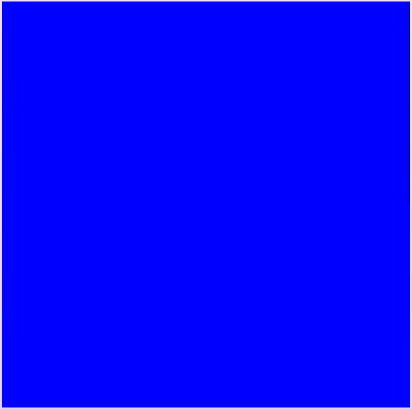
Density ρ



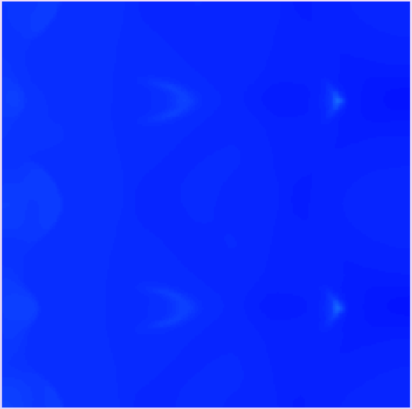
Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y



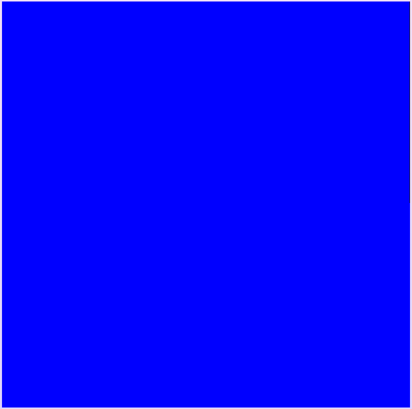
Density ρ



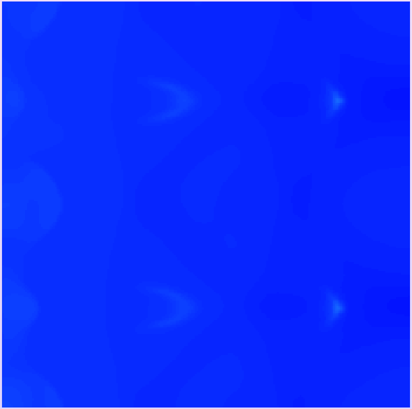
Geometry Play Skip

Compression of Vapor Bubbles

Mass Fraction y



Density ρ

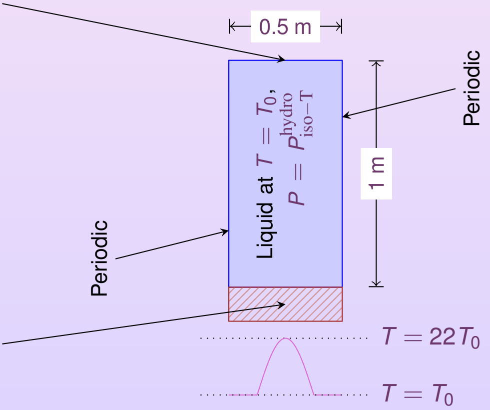


Geometry Play Skip

Nucleating Bubble

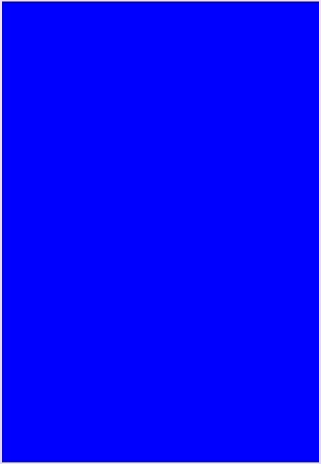
Pressure and temperature imposed
$P = P^{\text{ref}} > P^{\text{sat}}(T_0)$,
$T = T_0$

Wall,
temperature imposed

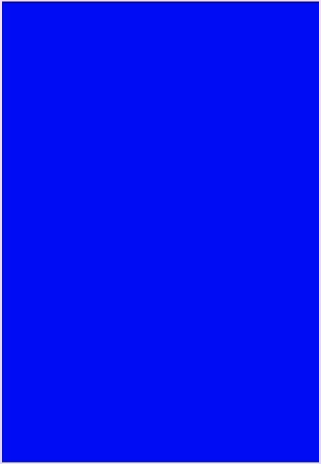


Nucleating Bubble

Mass Fraction y



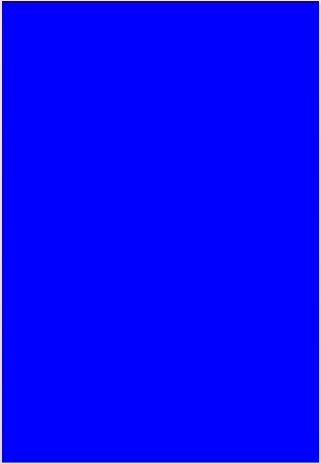
Temperature T



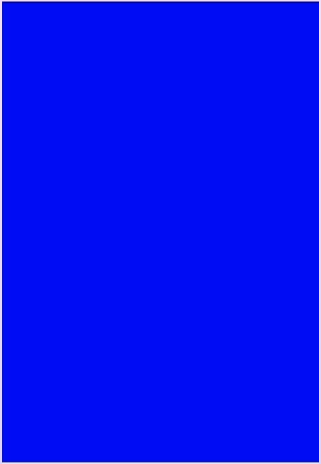
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

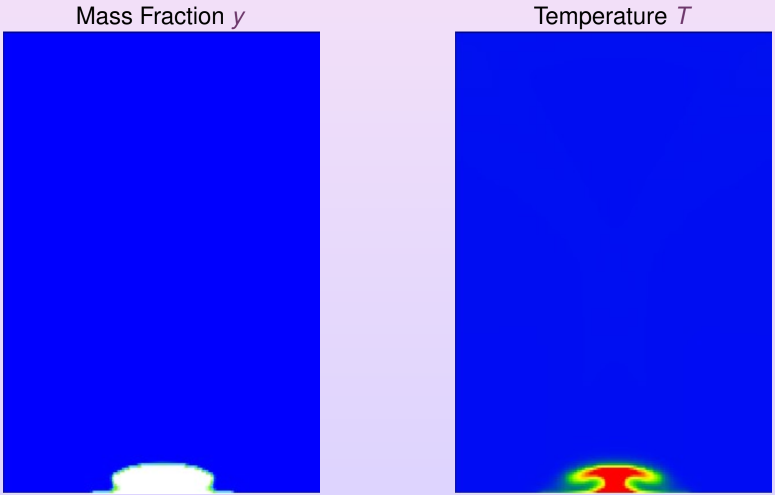


Temperature T



◂ Geometry ▸ Play ▸▸ Skip

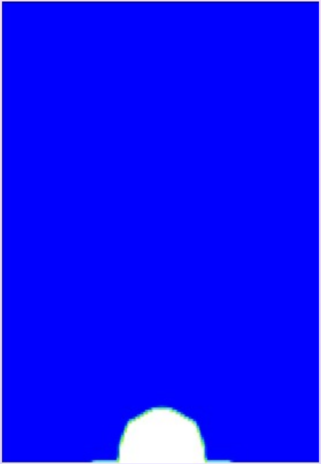
Nucleating Bubble



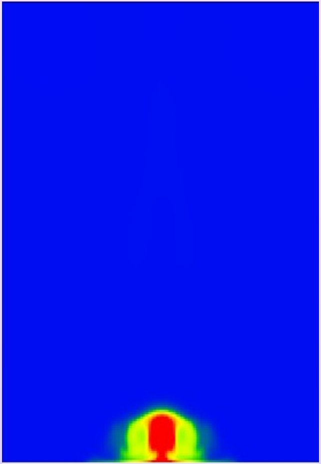
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



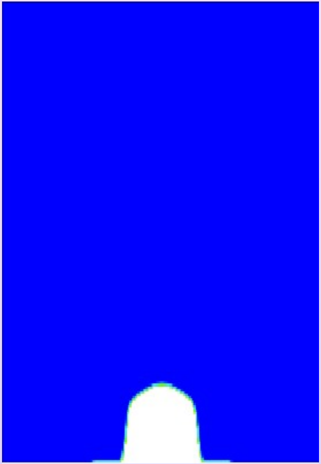
Temperature T



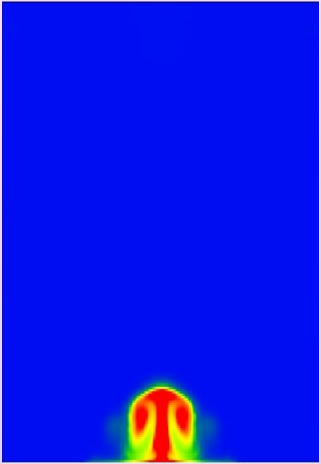
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



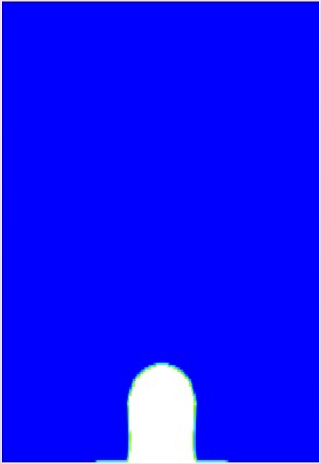
Temperature T



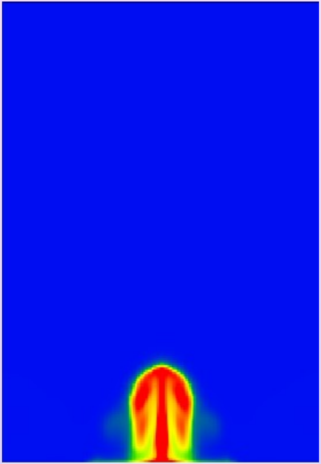
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



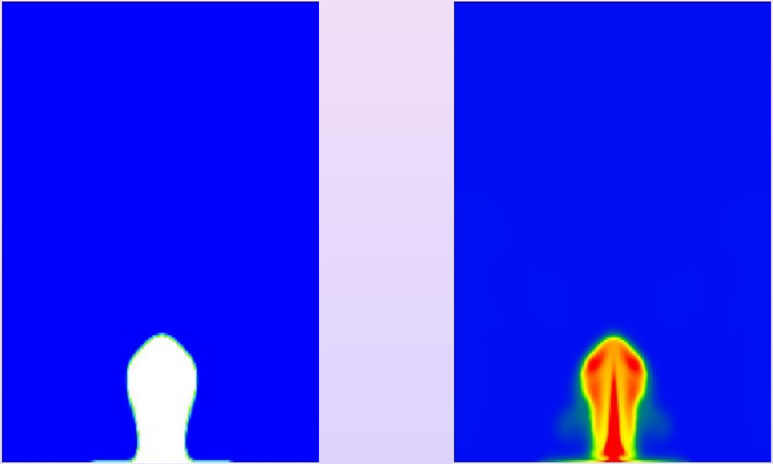
Temperature T



Geometry Play Skip

Nucleating Bubble

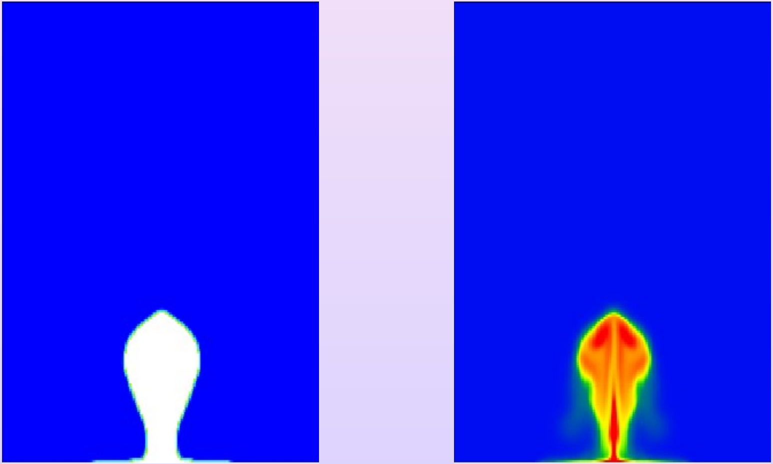
Mass Fraction y Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

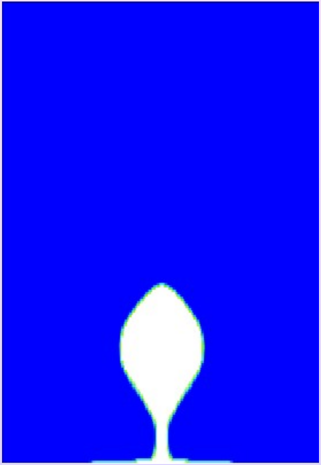
Mass Fraction y Temperature T



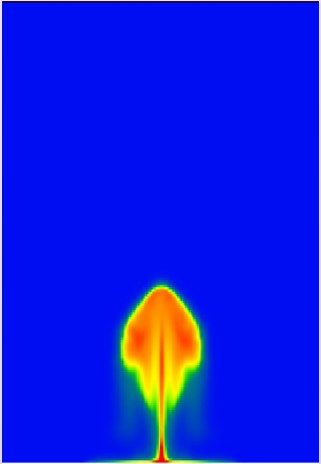
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



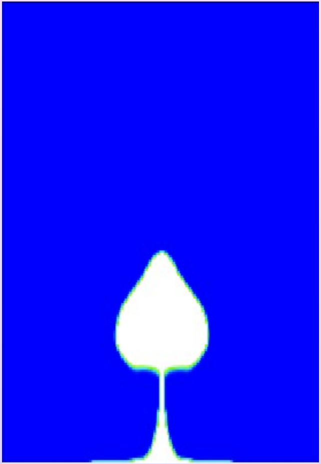
Temperature T



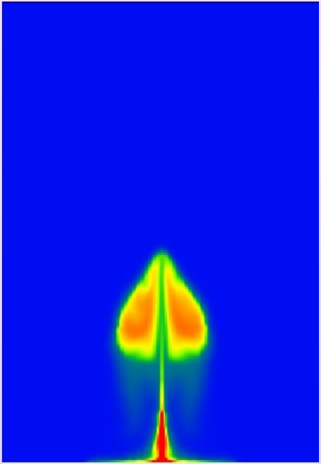
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



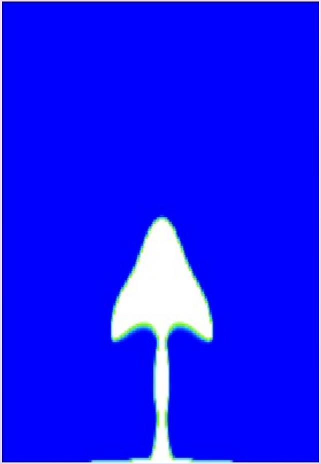
Temperature T



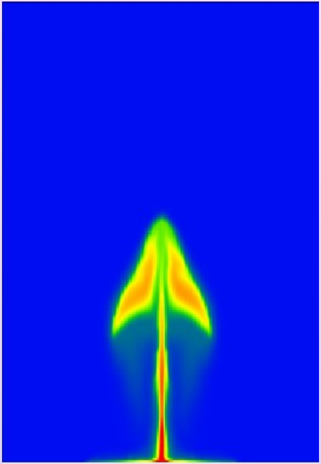
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

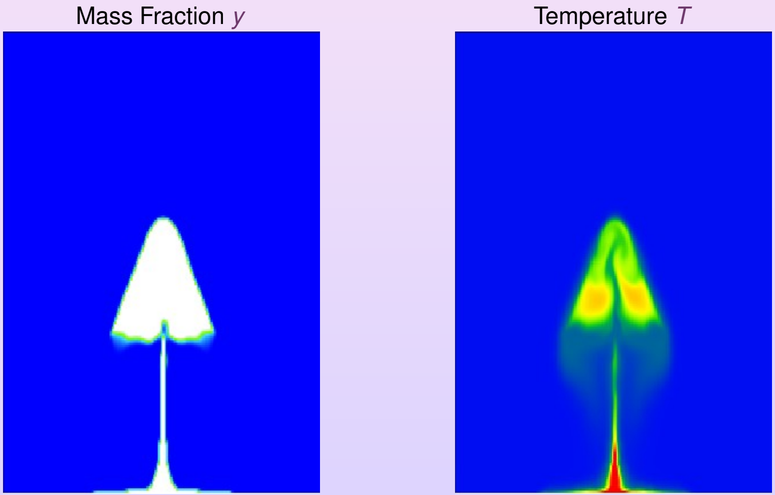


Temperature T



Geometry Play Skip

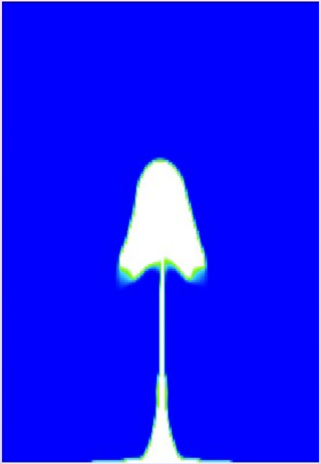
Nucleating Bubble



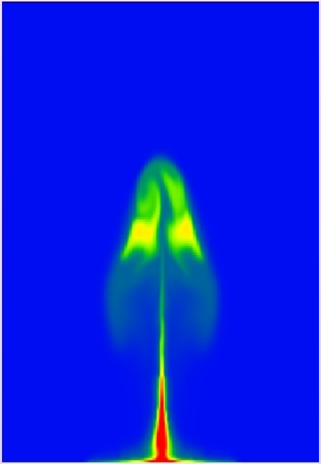
◀ Geometry ▶ Play ⏩ Skip

Nucleating Bubble

Mass Fraction y



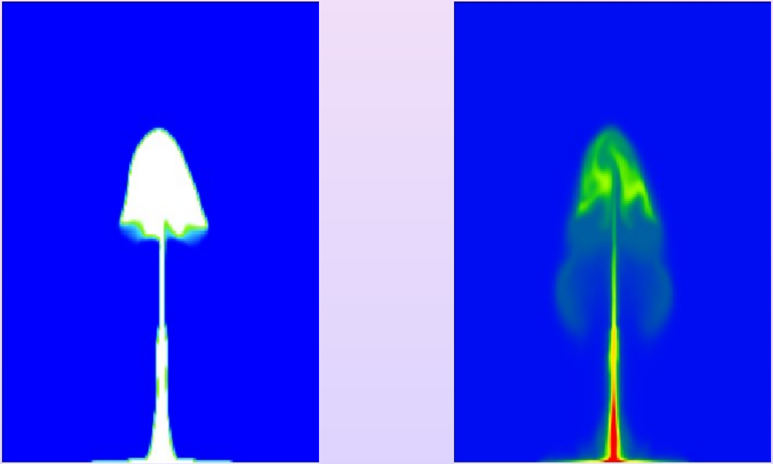
Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

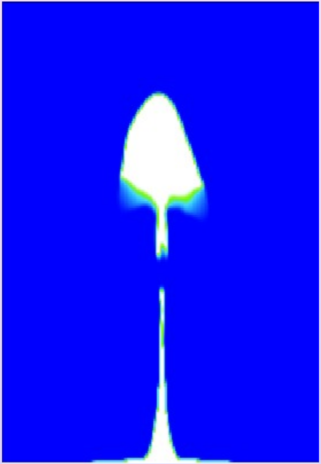
Mass Fraction y Temperature T



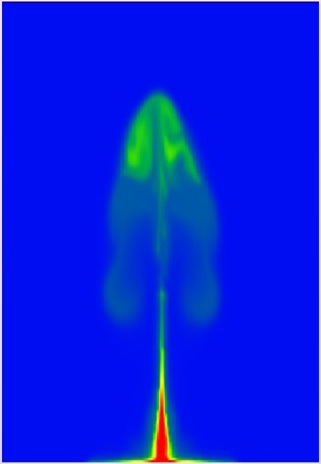
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



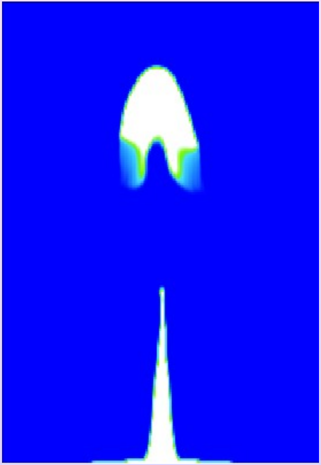
Temperature T



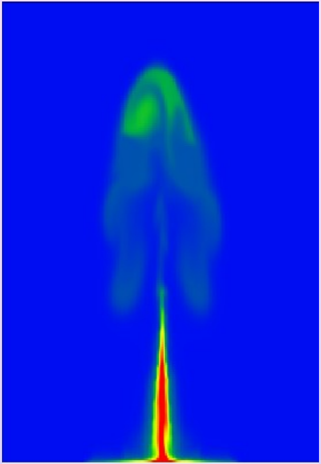
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



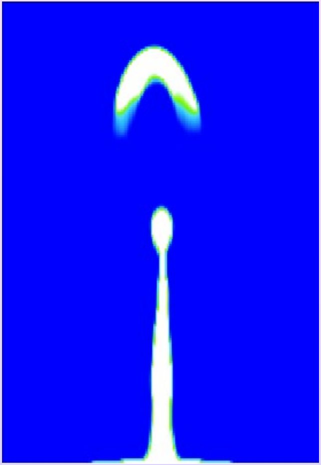
Temperature T



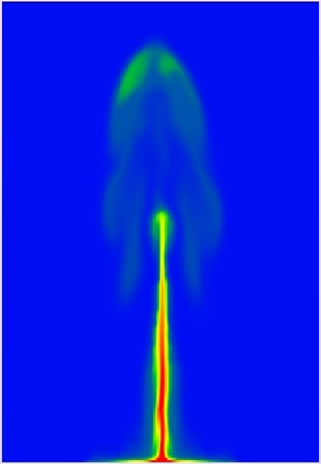
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



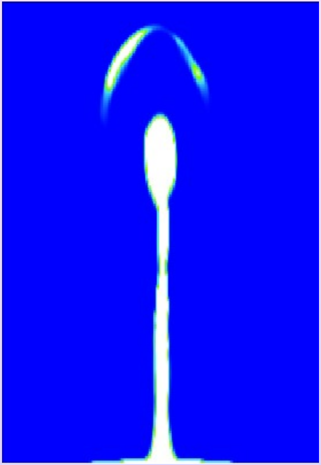
Temperature T



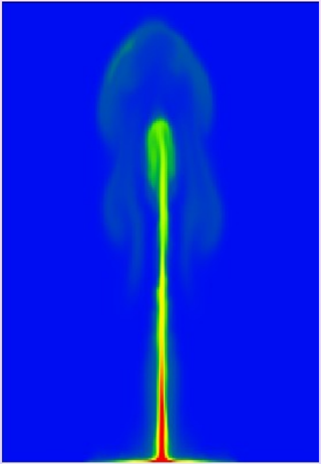
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



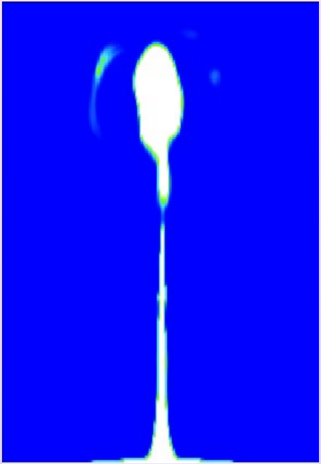
Temperature T



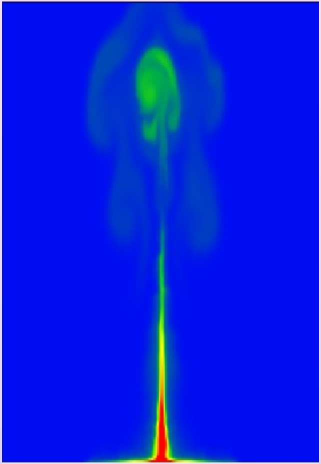
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



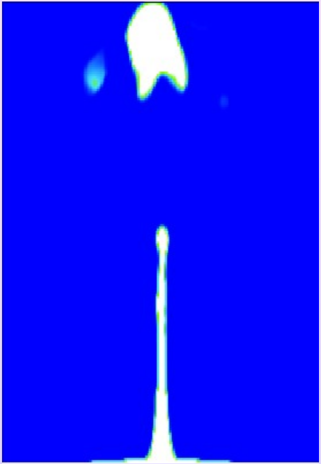
Temperature T



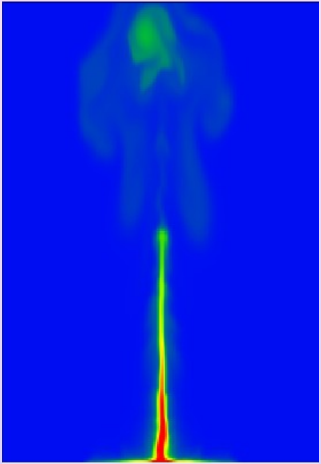
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



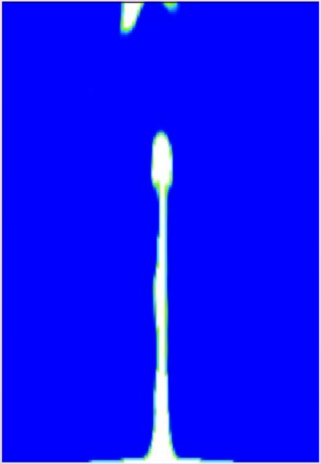
Temperature T



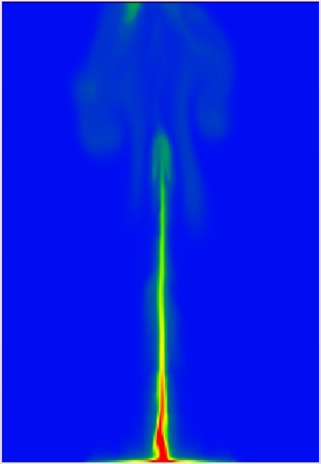
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



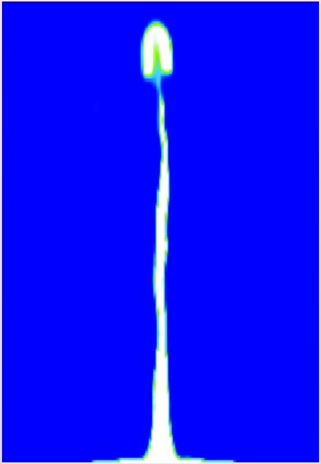
Temperature T



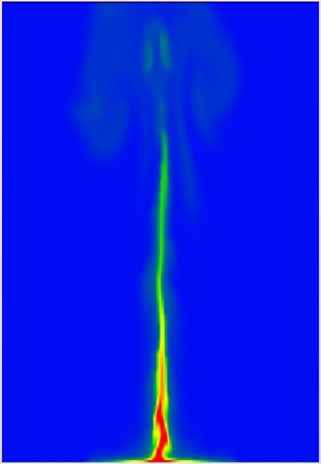
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



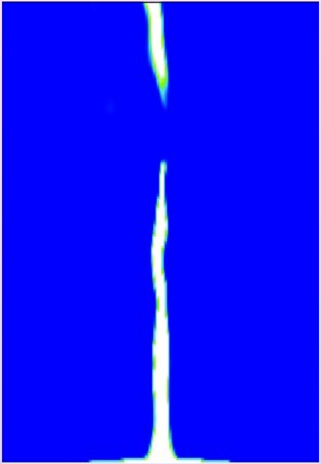
Temperature T



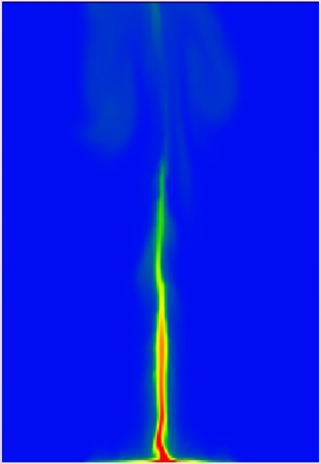
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



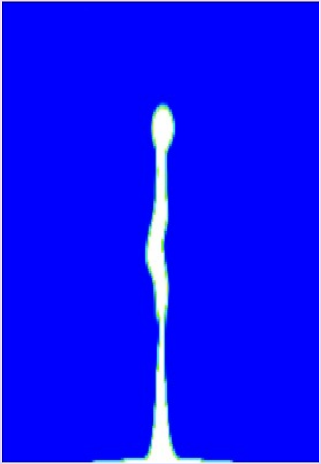
Temperature T



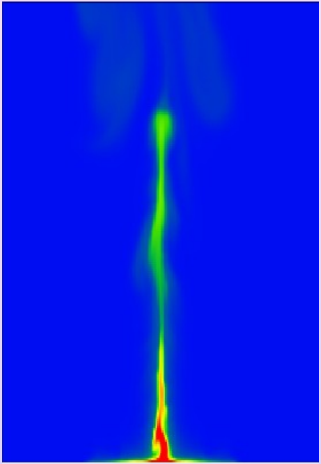
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

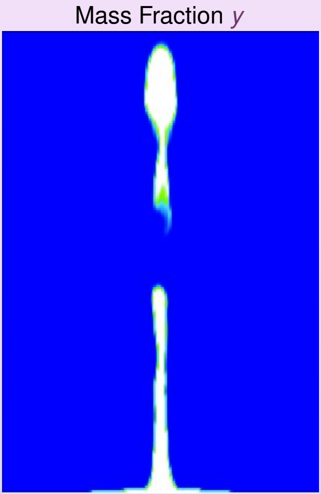


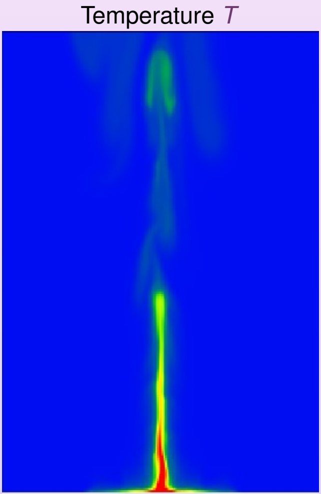
Temperature T



Geometry Play Skip

Nucleating Bubble

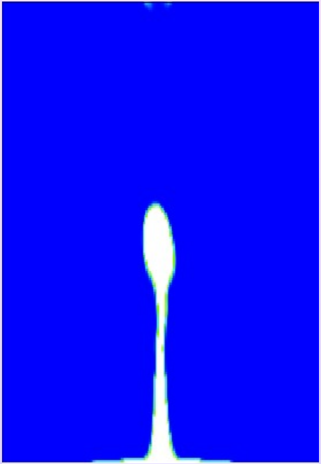




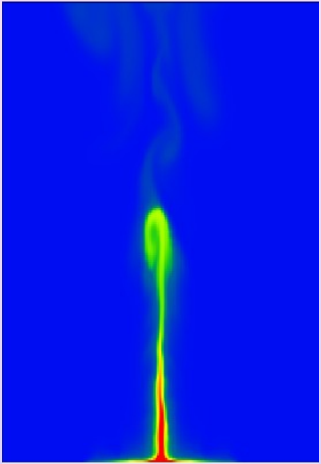
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



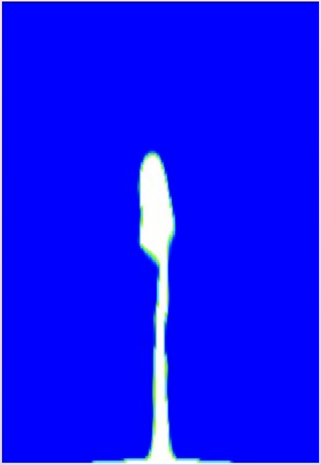
Temperature T



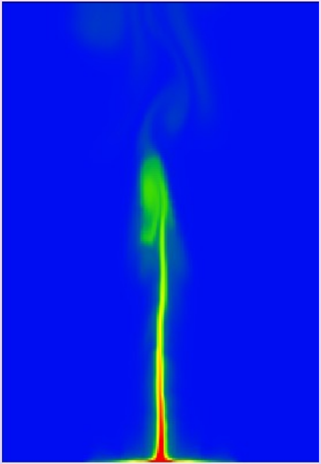
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



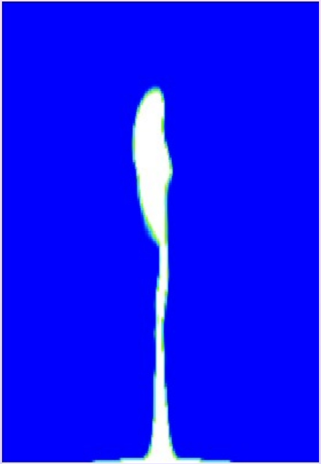
Temperature T



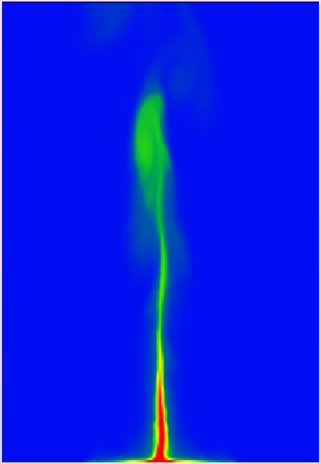
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



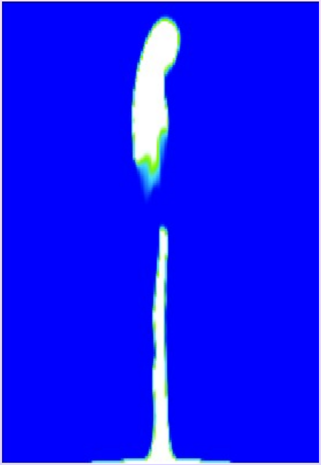
Temperature T



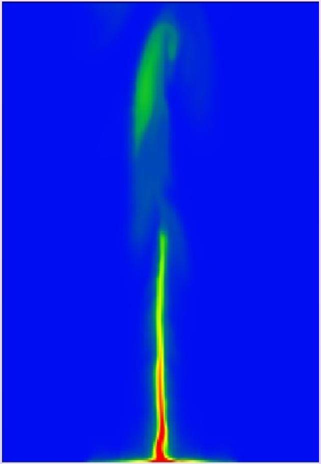
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



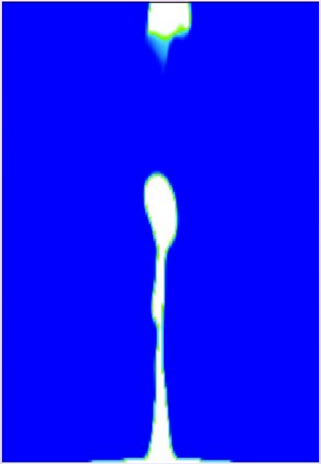
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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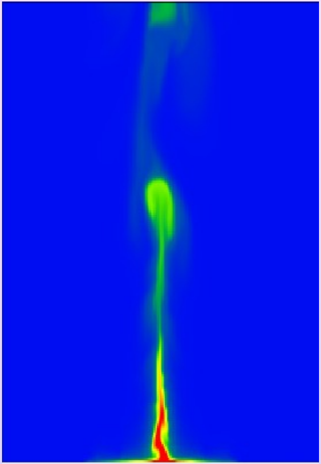
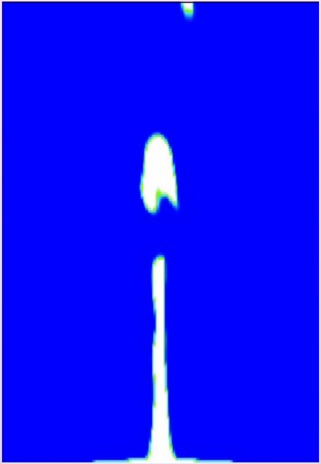


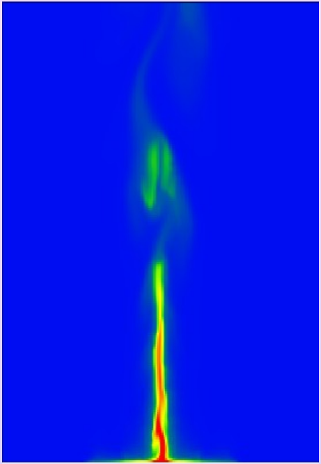
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

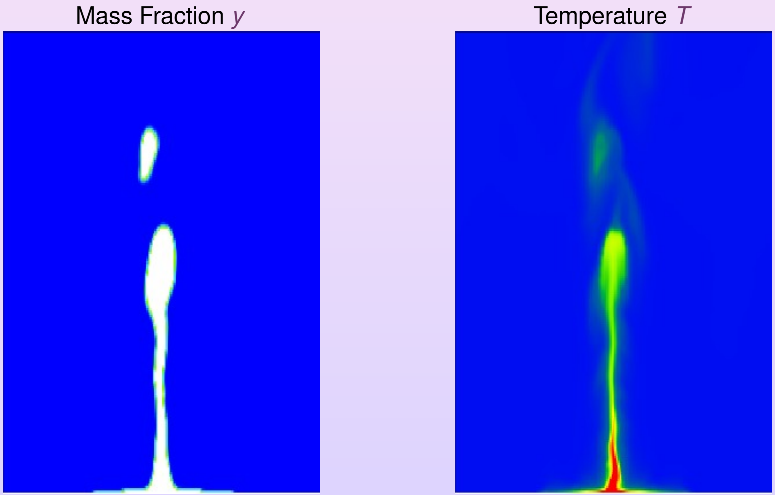


Temperature T



Geometry Play Skip

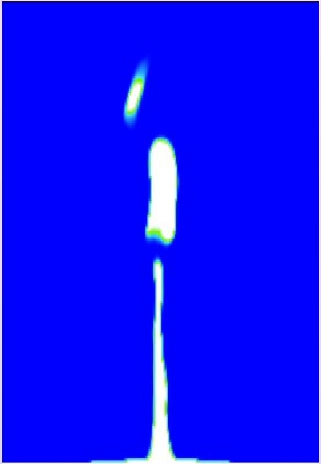
Nucleating Bubble



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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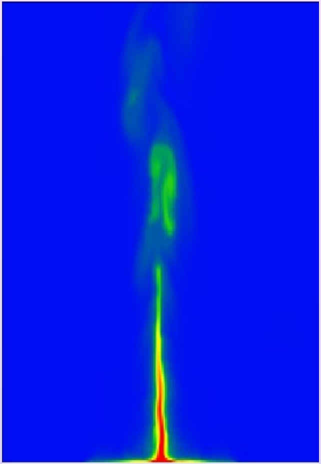
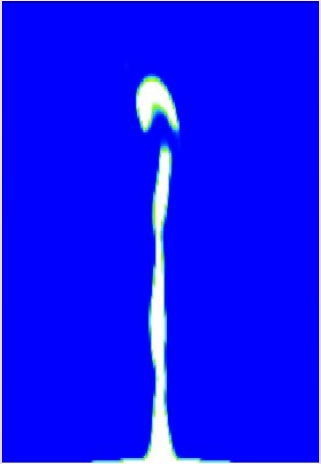


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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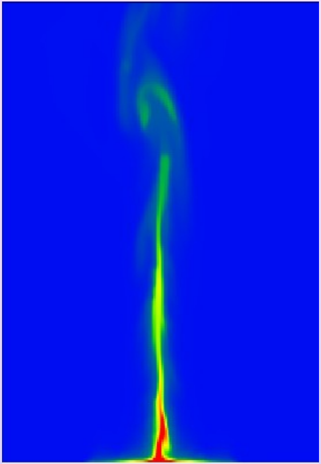
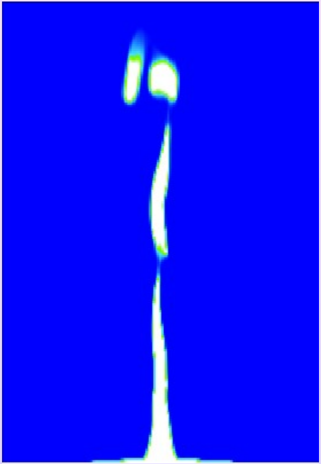


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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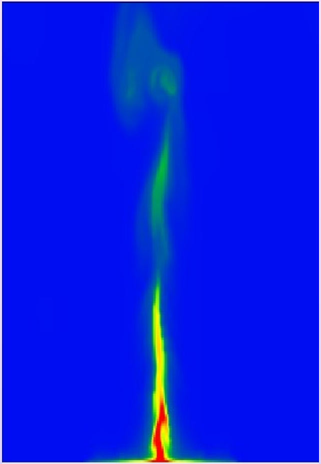
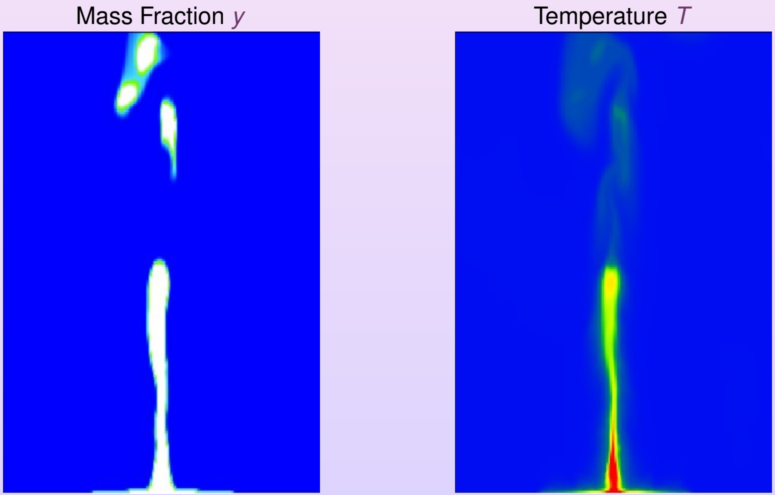


◂ Geometry ▸ Play ▸▸ Skip

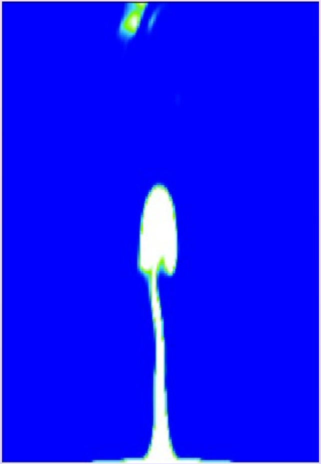
Nucleating Bubble



◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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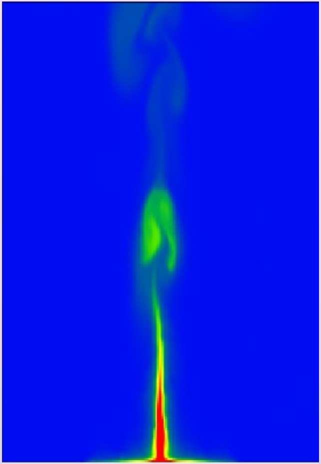
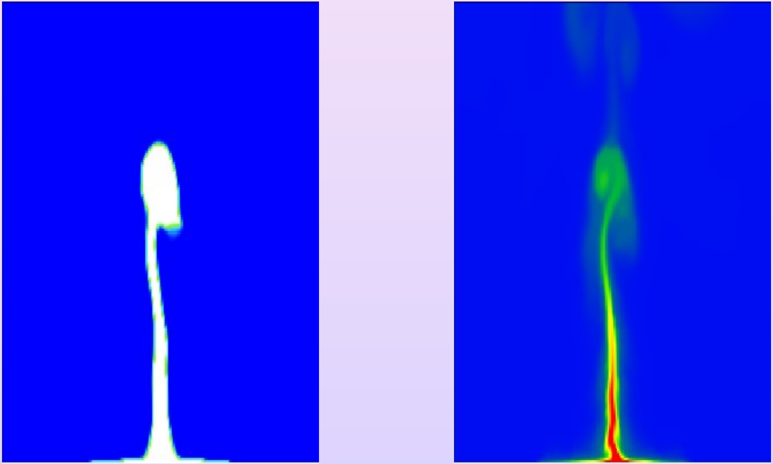


Geometry Play Skip

Nucleating Bubble

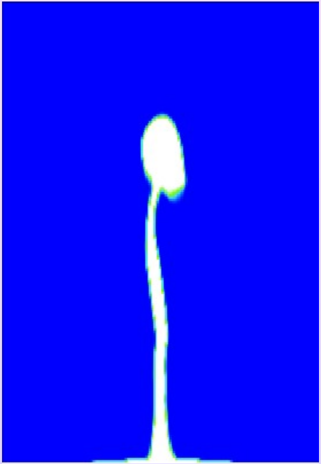
Mass Fraction y Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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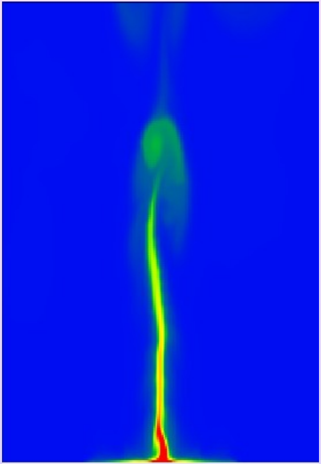
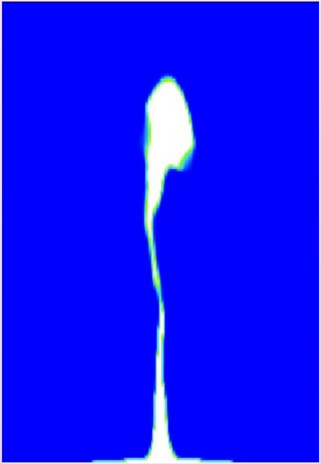


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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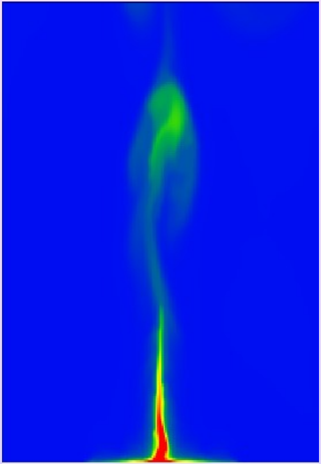
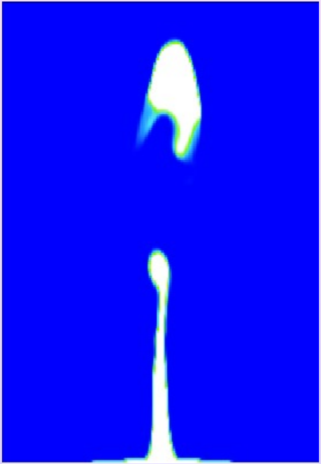


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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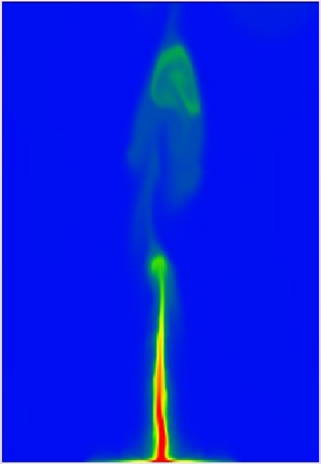
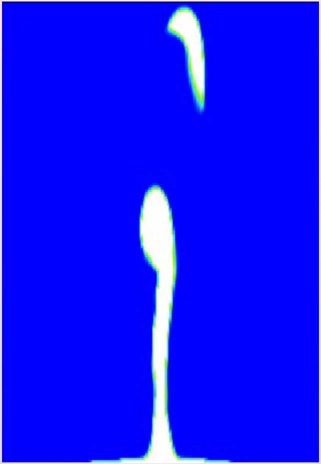


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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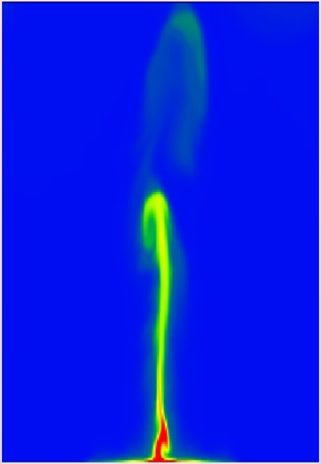
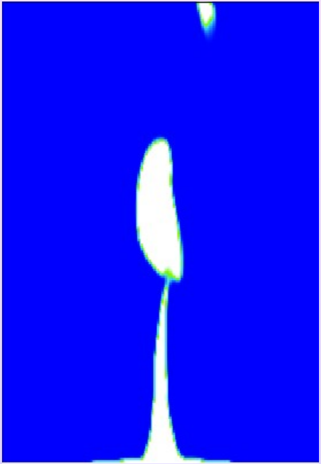


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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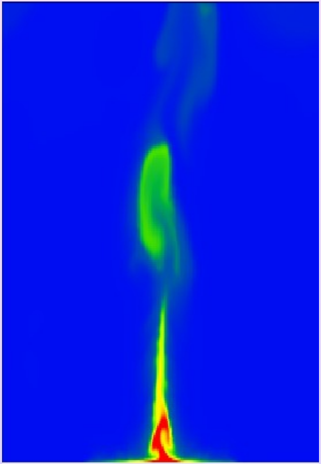
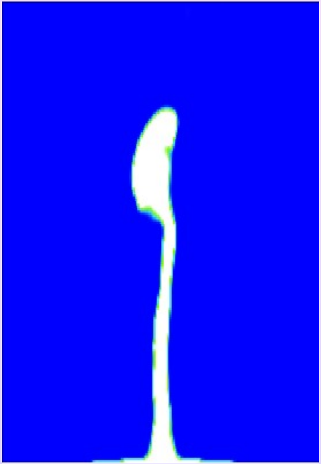


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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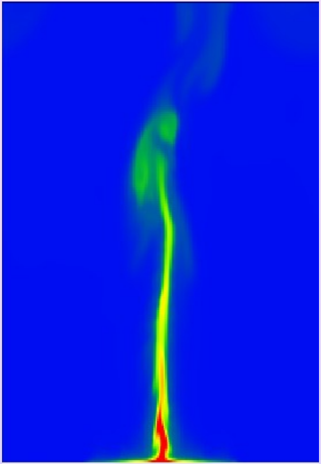
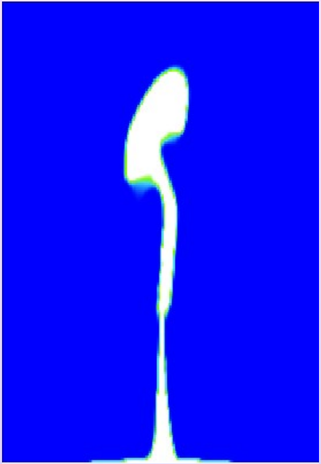


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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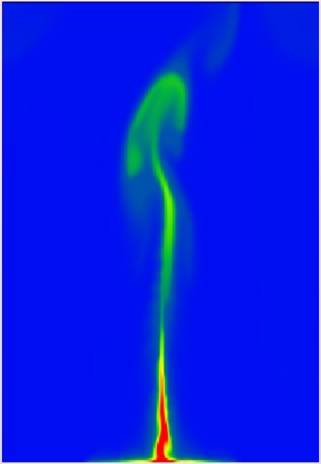
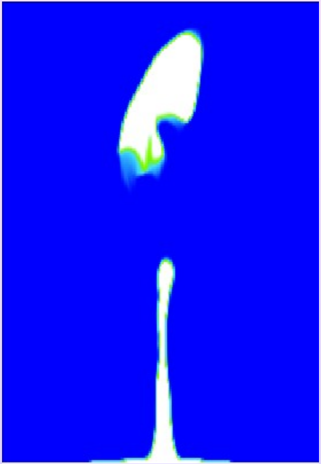


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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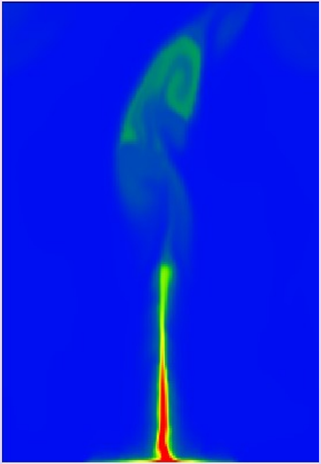
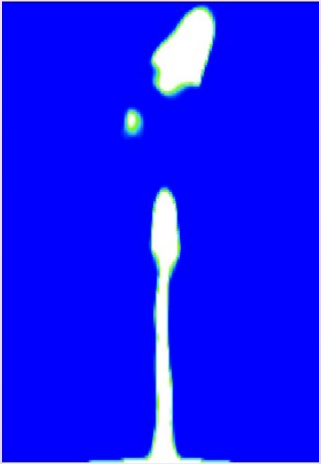


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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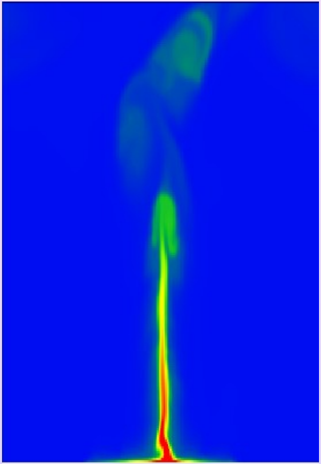
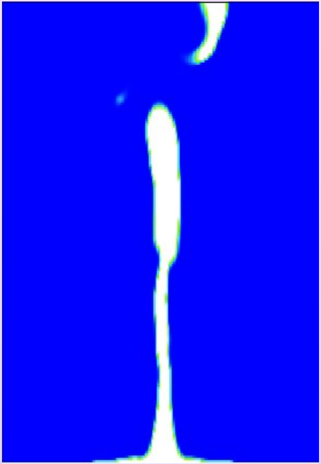


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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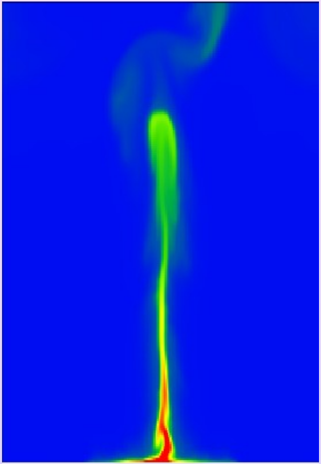
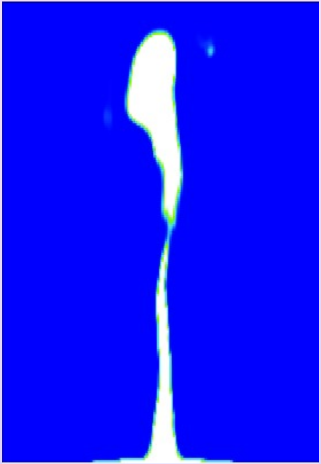


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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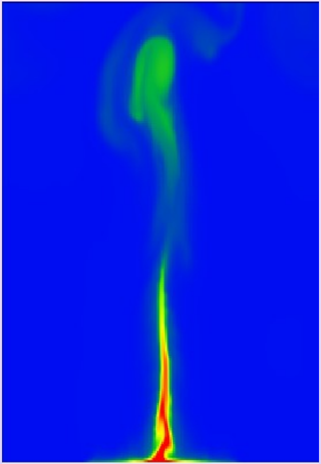
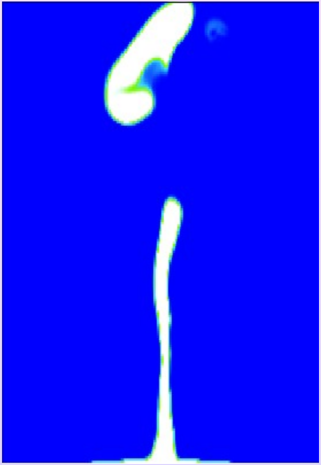


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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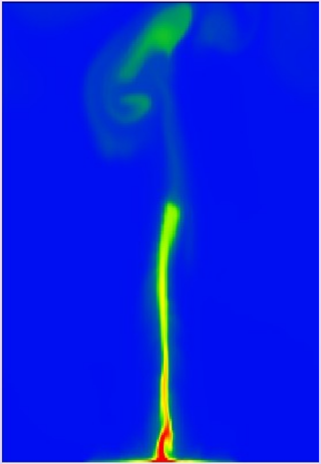
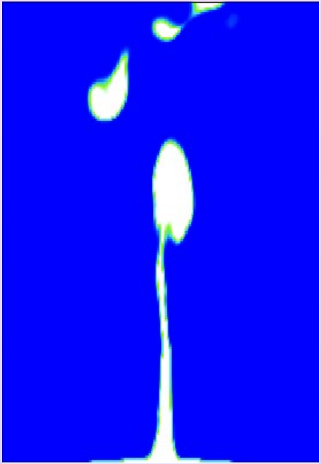


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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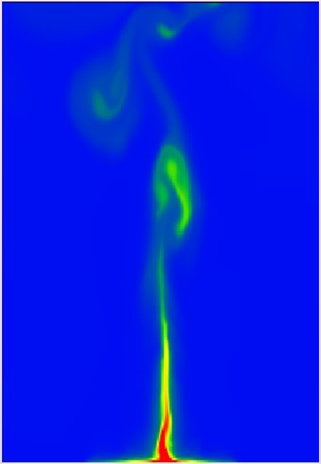
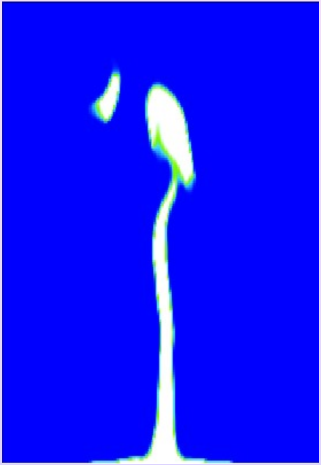


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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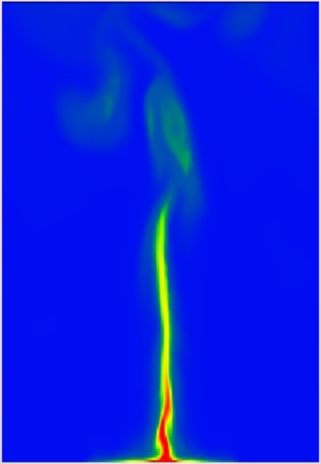
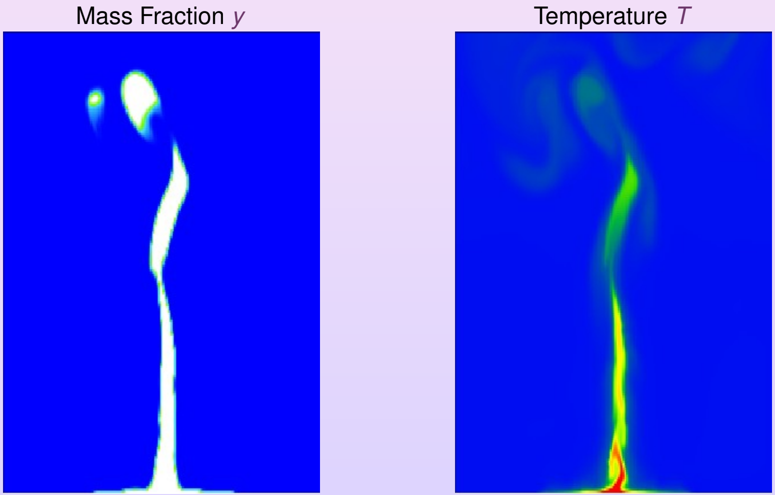


Geometry Play Skip

Nucleating Bubble

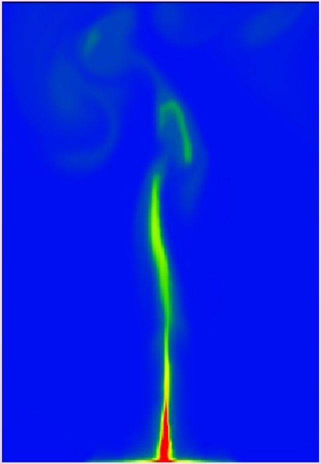


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

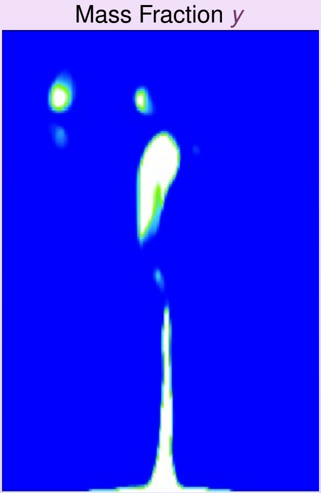
Mass Fraction y

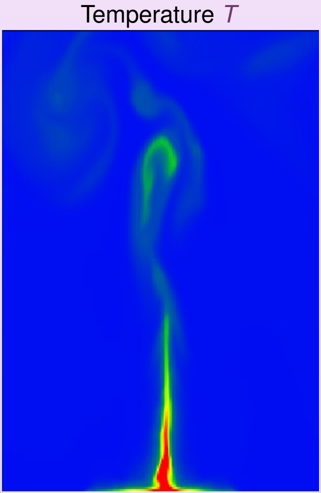
Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

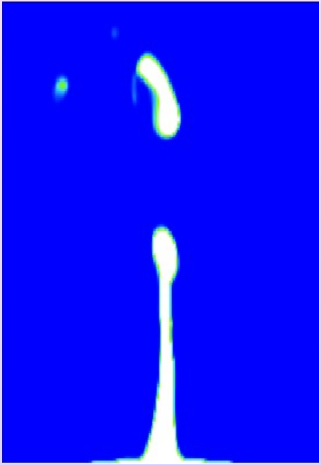




Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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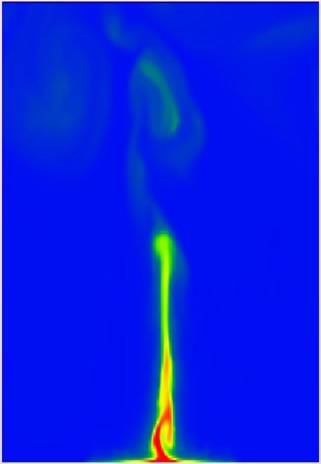
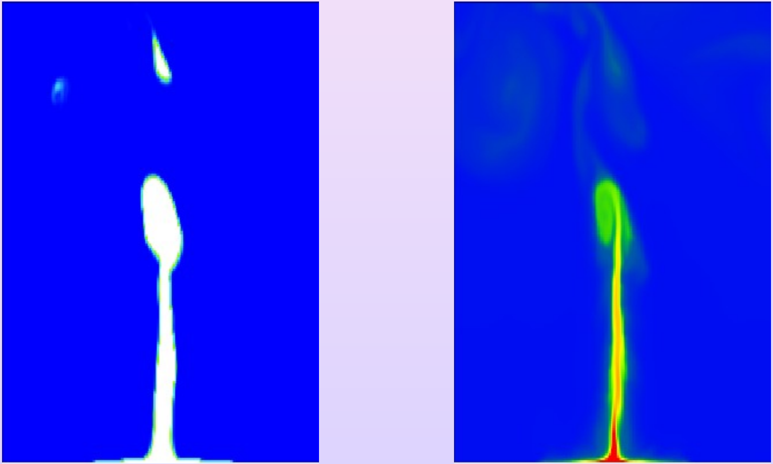


Geometry Play Skip

Nucleating Bubble

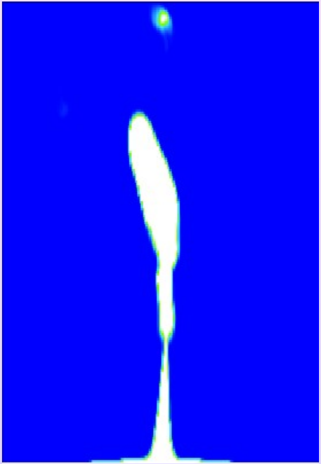
Mass Fraction y | Temperature T



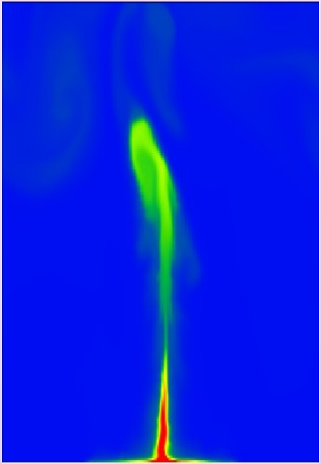
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



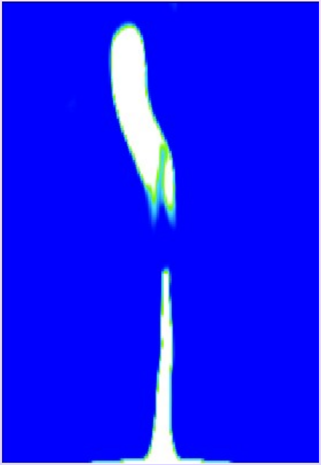
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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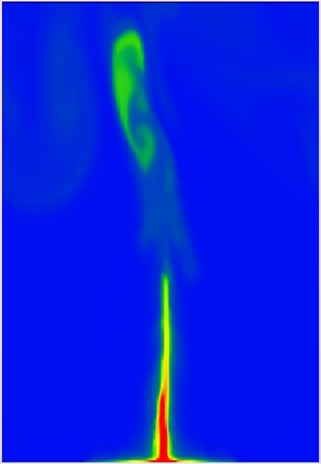
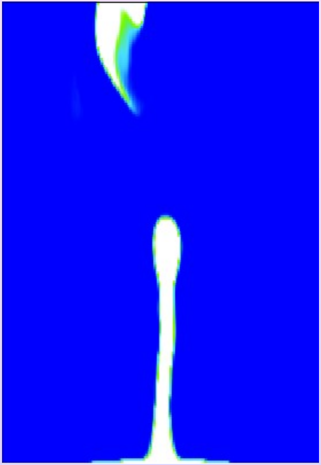


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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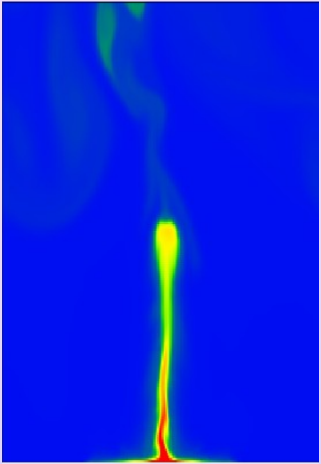
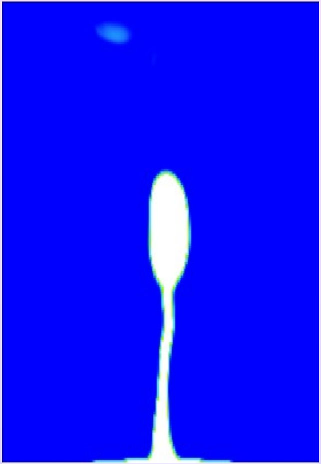


◀ Geometry ▶ Play ▶▶ Skip

Nucleating Bubble

Mass Fraction y



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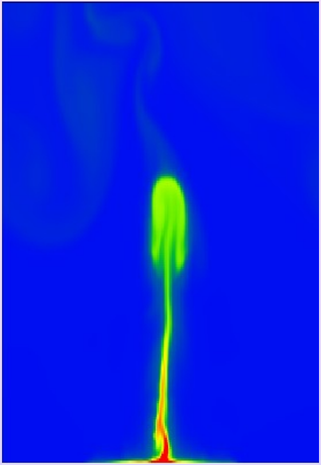
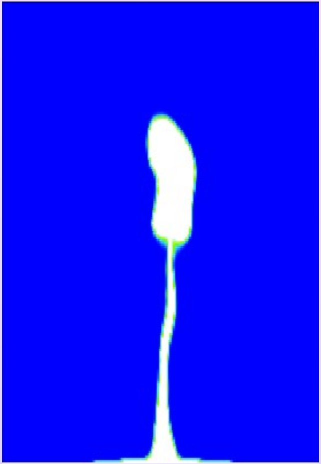


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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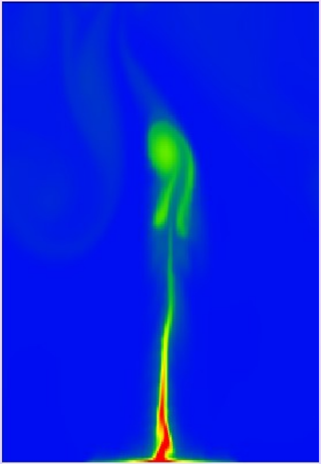
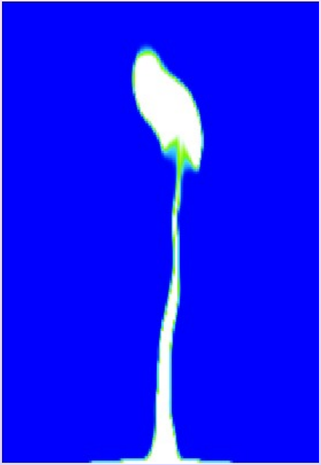


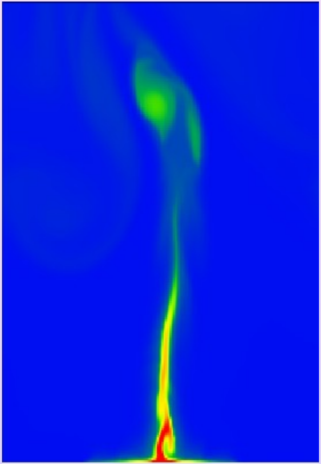
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



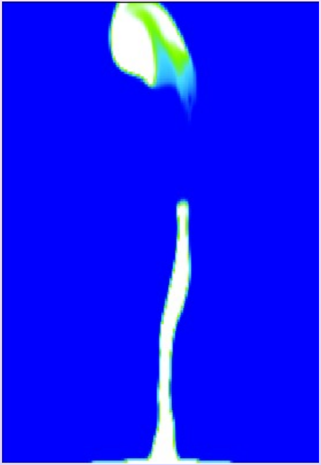
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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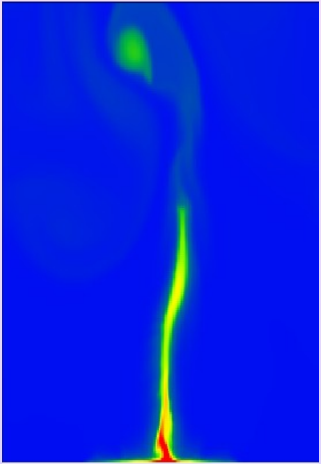
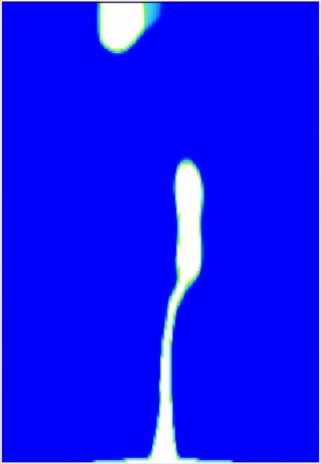


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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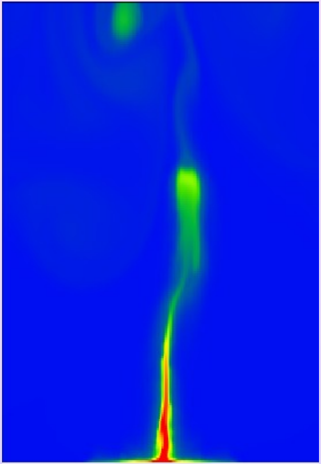
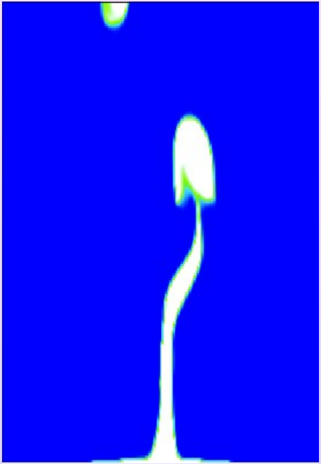


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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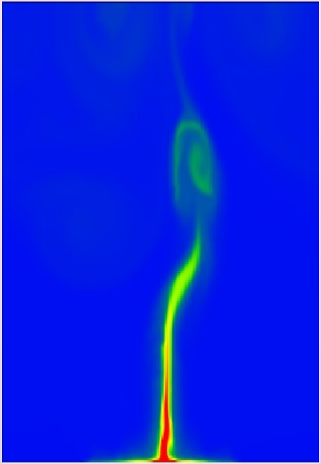
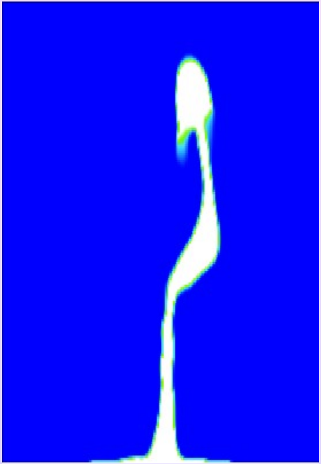


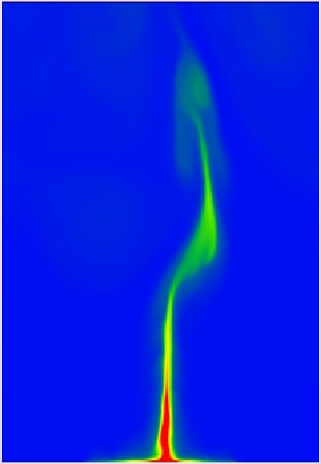
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



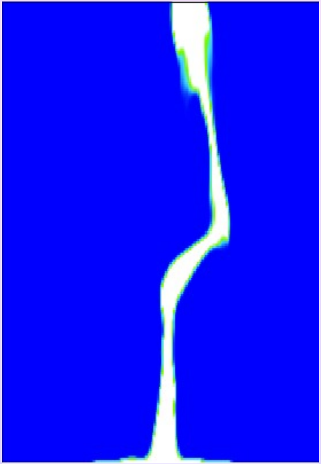
Temperature T



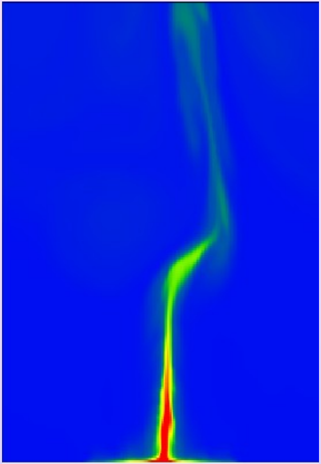
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

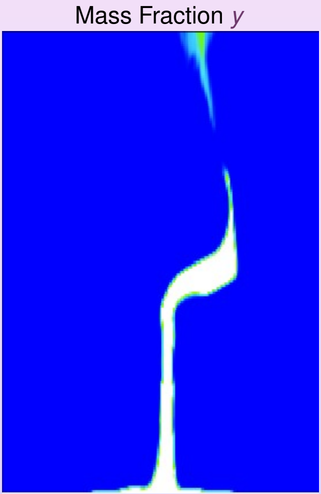


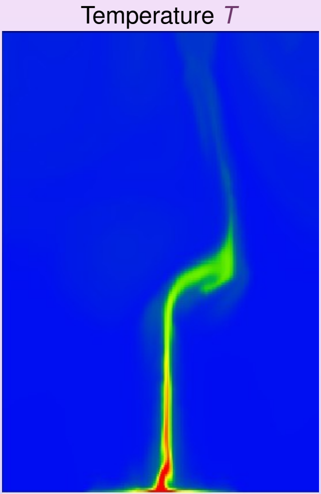
Temperature T



Geometry Play Skip

Nucleating Bubble

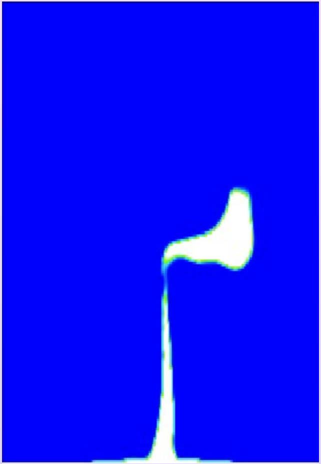




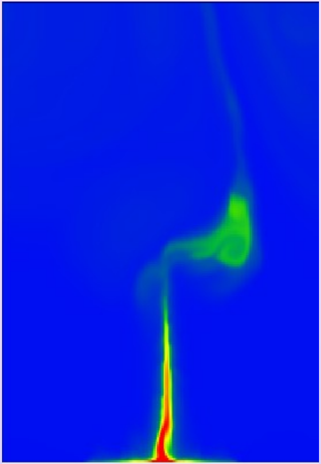
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



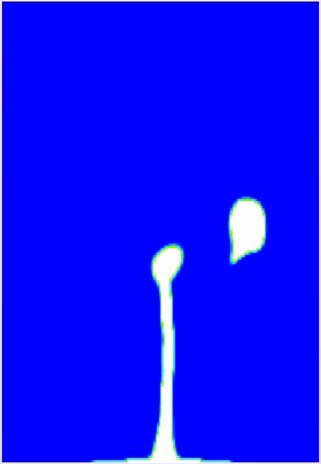
Temperature T



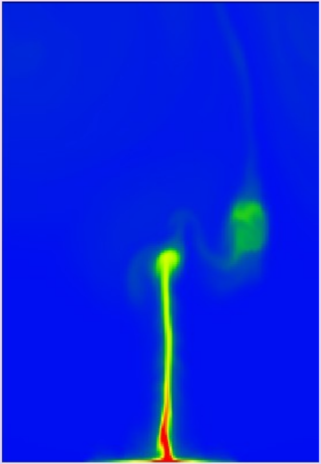
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



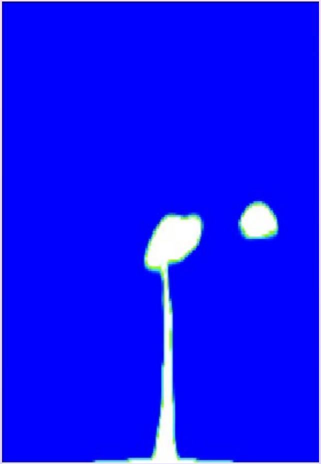
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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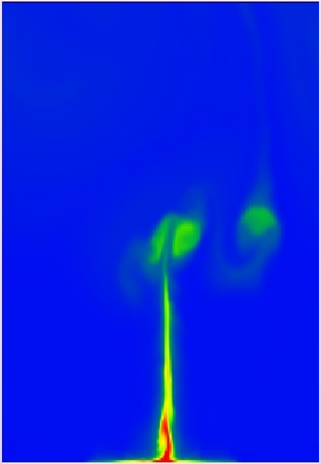
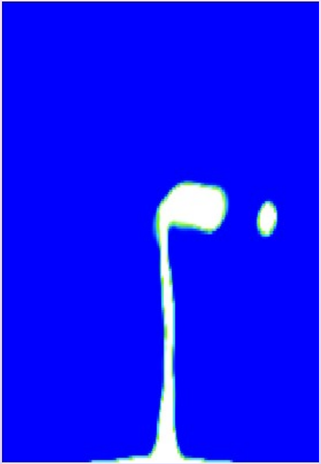


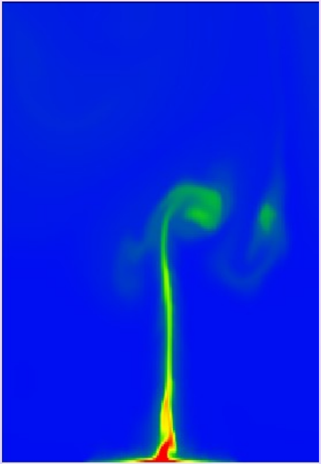
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



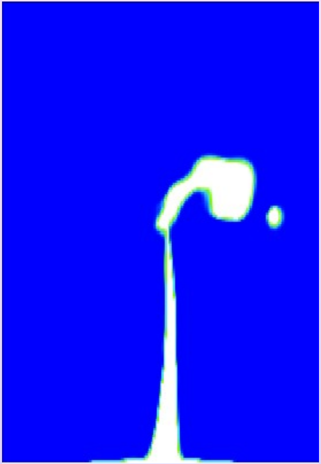
Temperature T



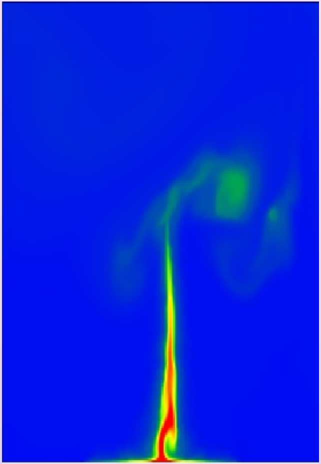
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



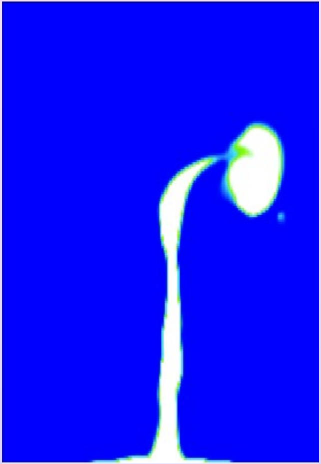
Temperature T



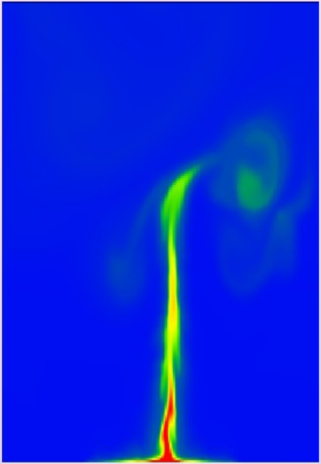
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

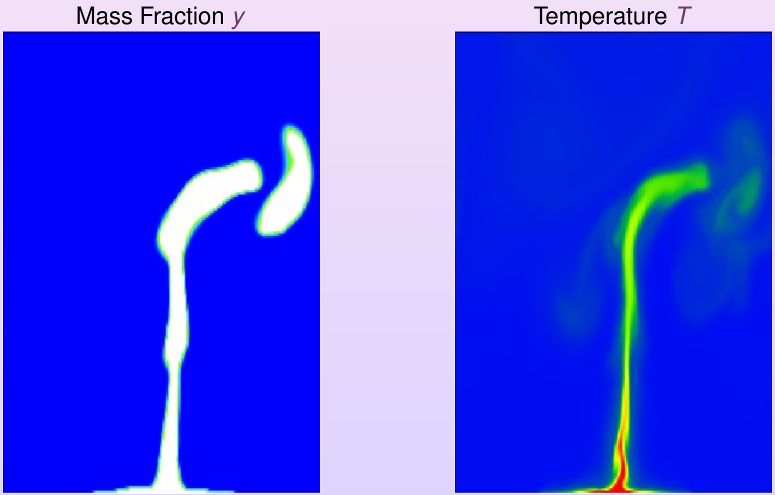


Temperature T



◂ Geometry ▸ Play ▸▸ Skip

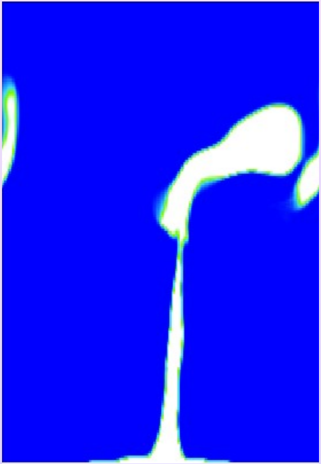
Nucleating Bubble



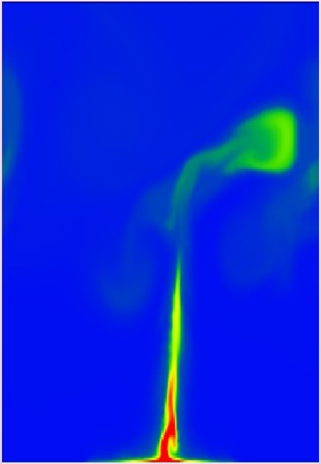
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



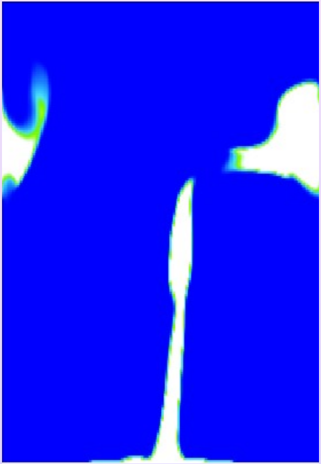
Temperature T



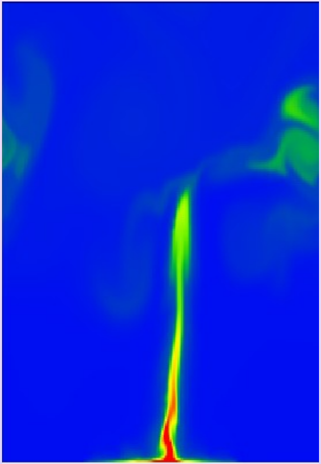
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

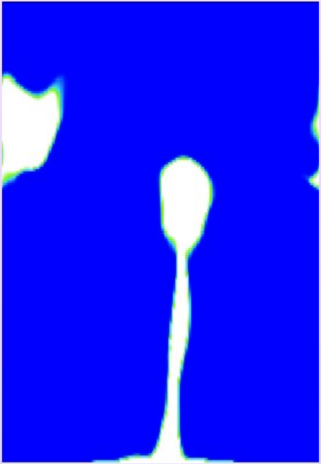


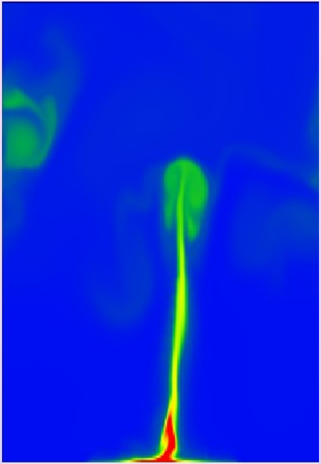
Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

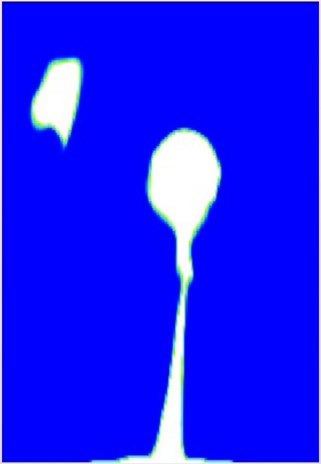




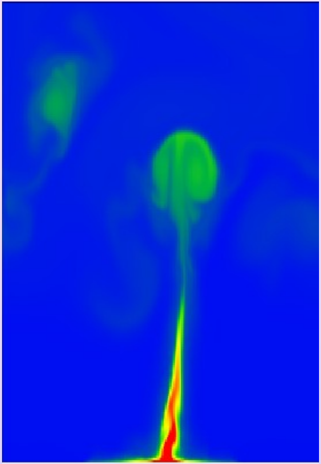
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



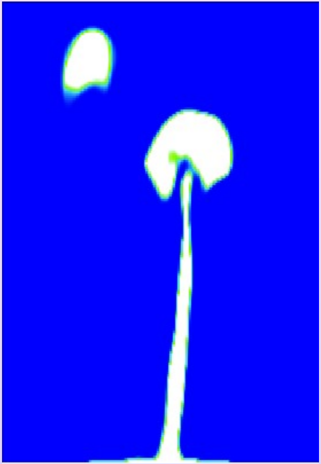
Temperature T



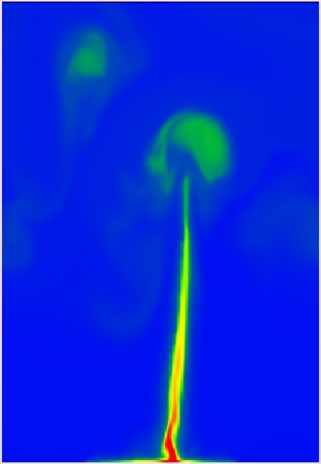
◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



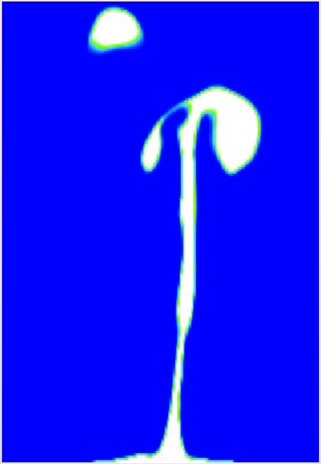
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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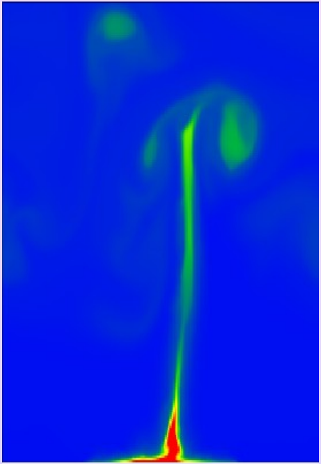
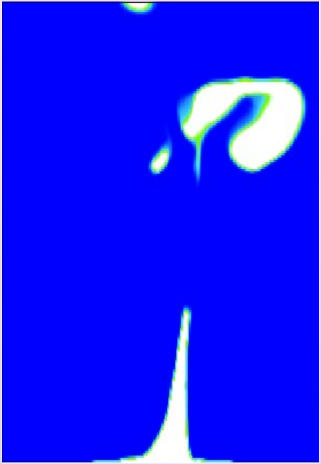


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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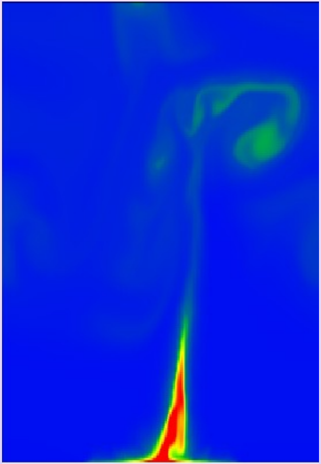
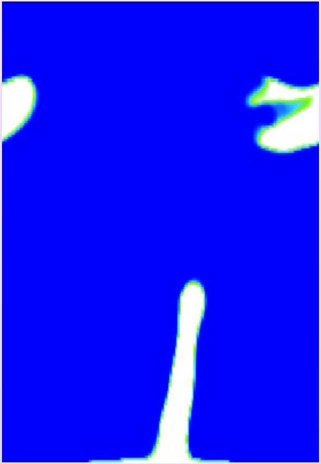


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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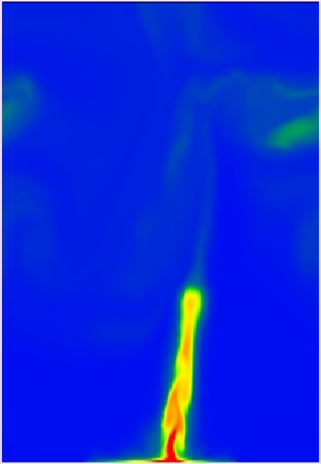
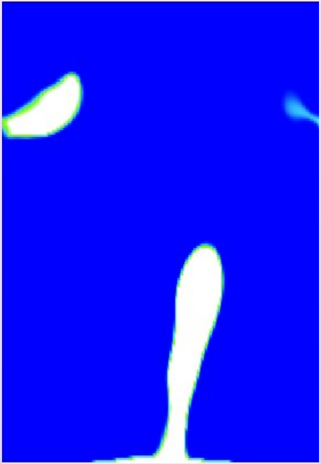


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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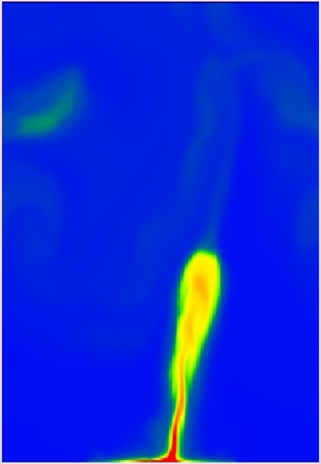
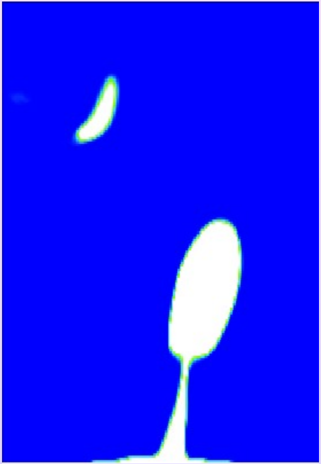


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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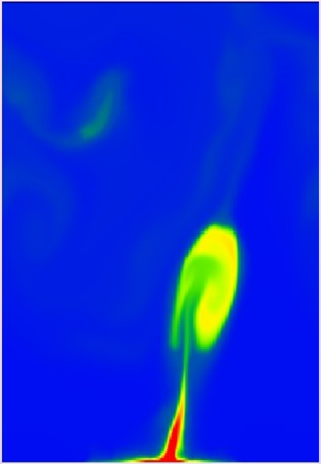
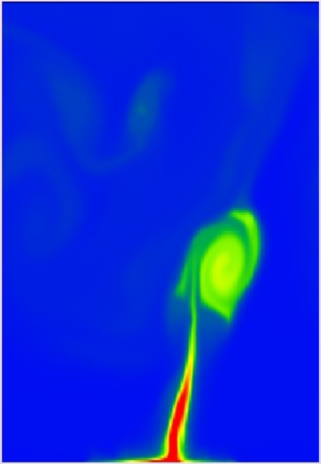


Geometry Play Skip

Nucleating Bubble

Mass Fraction y

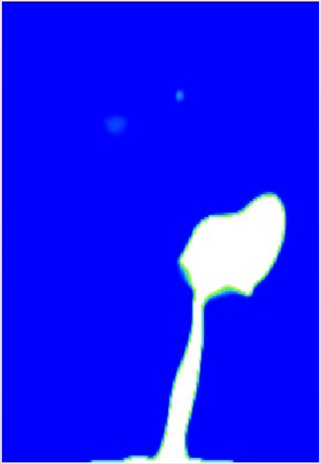
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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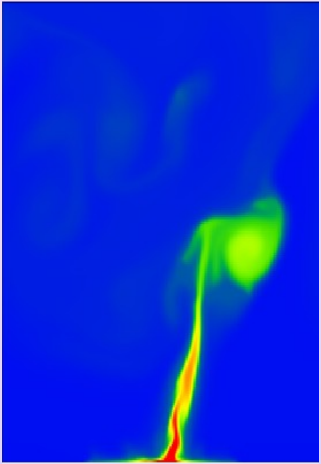
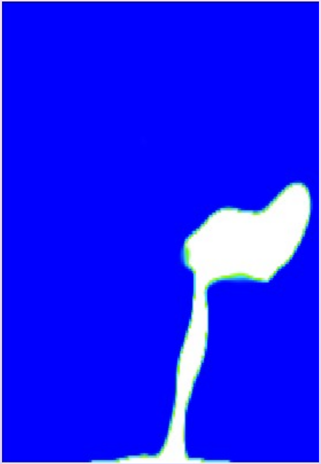


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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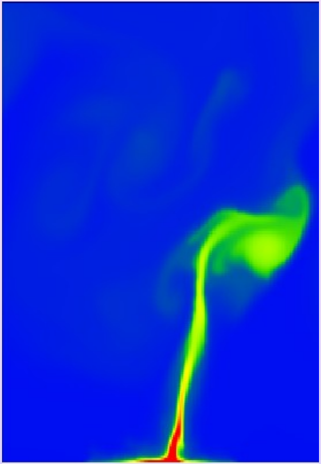
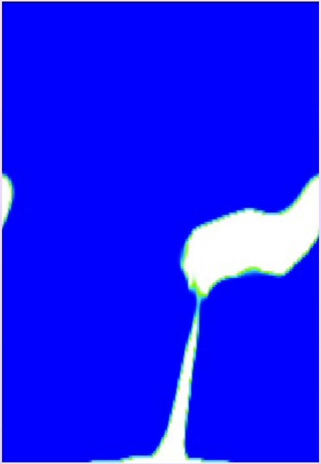


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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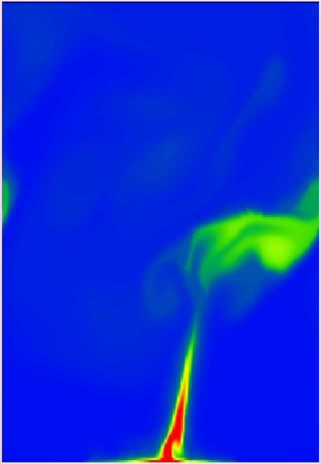
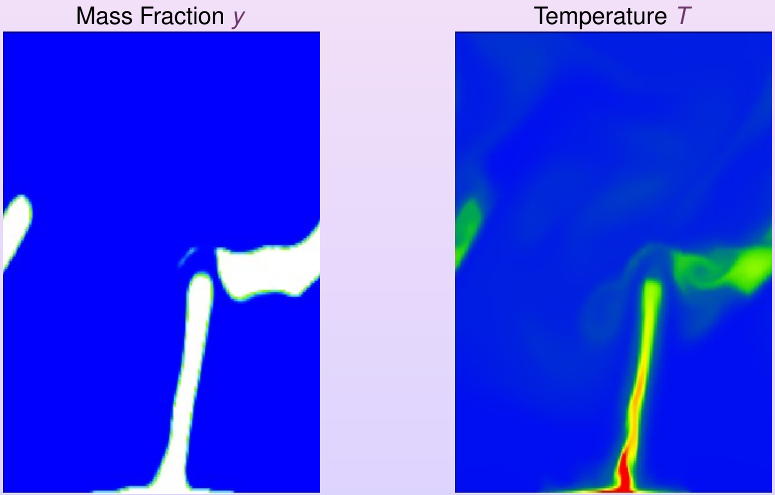


Geometry Play Skip

Nucleating Bubble

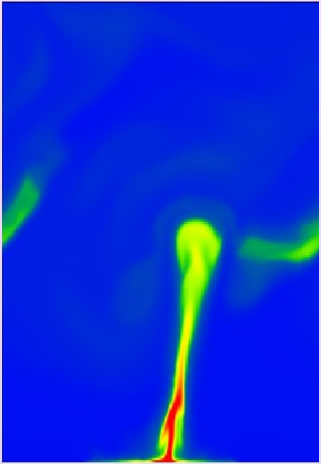


◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y

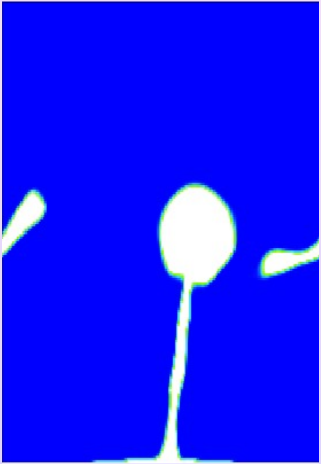
Temperature T



Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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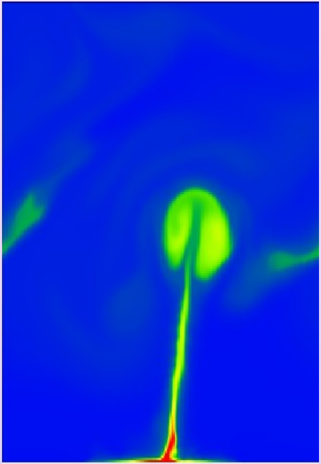
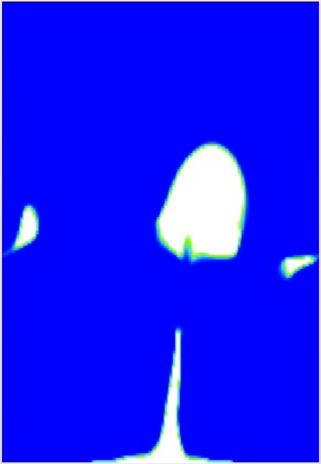


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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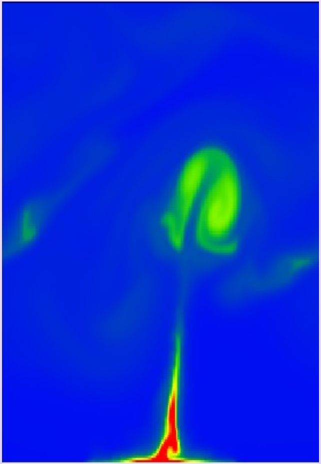
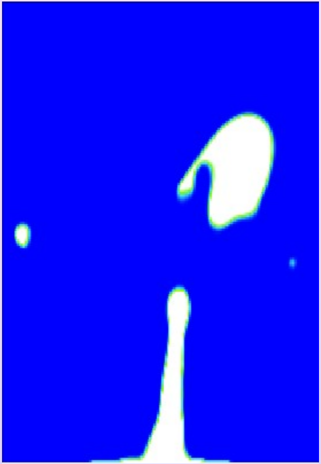


Geometry Play Skip

Nucleating Bubble

Mass Fraction y



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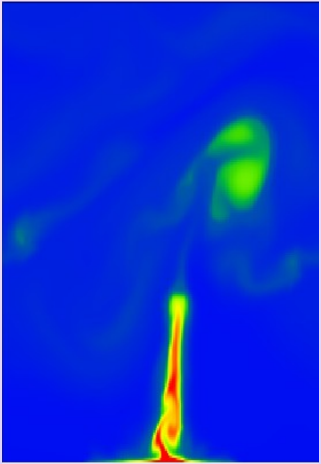
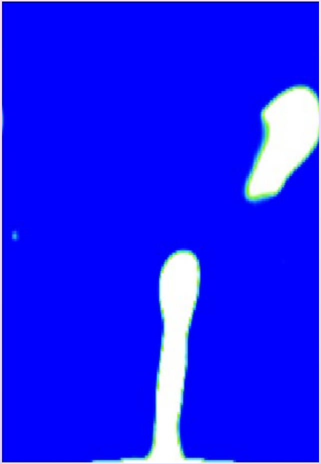


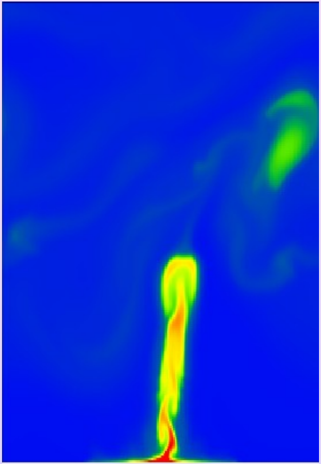
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



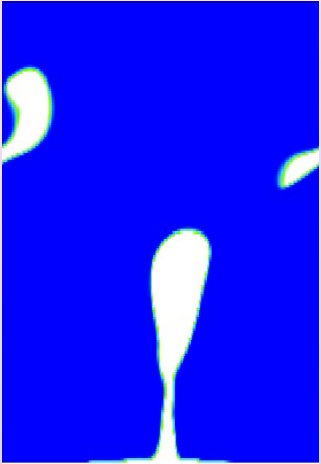
Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Nucleating Bubble

Mass Fraction y



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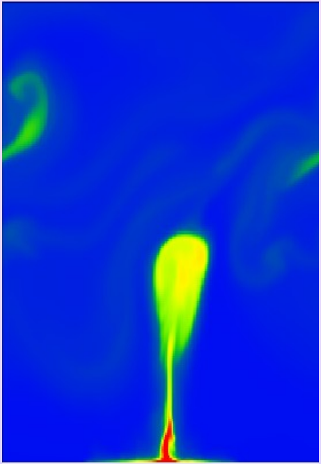
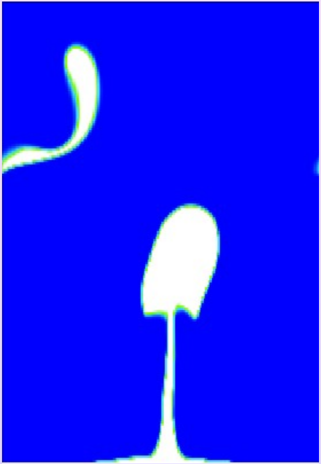


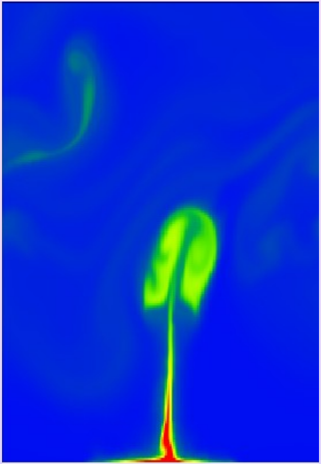
Geometry Play Skip

Nucleating Bubble

Mass Fraction y

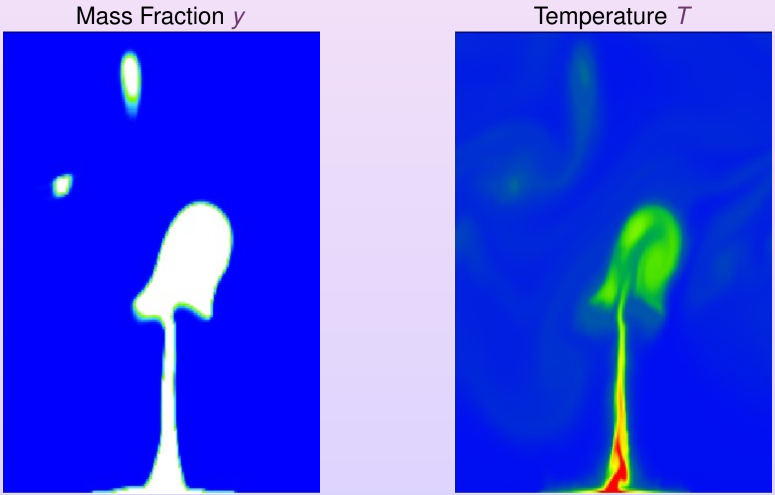


Temperature T



◀ Geometry ▶ Play ▶▶ Skip

Nucleating Bubble

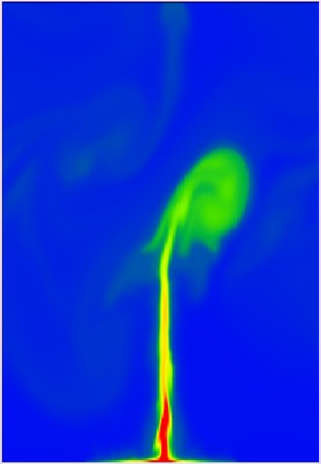


Geometry Play Skip

Nucleating Bubble

Mass Fraction y

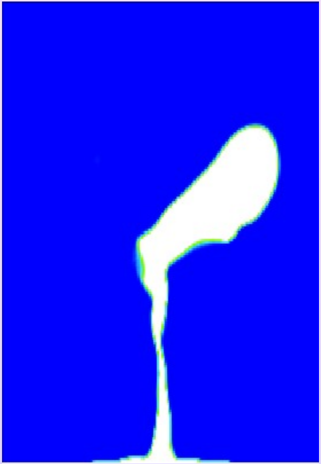
Temperature T



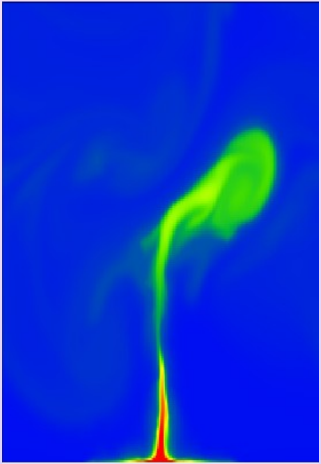
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



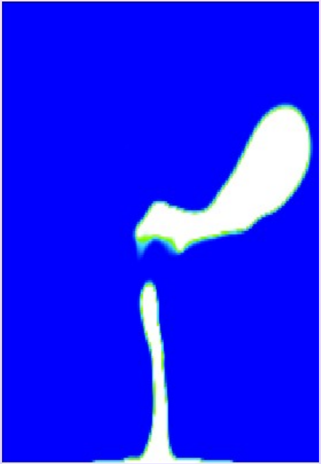
Temperature T



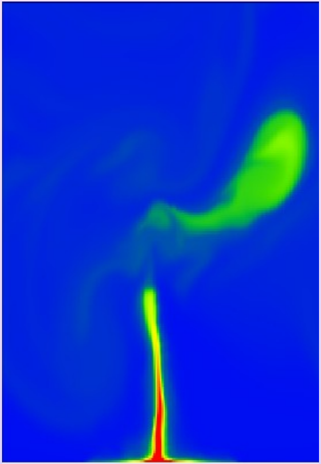
Geometry Play Skip

Nucleating Bubble

Mass Fraction y



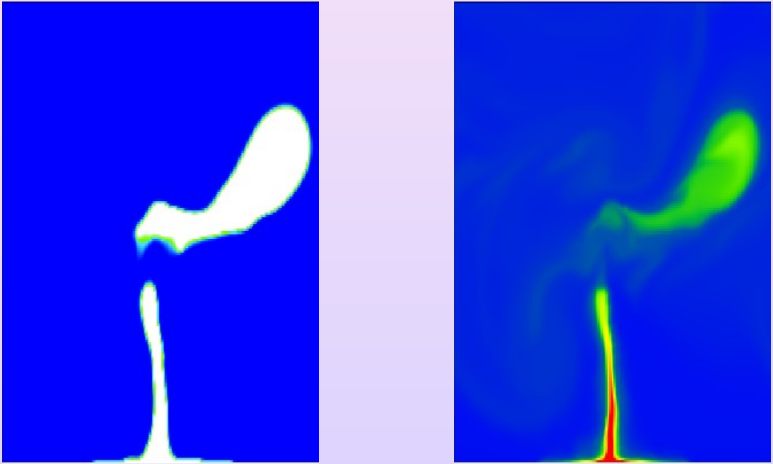
Temperature T



Geometry Play Skip

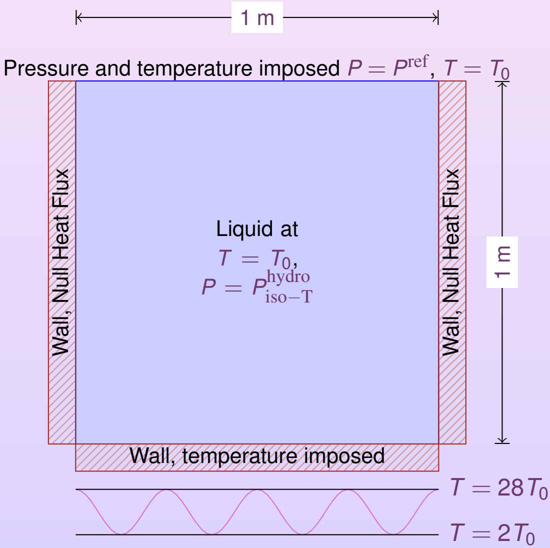
Nucleating Bubble

Mass Fraction y Temperature T



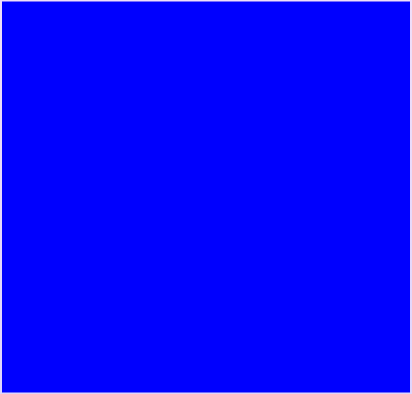
◂ Geometry ▸ Play ▸▸ Skip

Film

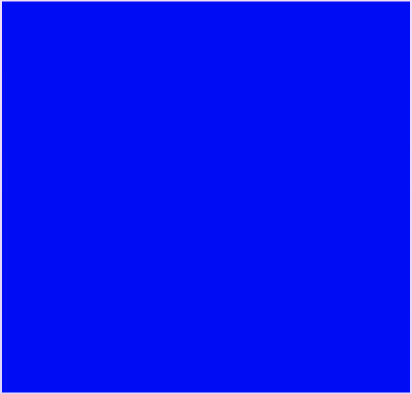

1 m
Pressure and temperature imposed $P = P^{\text{ref}}$, $T = T_0$
Wall, Null Heat Flux
Liquid at
$T = T_0$,
$P = P^{\text{hydro}}_{\text{iso}-\text{T}}$
Wall, Null Heat Flux
1 m
Wall, temperature imposed
$T = 28T_0$
$T = 2T_0$

Film

Mass Fraction y



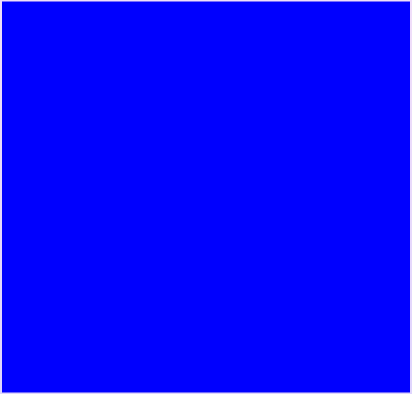
Temperature T



Geometry Play Skip

Film

Mass Fraction y



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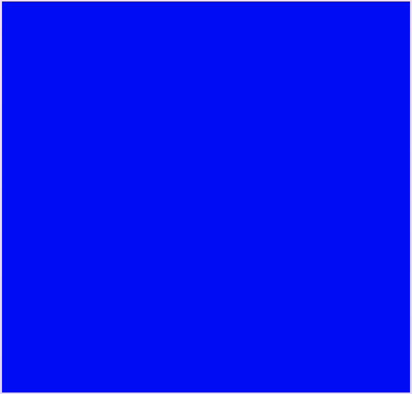

◂ Geometry ▸ Play ▸▸ Skip

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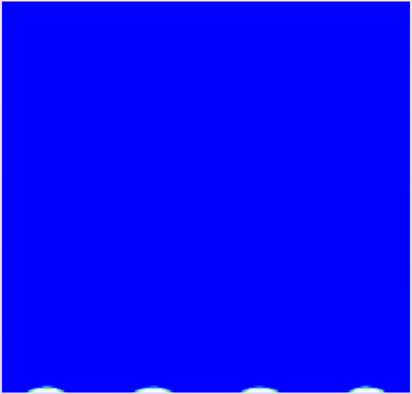

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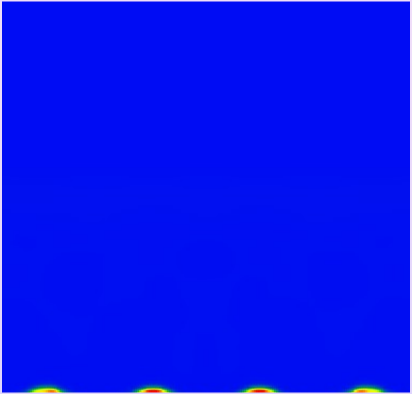

◀ Geometry ▶ Play ▶▶ Skip

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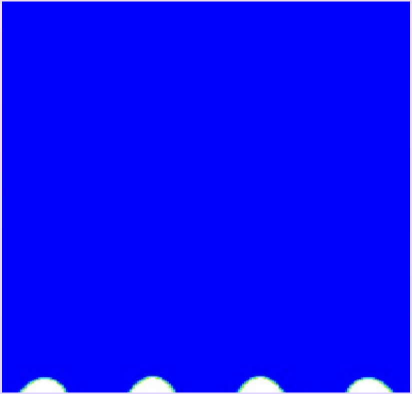

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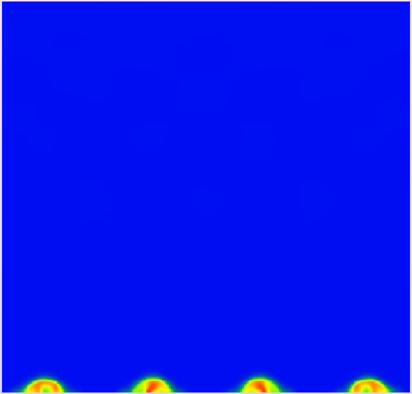

Geometry Play Skip

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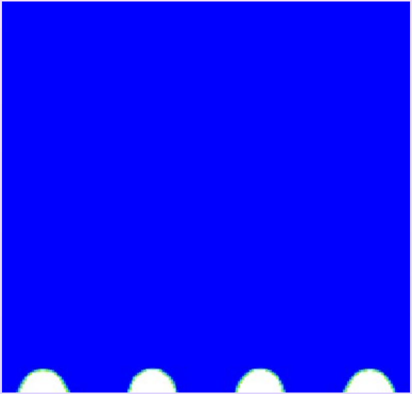

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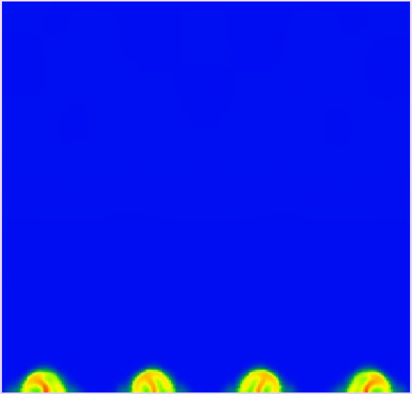

◂ Geometry ▸ Play ▸▸ Skip

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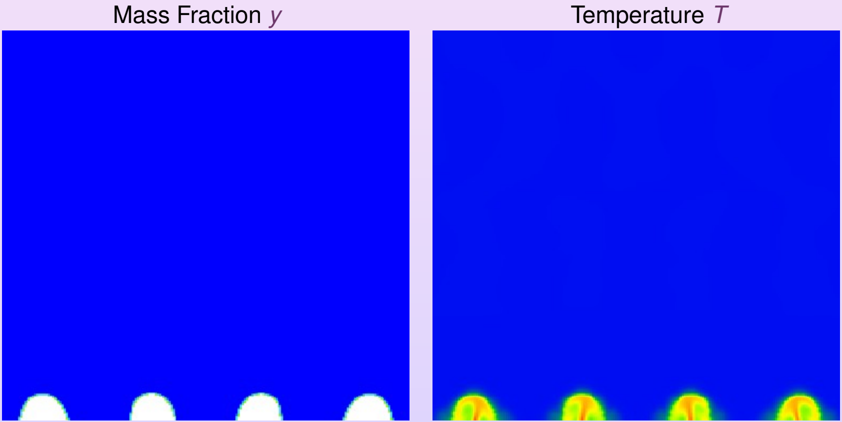

◂ Geometry ▸ Play ▸▸ Skip

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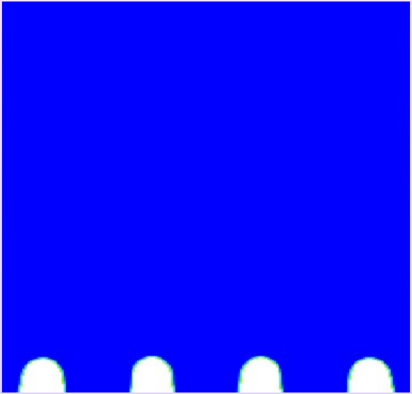

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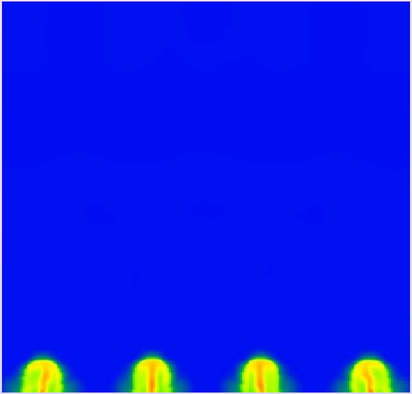

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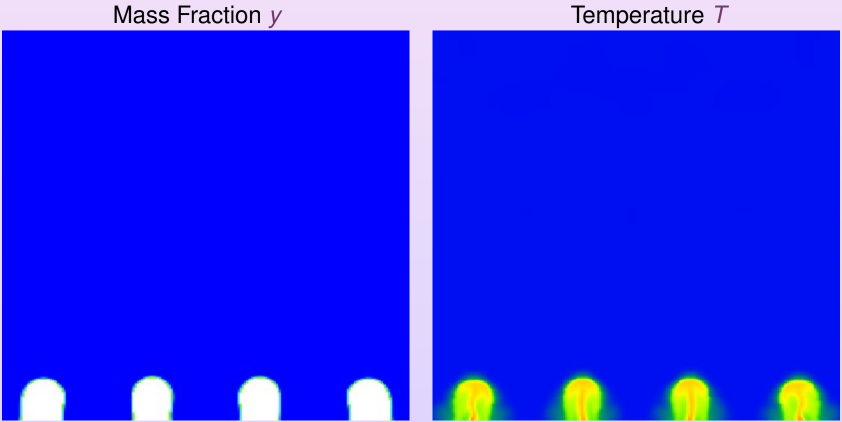

Geometry Play Skip

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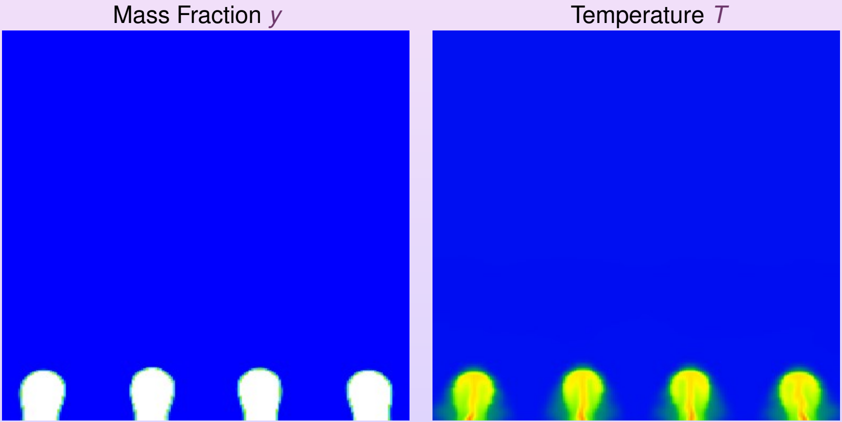
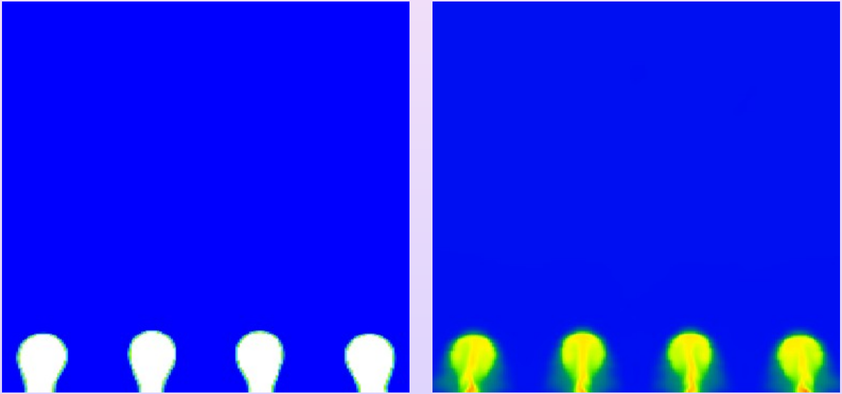


◂ Geometry ▸ Play ▸▸ Skip

Film

Mass Fraction y Temperature T



◂ Geometry ▸ Play ▸▸ Skip

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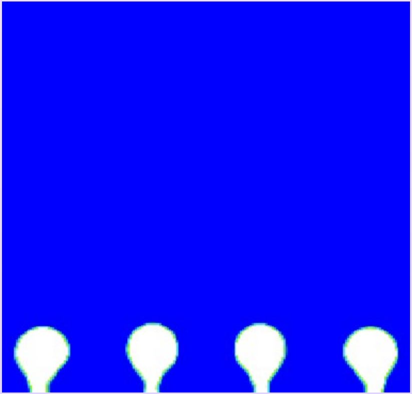

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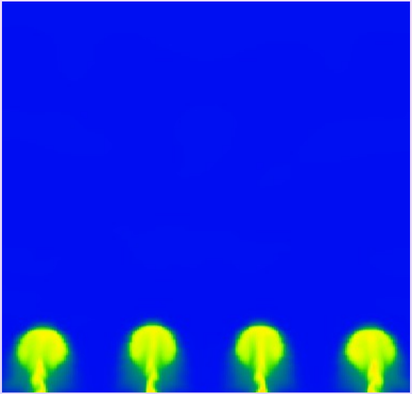

◂ Geometry ▸ Play ▸▸ Skip

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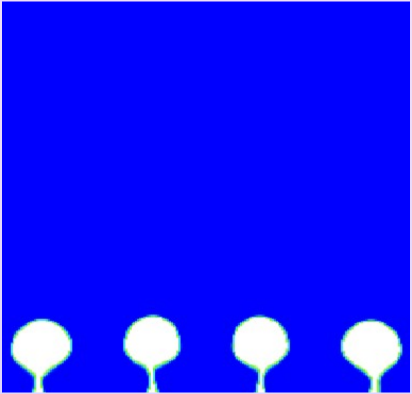

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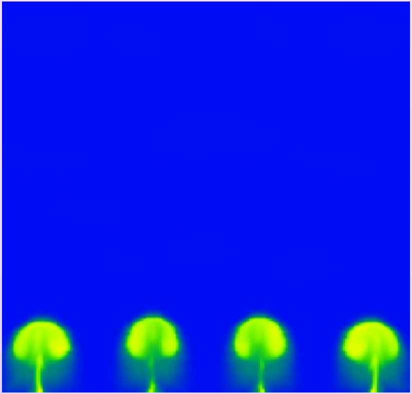
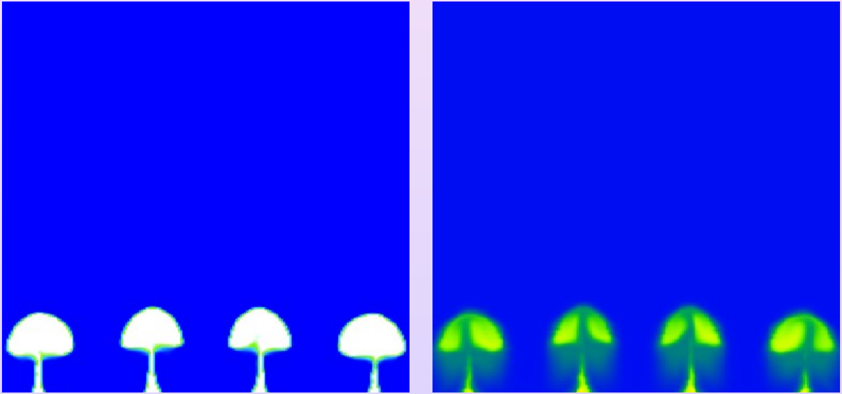


◂ Geometry ▸ Play ▸▸ Skip

Film

Mass Fraction y Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Film

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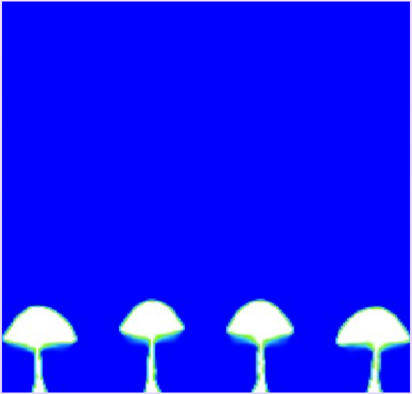

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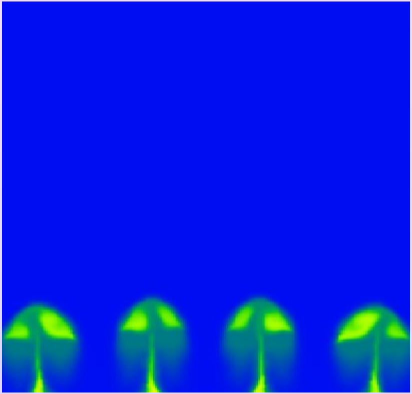

◀ Geometry ▶ Play ▶▶ Skip

Film

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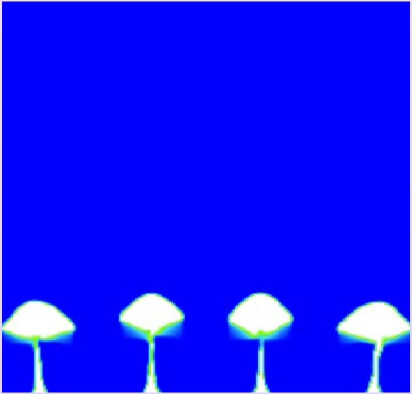

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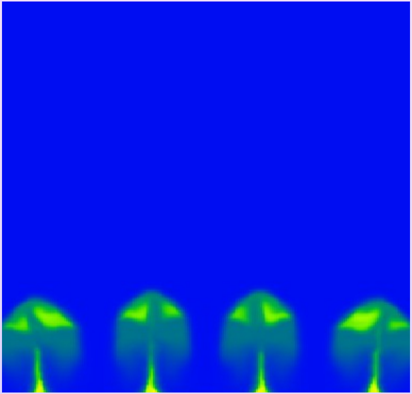
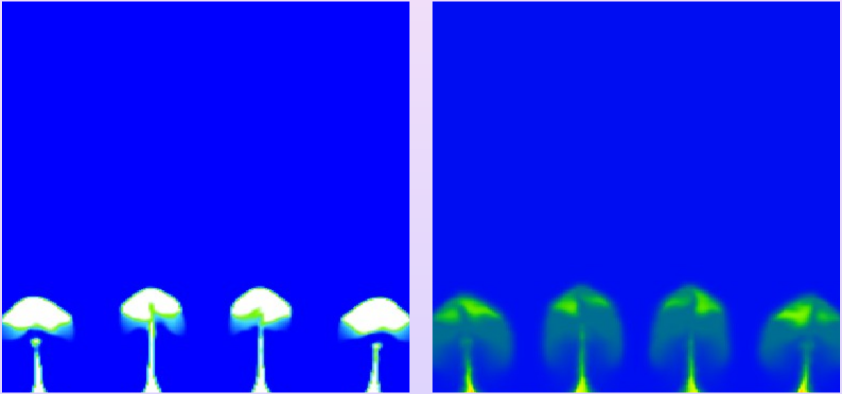


◂ Geometry ▸ Play ▸▸ Skip

Film

Mass Fraction y | Temperature T



◂ Geometry ▸ Play ▸▸ Skip

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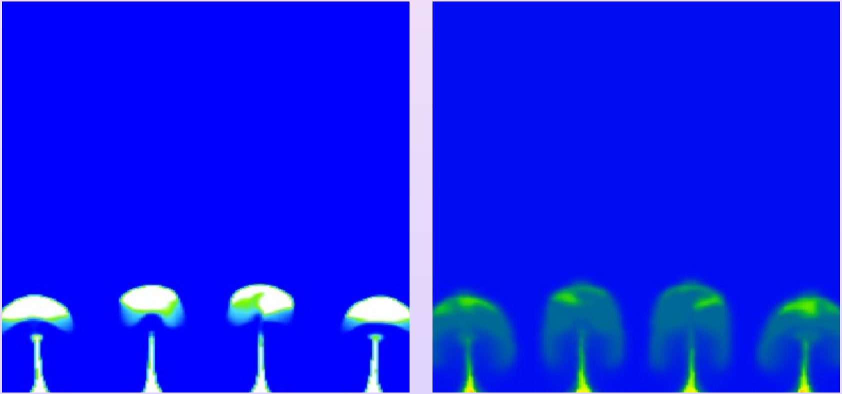

◂ Geometry ▸ Play ▸▸ Skip

Film

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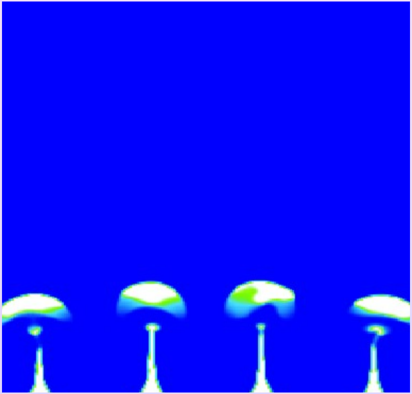

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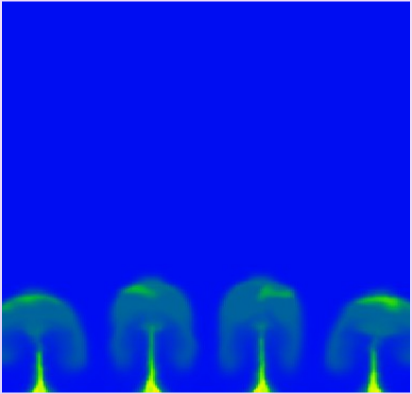

◀ Geometry ▶ Play ▶▶ Skip

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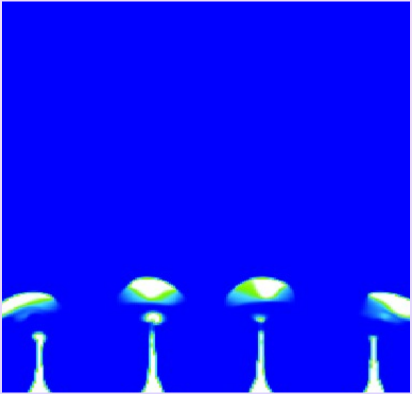

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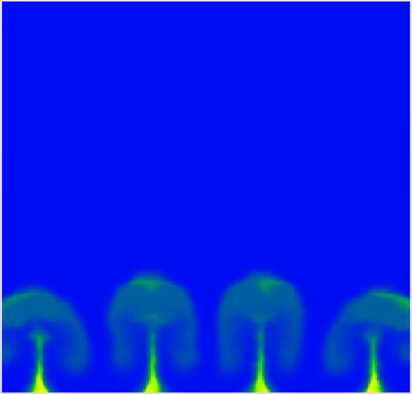

◂ Geometry ▸ Play ▸▸ Skip

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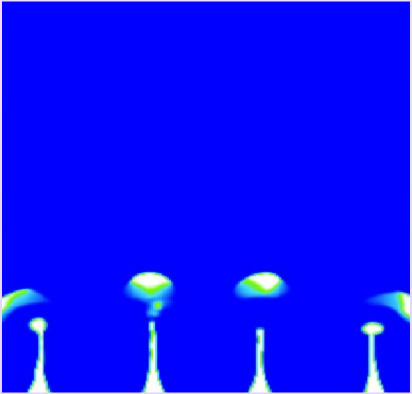

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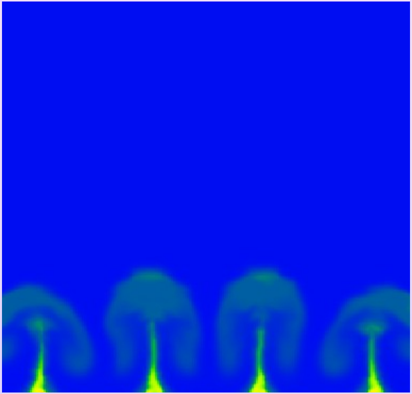

Geometry Play Skip

Film

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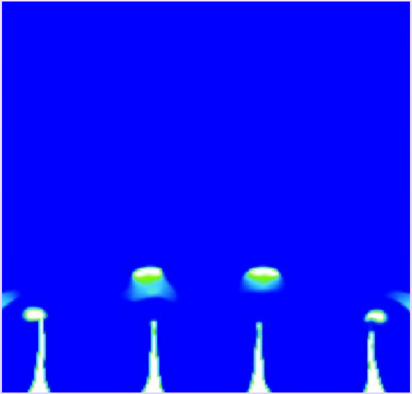

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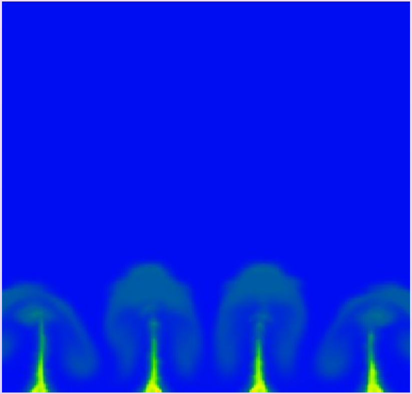

Geometry Play Skip

Film

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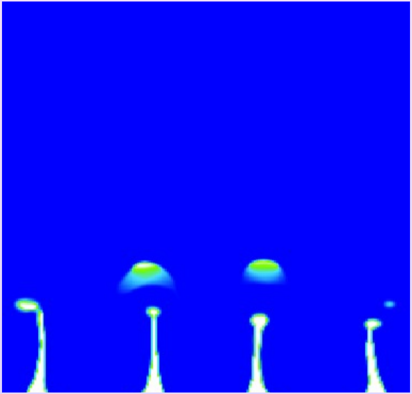

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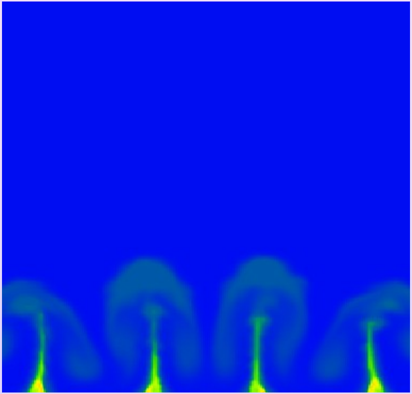
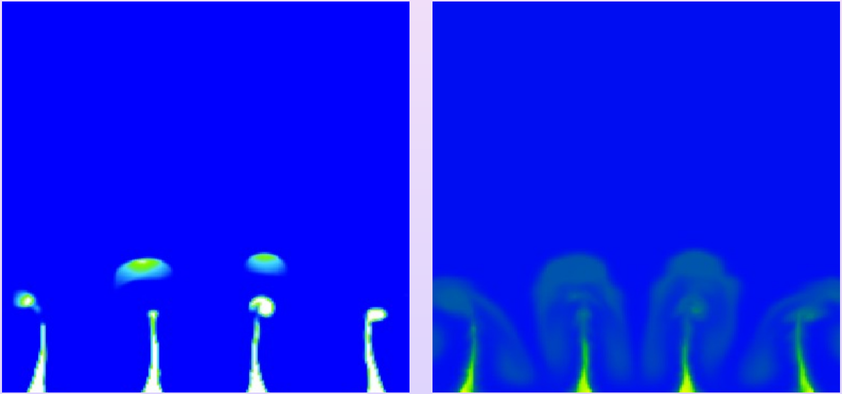


Geometry Play Skip

Film

Mass Fraction y Temperature T



Geometry Play Skip

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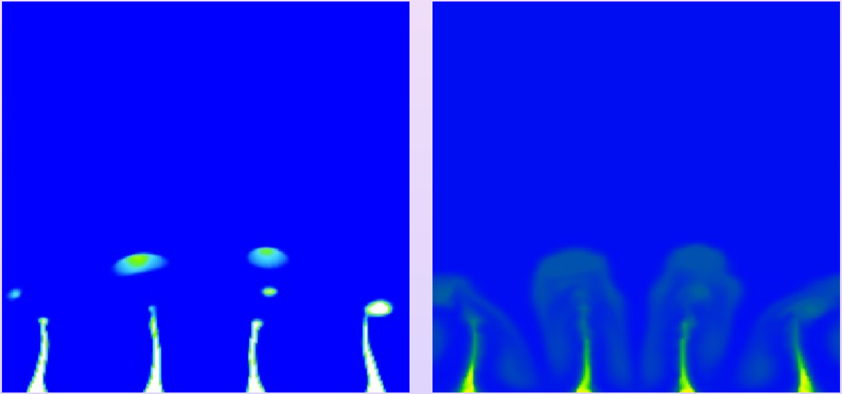

Geometry Play Skip

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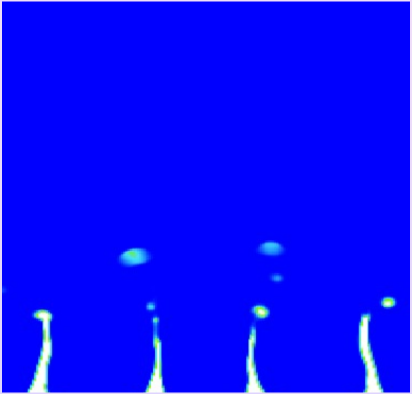

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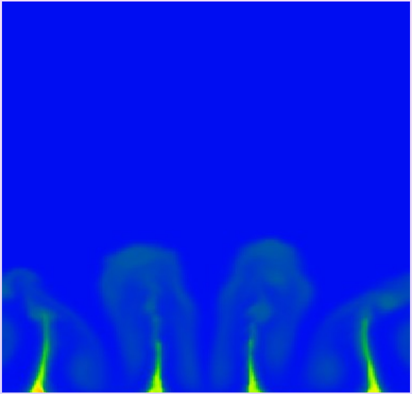

Geometry Play Skip

Film

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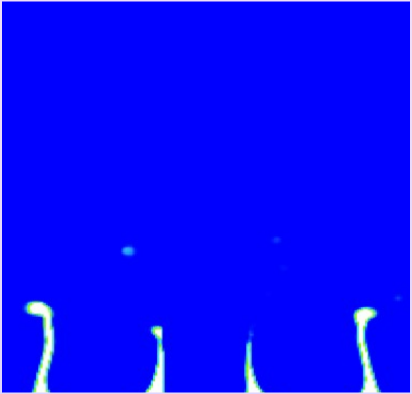

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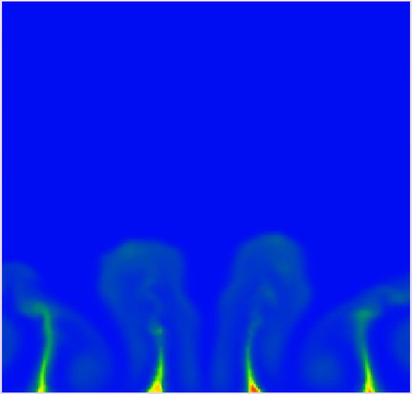

Geometry Play Skip

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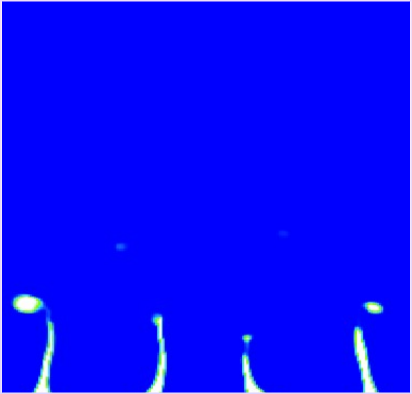

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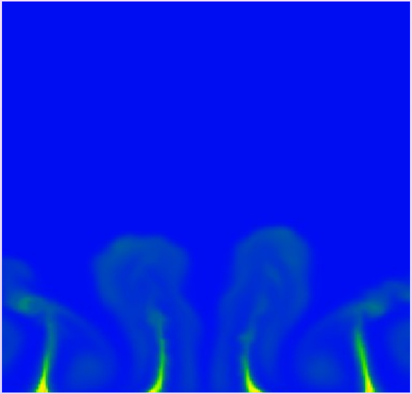

Geometry Play Skip

Film

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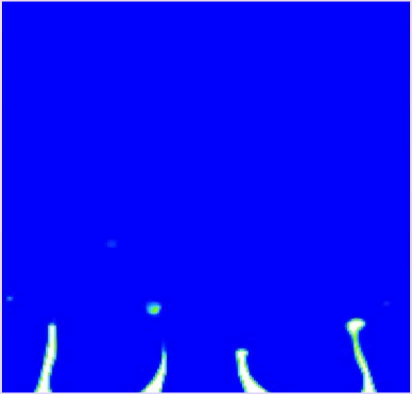

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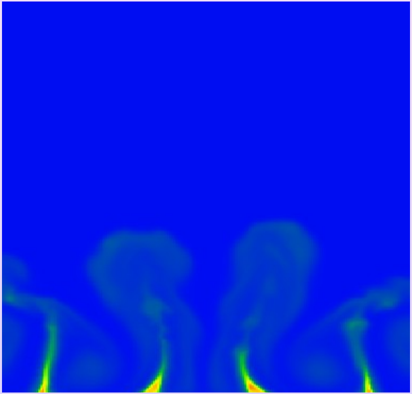

Geometry · Play · Skip

Film

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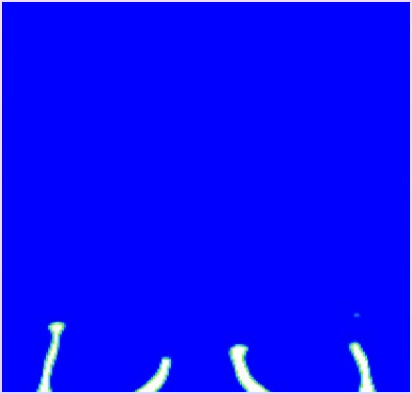

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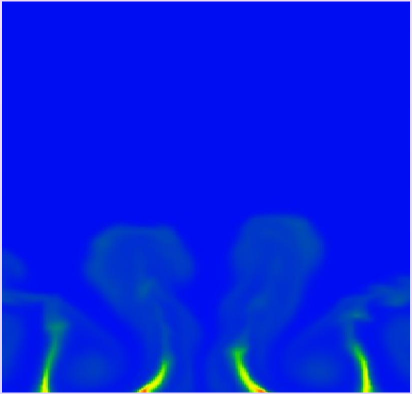

Geometry Play Skip

Film

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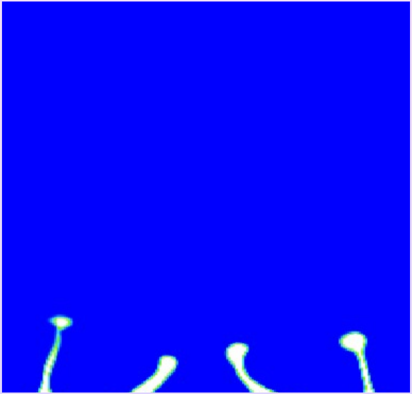

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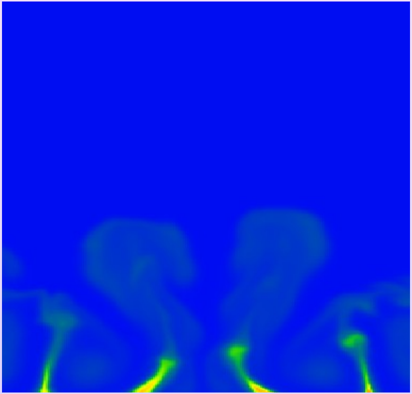
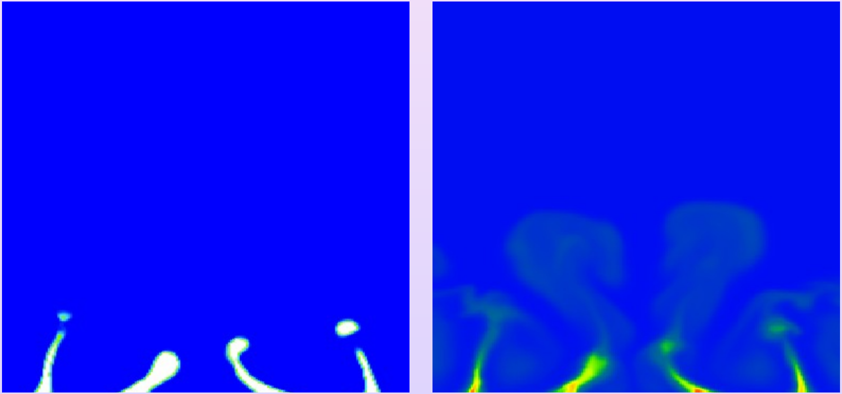


◀ Geometry ▶ Play ▶▶ Skip

Film

Mass Fraction y Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Film

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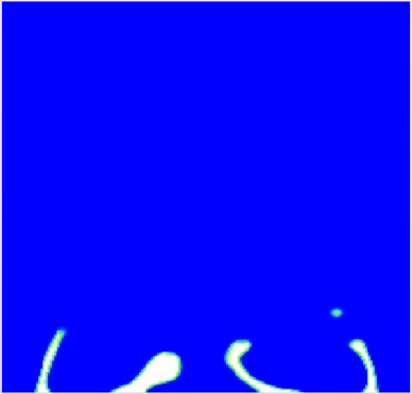

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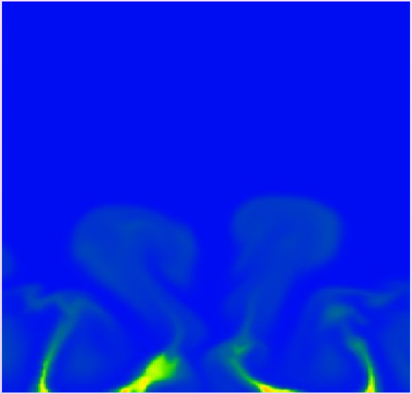

Geometry Play Skip

Film

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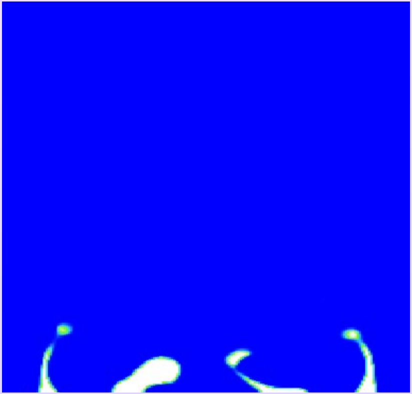

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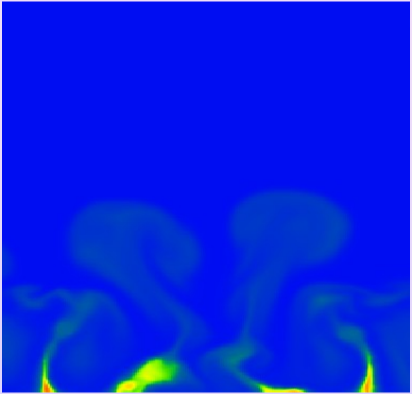

◀ Geometry ▶ Play ▶▶ Skip

Film

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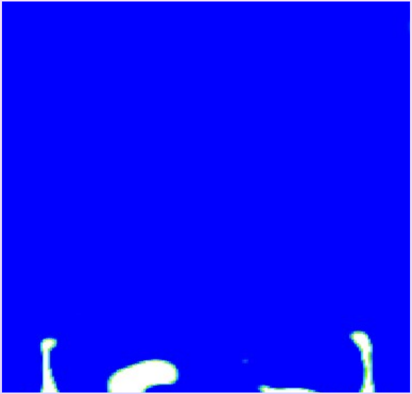

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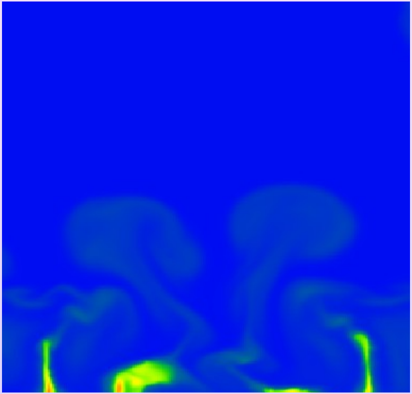
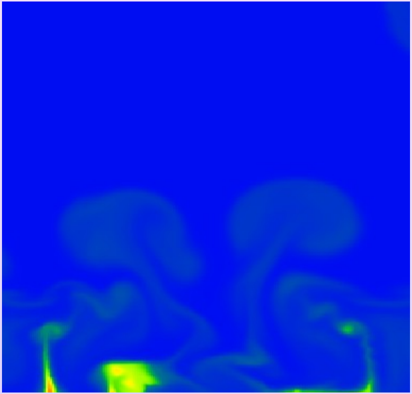


Geometry Play Skip

Film

Mass Fraction y

Temperature T



◀ Geometry ▶ Play ▶▶ Skip

Film

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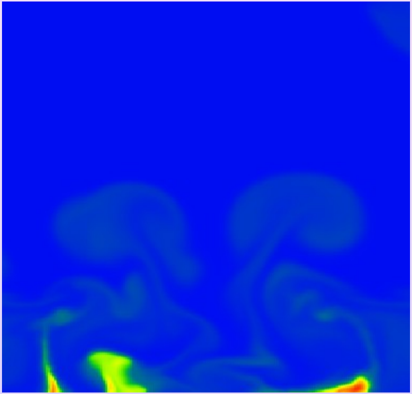
Temperature T



◀ Geometry ▶ Play ▶▶ Skip

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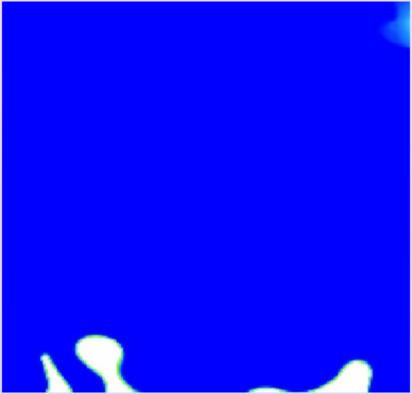

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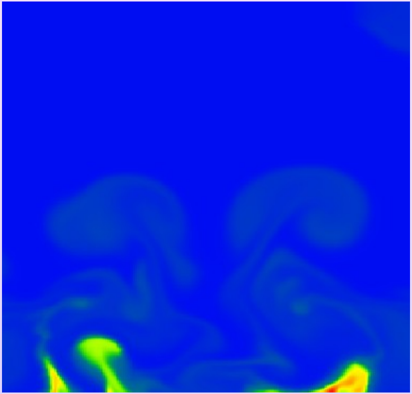
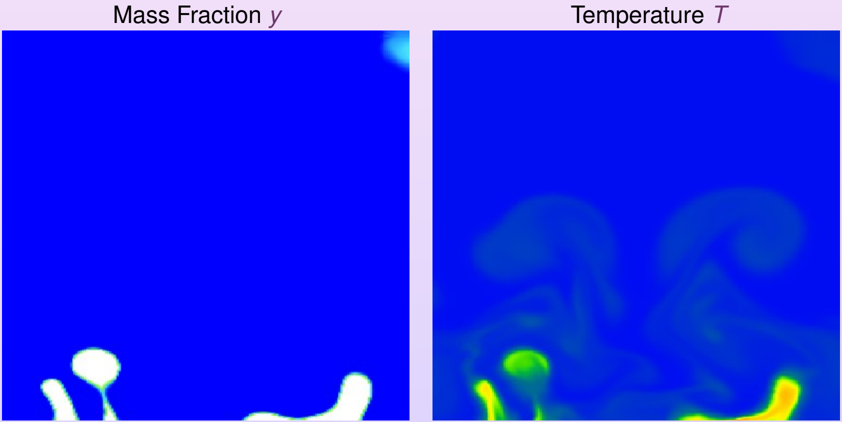


◂ Geometry ▸ Play ▸▸ Skip

Film



◂ Geometry ▸ Play ▸▸ Skip

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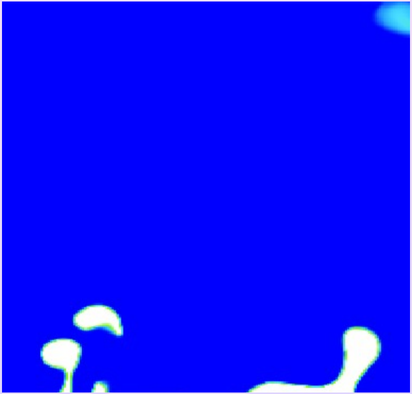

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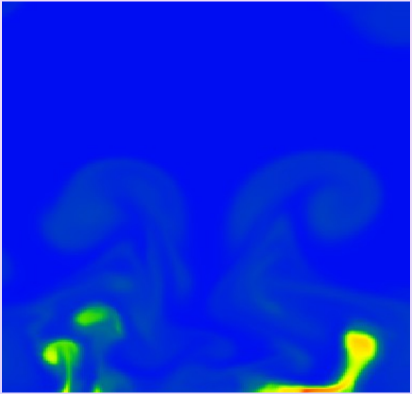

Geometry Play Skip

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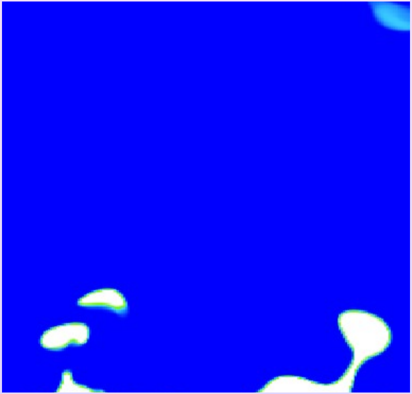

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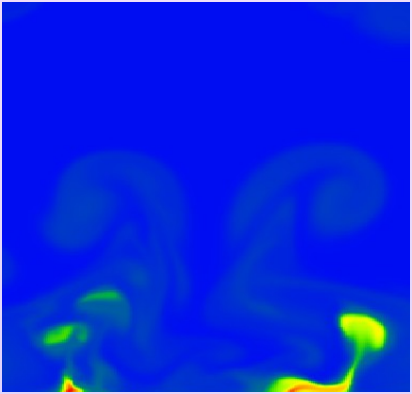

◂ Geometry ▸ Play ▸▸ Skip

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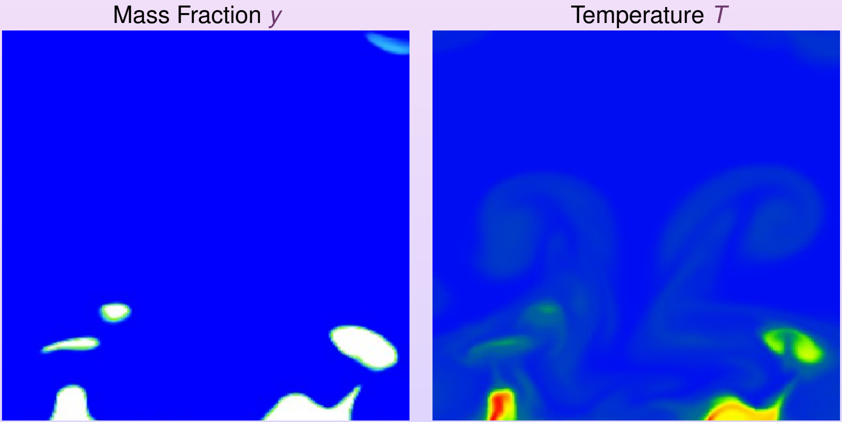

◀ Geometry ▶ Play ▶▶ Skip

Film

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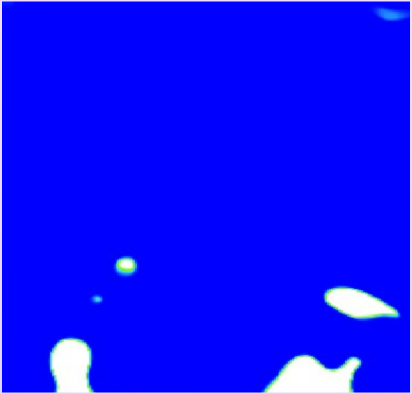

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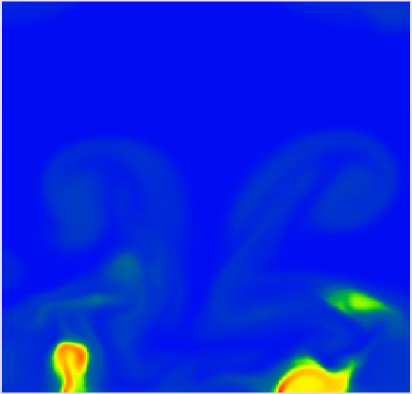
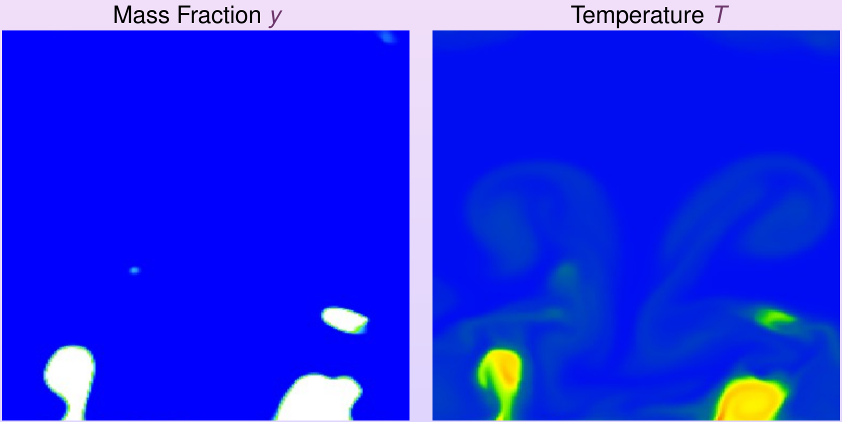


Geometry Play Skip

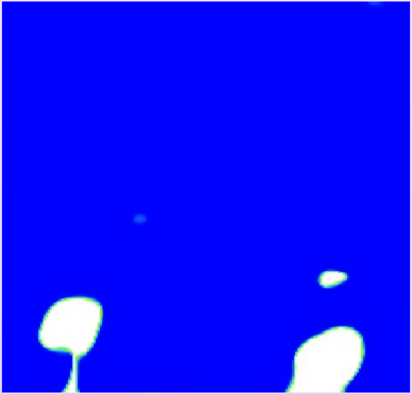
Film



◀ Geometry ▶ Play ▶▶ Skip

Film

Mass Fraction y



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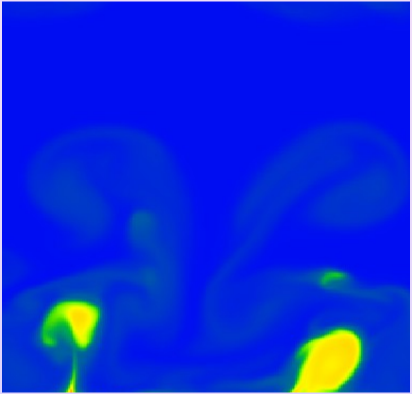

Geometry Play Skip

Film

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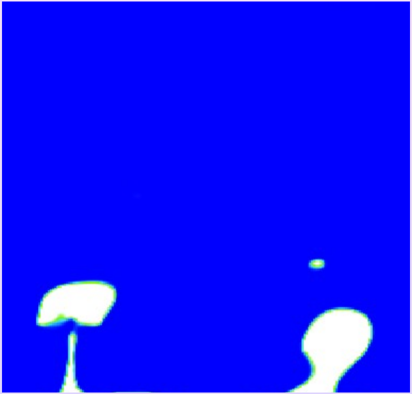

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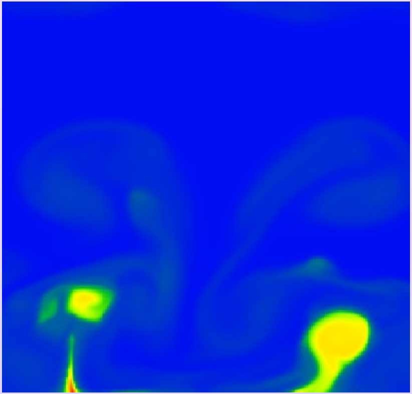

◂ Geometry ▸ Play ▸▸ Skip

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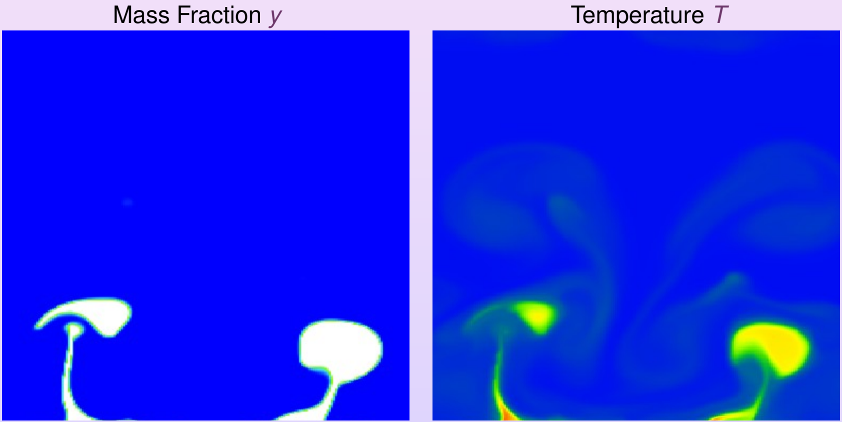

◂ Geometry ▸ Play ▸▸ Skip

Film

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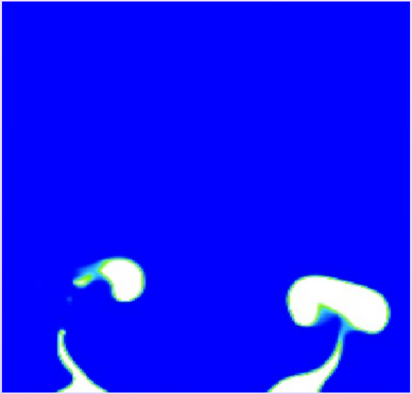

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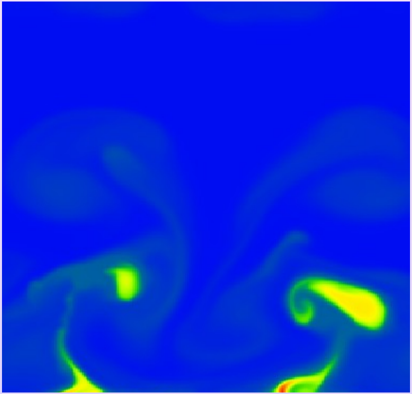

Geometry Play Skip

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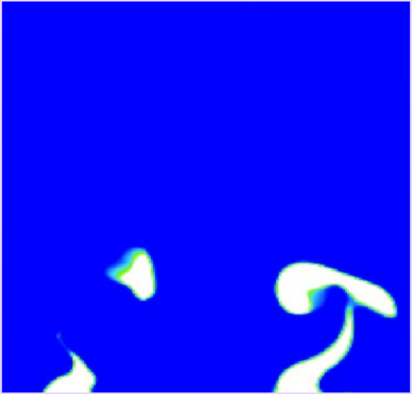

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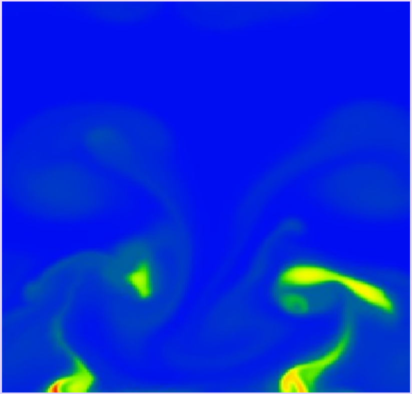

◂ Geometry ▸ Play ▸▸ Skip

Film

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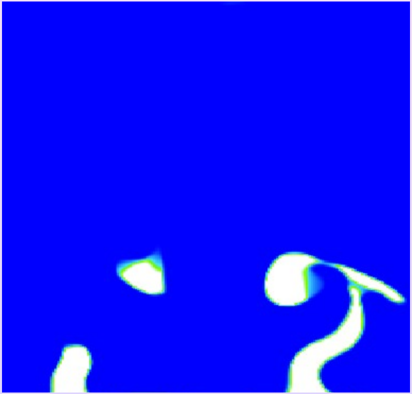

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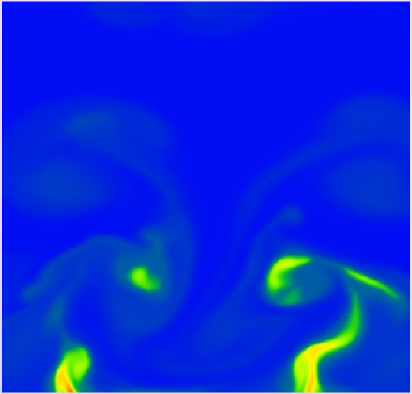

◂ Geometry ▸ Play ▸▸ Skip

Film

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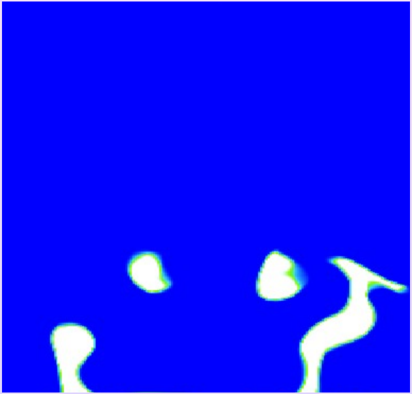

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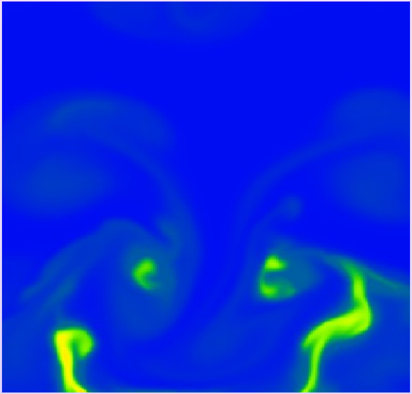

◂ Geometry ▸ Play ▸▸ Skip

Film

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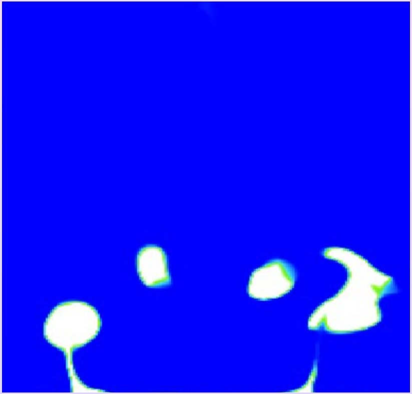

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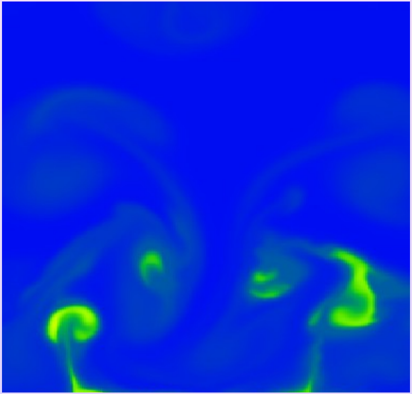

Geometry Play Skip

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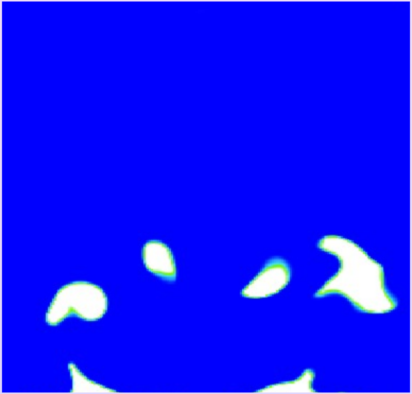

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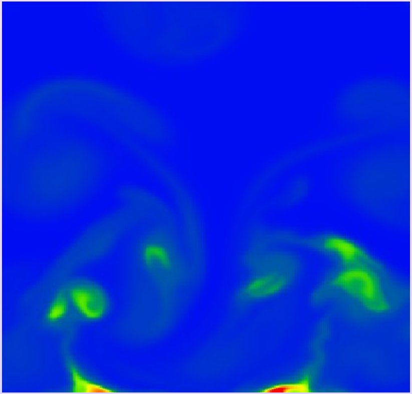

◂ Geometry ▸ Play ▸▸ Skip

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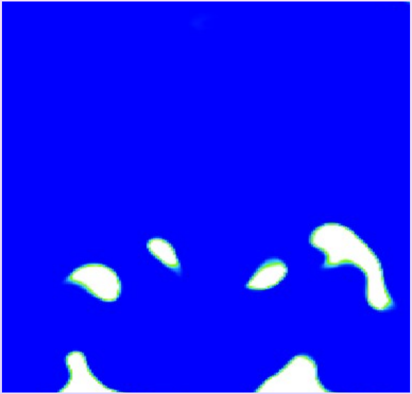

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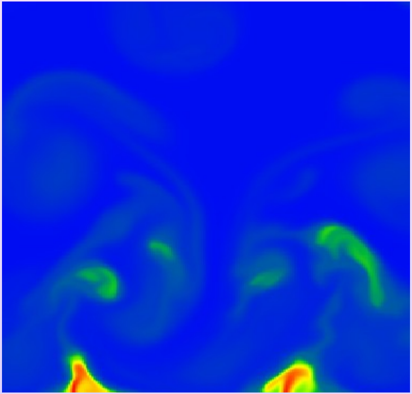

◀ Geometry ▶ Play ▶▶ Skip

Film

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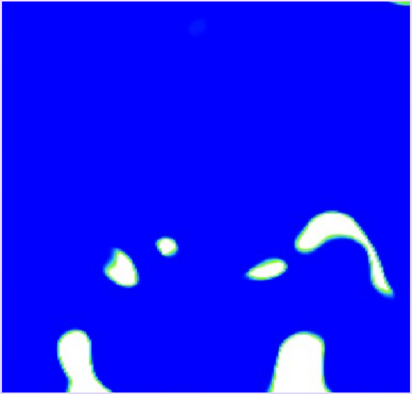

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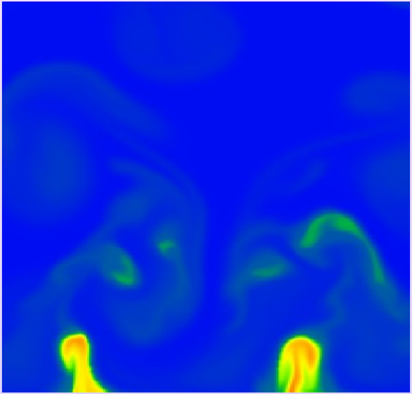
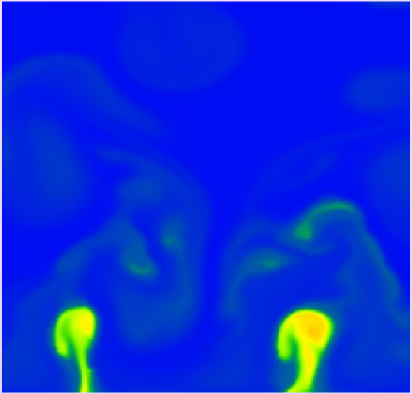


◂ Geometry ▸ Play ▸▸ Skip

Film

Mass Fraction y

Temperature T



Geometry Play Skip

Film

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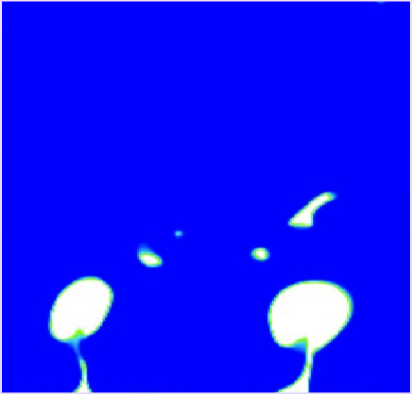

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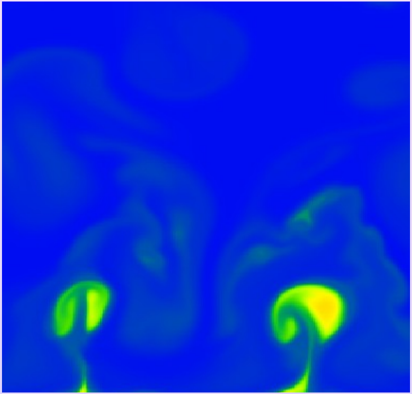
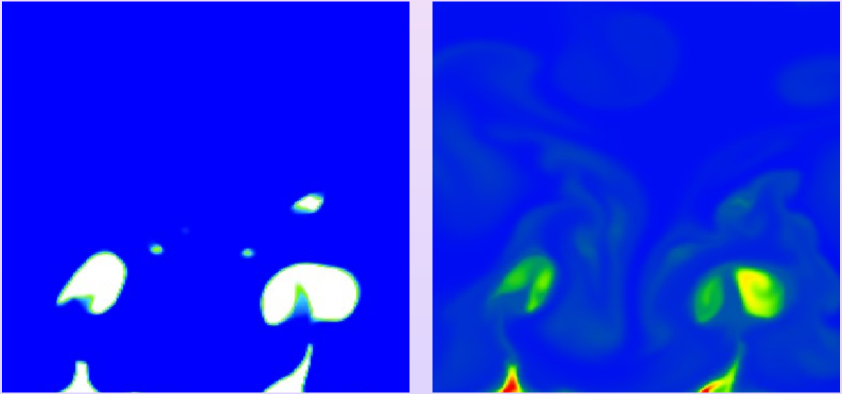


Geometry Play Skip

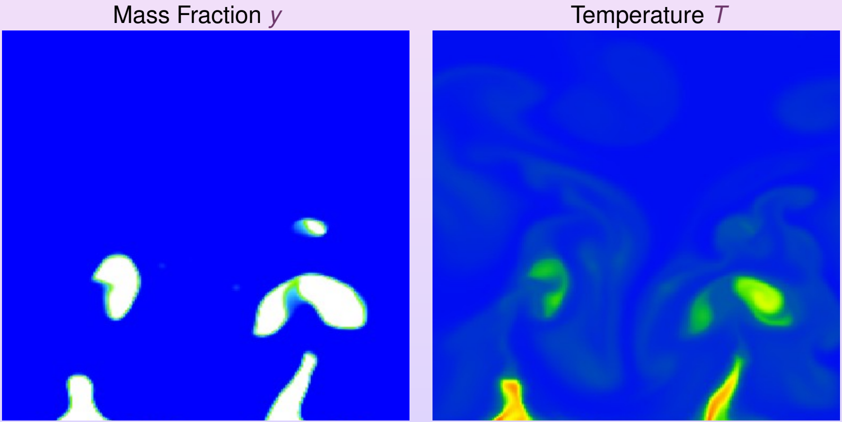
Film

Mass Fraction y Temperature T



◂ Geometry ▸ Play ▸▸ Skip

Film



◂ Geometry ▸ Play ▸▸ Skip

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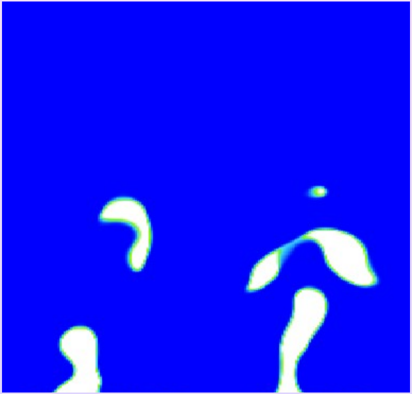

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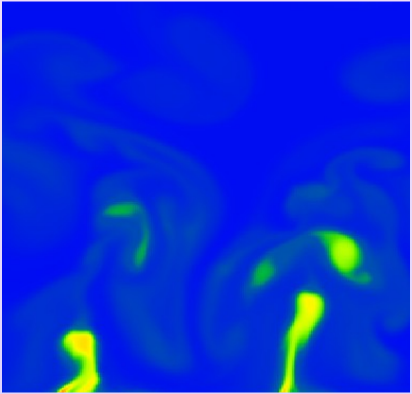

◂ Geometry ▸ Play ▸▸ Skip

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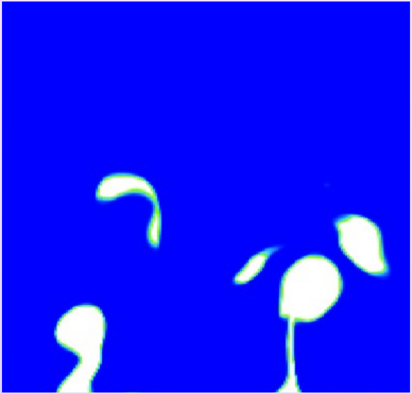

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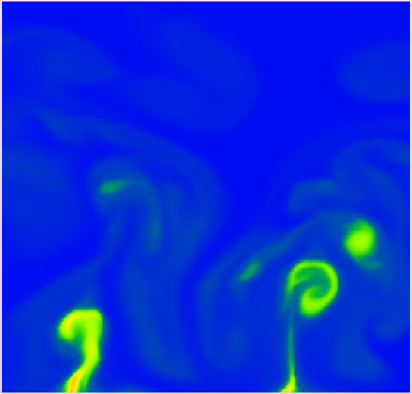

◂ Geometry ▸ Play ▸▸ Skip

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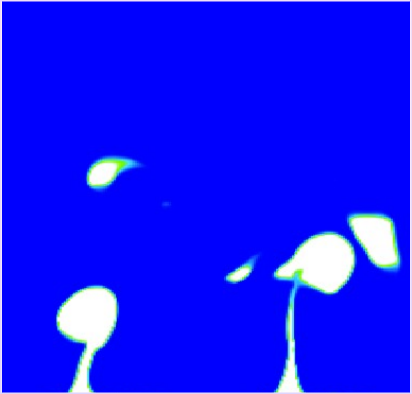

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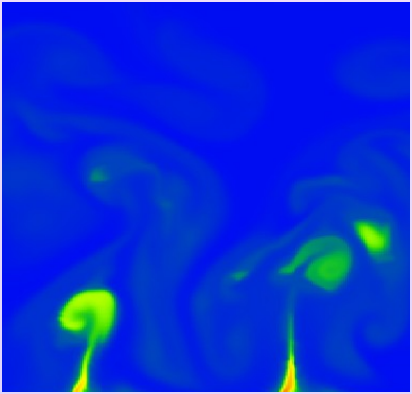

Geometry Play Skip

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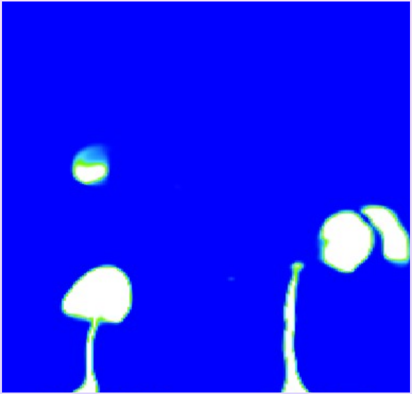

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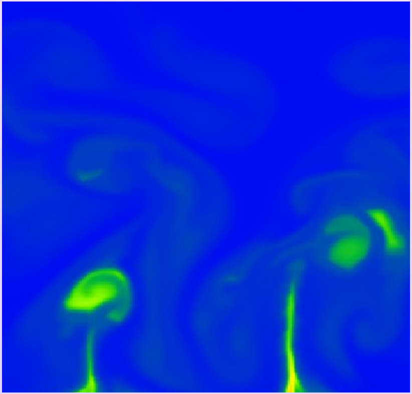

◀ Geometry ▶ Play ▶▶ Skip

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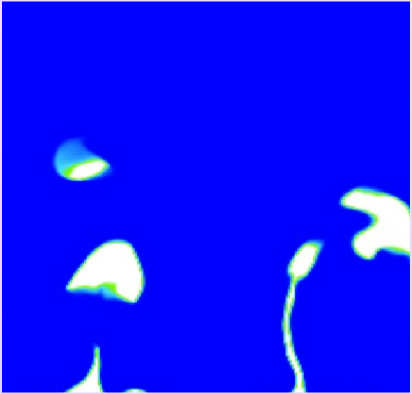

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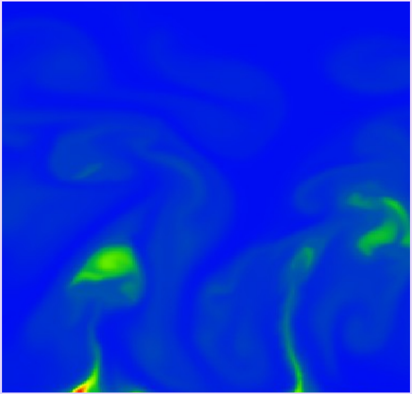

Geometry Play Skip

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Mass Fraction y

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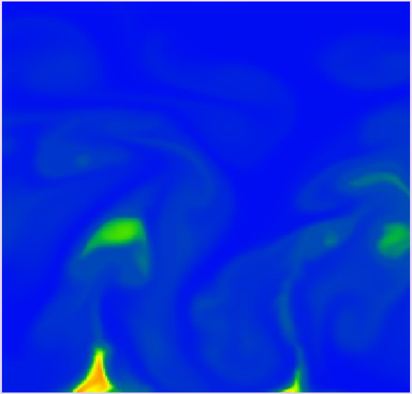

Geometry Play Skip

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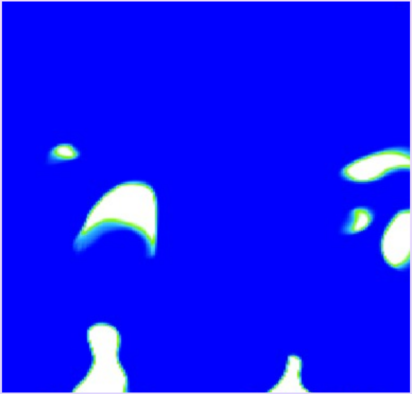

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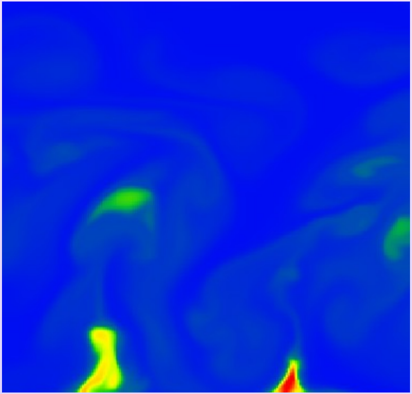

◂ Geometry ▸ Play ▸▸ Skip

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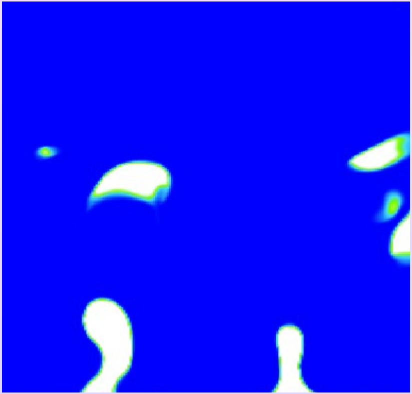

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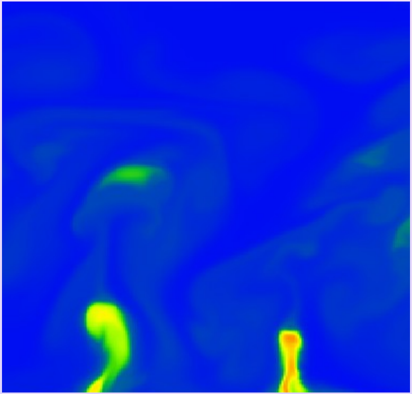

◂ Geometry ▸ Play ▸▸ Skip

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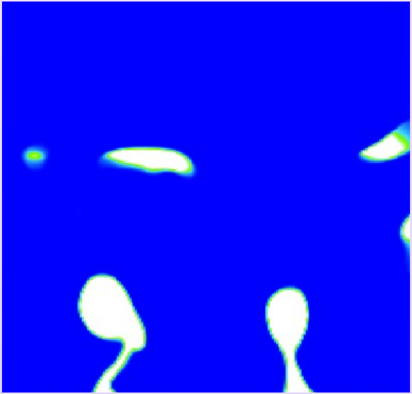

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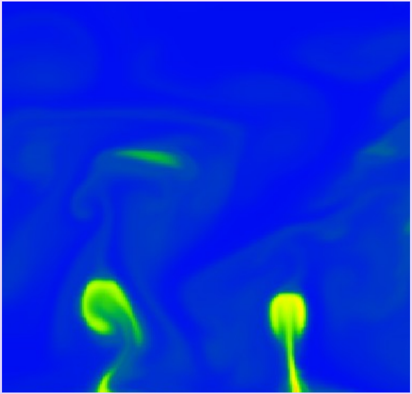

Geometry Play Skip

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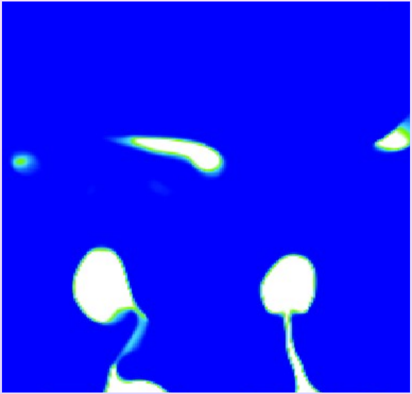

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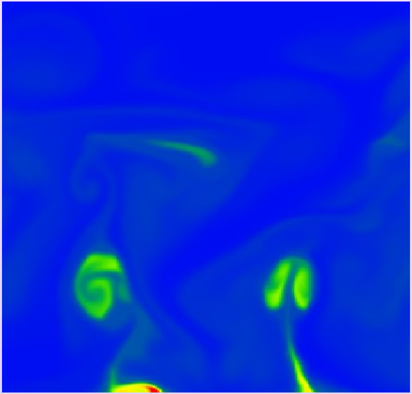

◀ Geometry ▶ Play ⏩ Skip

Film

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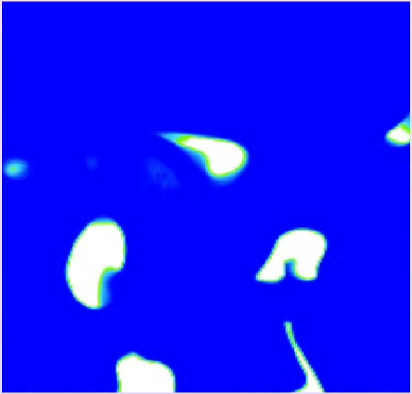

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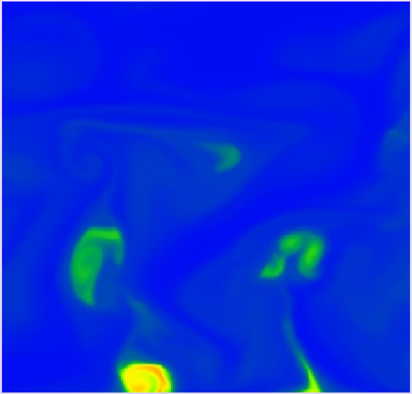

Geometry Play Skip

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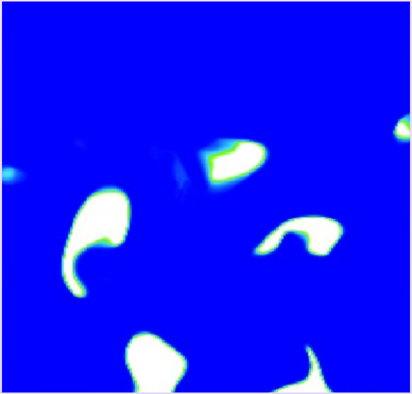

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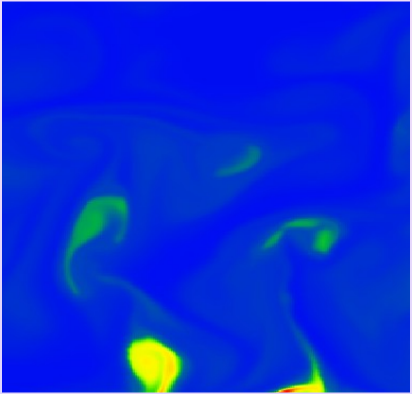

◂ Geometry ▸ Play ▸▸ Skip

Film

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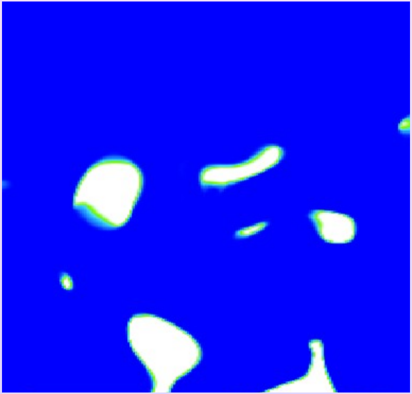

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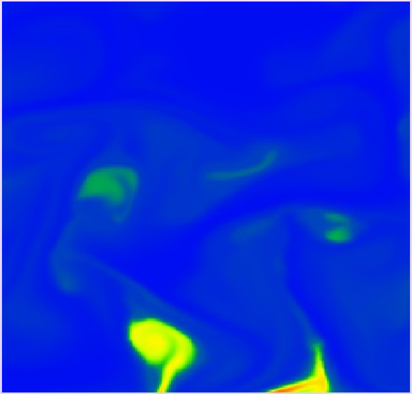
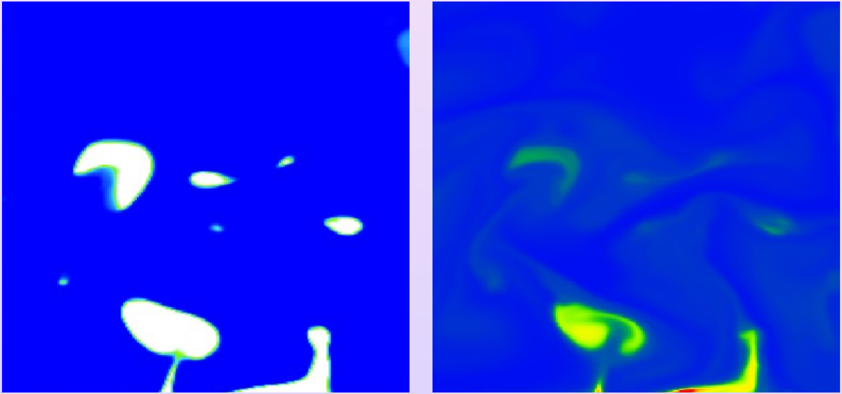


◂ Geometry ▸ Play ▸▸ Skip

Film

Mass Fraction y Temperature T



◀ Geometry ▶ Play ▶▶ Skip

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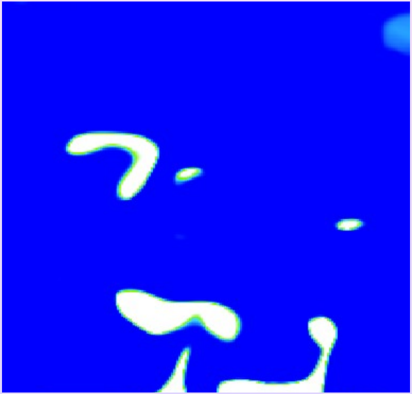

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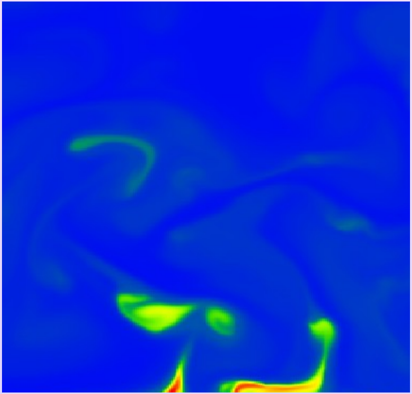

Geometry Play Skip

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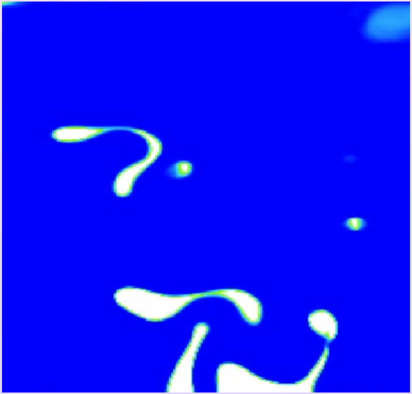

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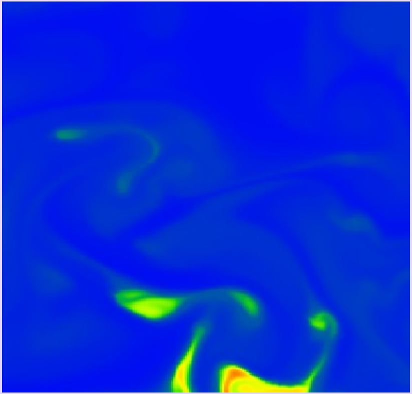

Geometry Play Skip

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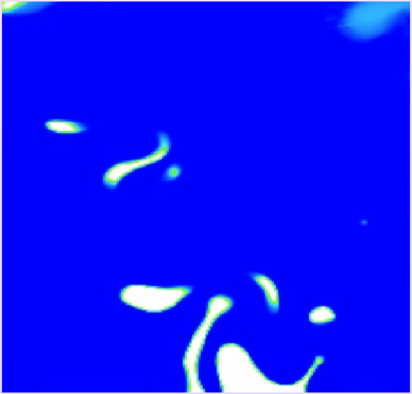

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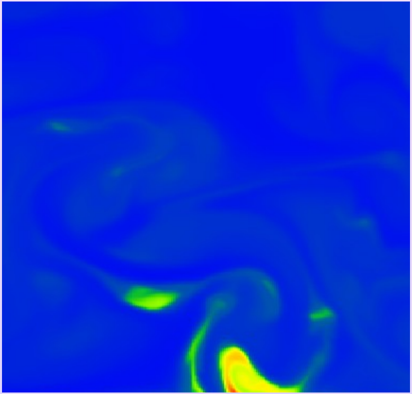

◂ Geometry ▸ Play ▸▸ Skip

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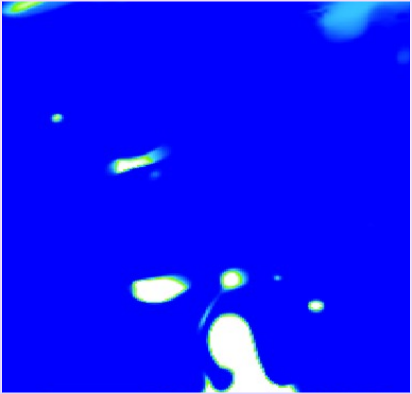

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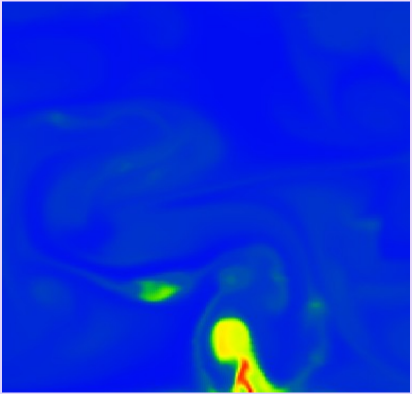

◀ Geometry ▶ Play ▶▶ Skip

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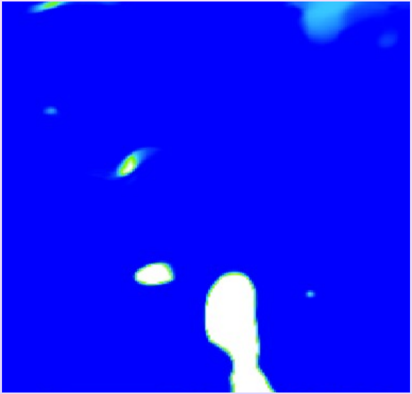

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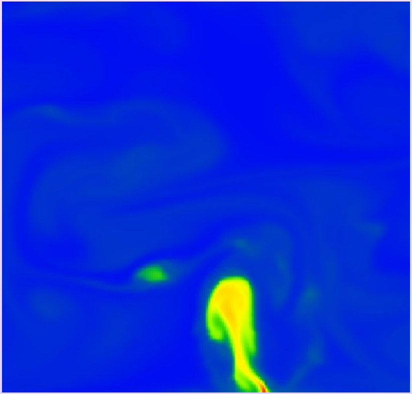

Geometry Play Skip

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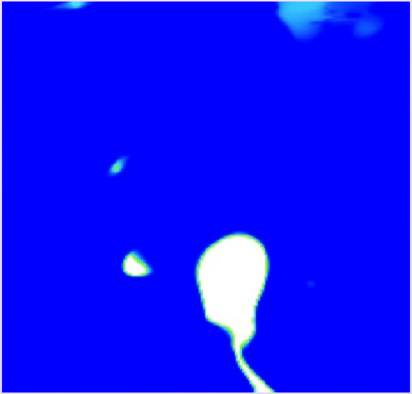

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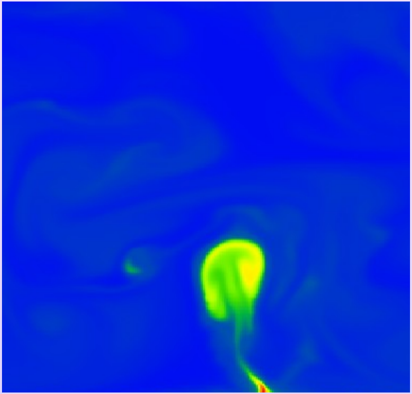

◂ Geometry ▸ Play ▸▸ Skip

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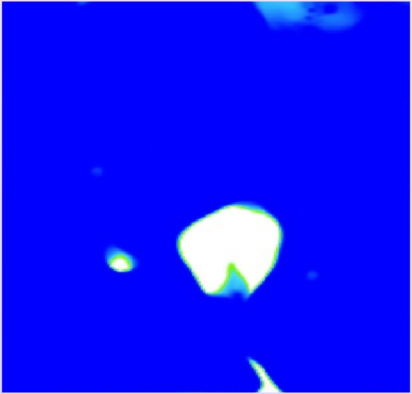

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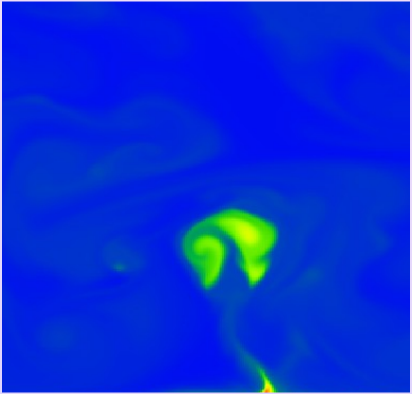

Geometry Play Skip

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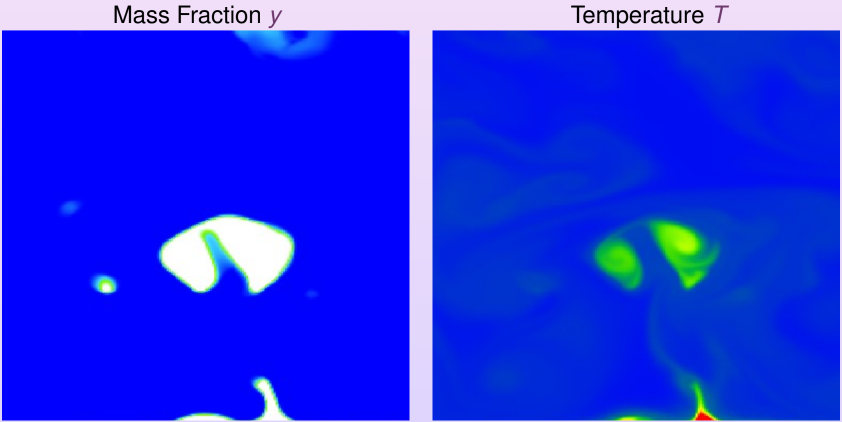

◂ Geometry ▸ Play ▸▸ Skip

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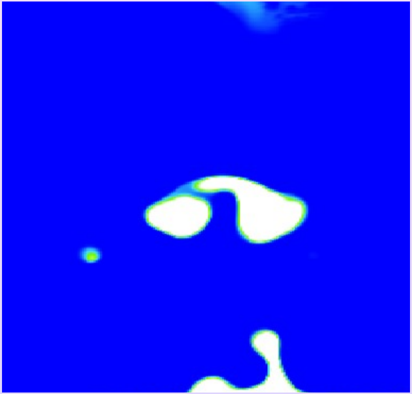
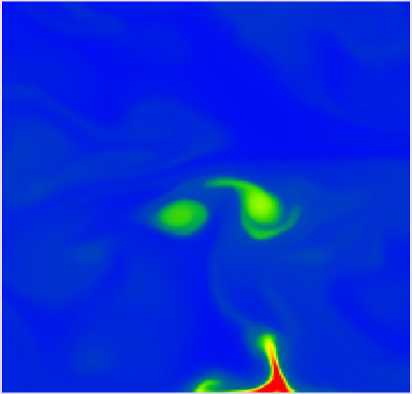


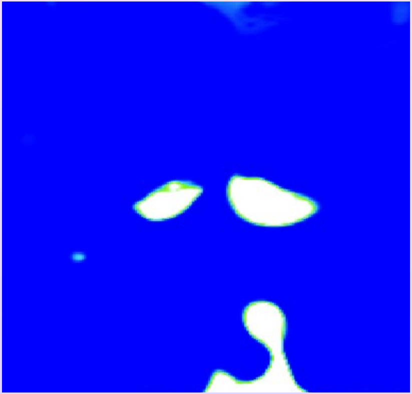
Temperature T



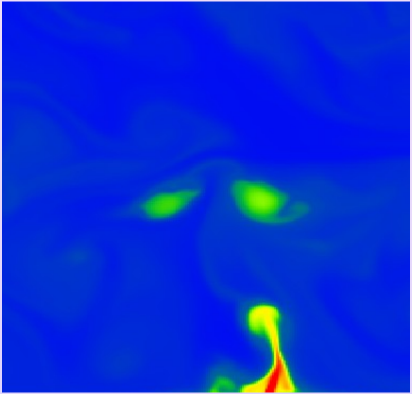
Geometry Play Skip

Film

Mass Fraction y



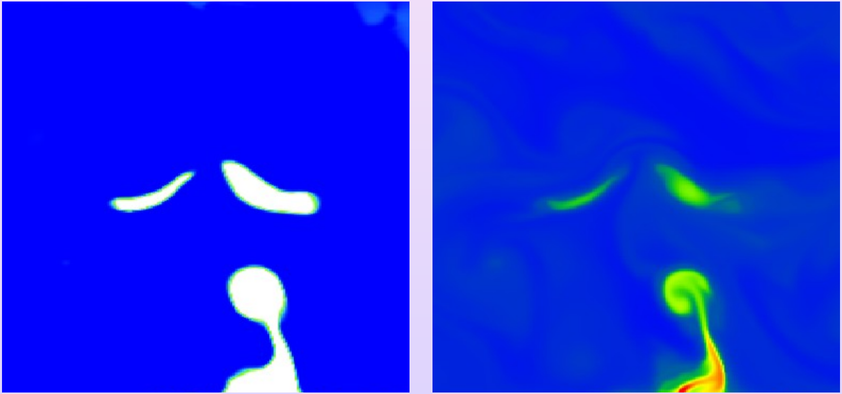
Temperature T



Geometry Play Skip

Film

Mass Fraction y Temperature T



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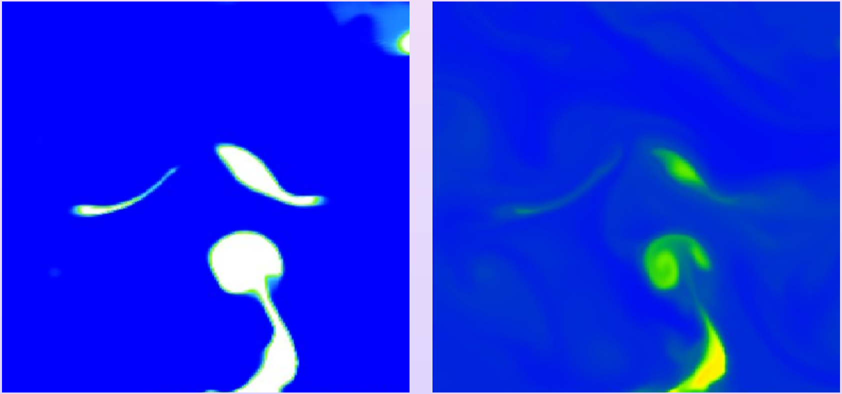

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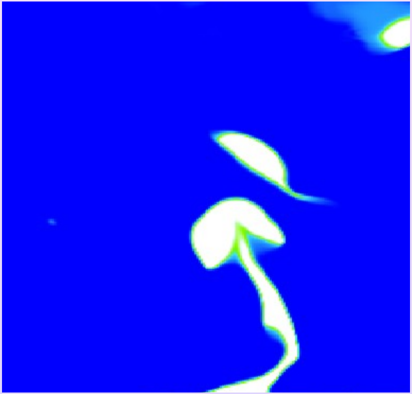

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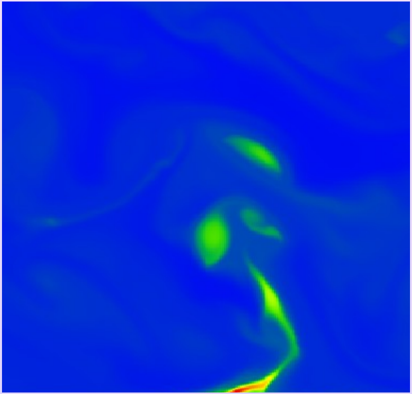

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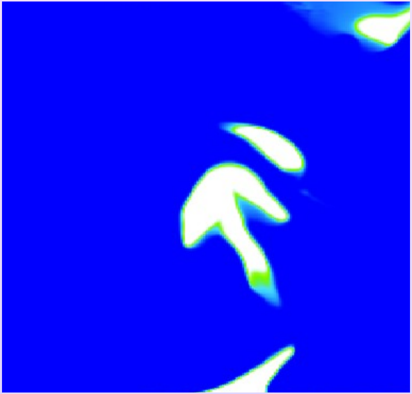

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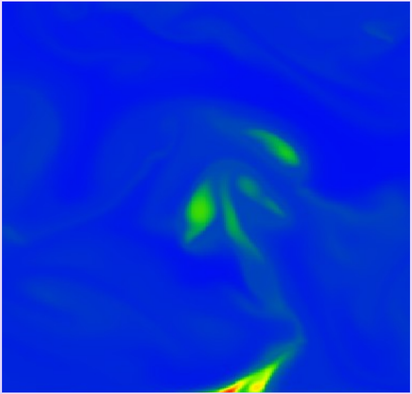

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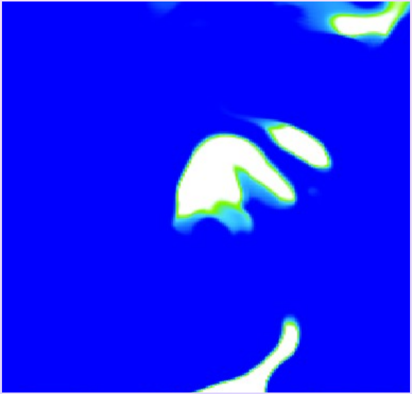

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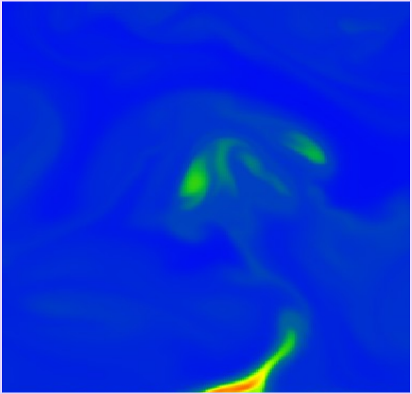

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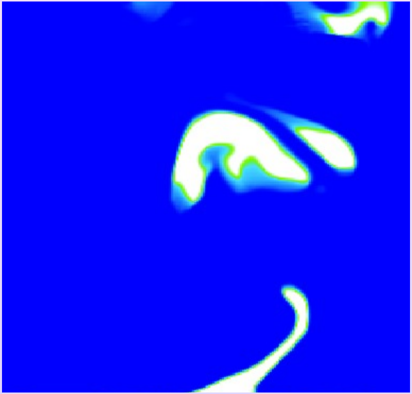

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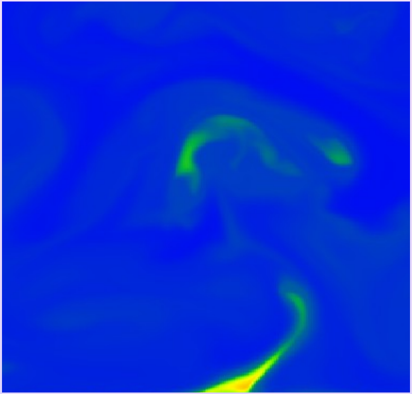

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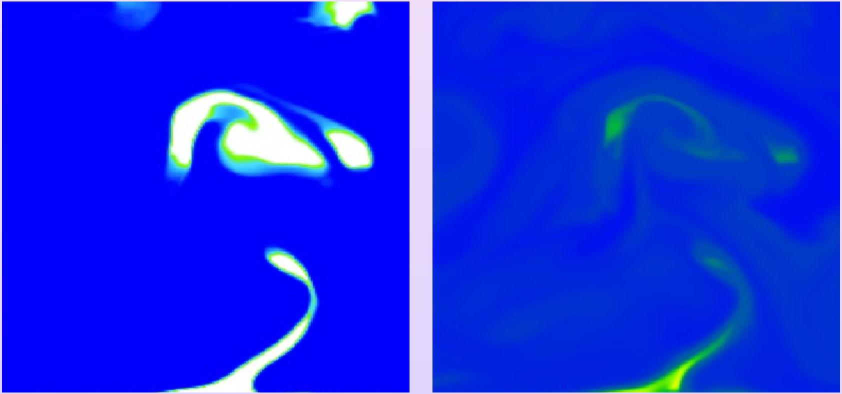
Mass Fraction y Temperature T



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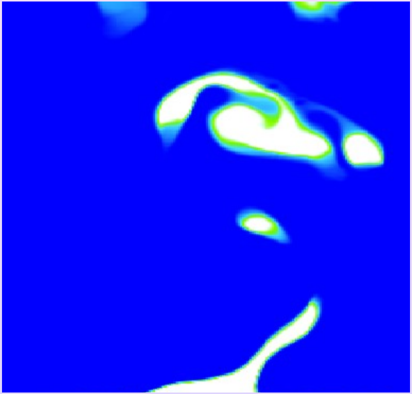

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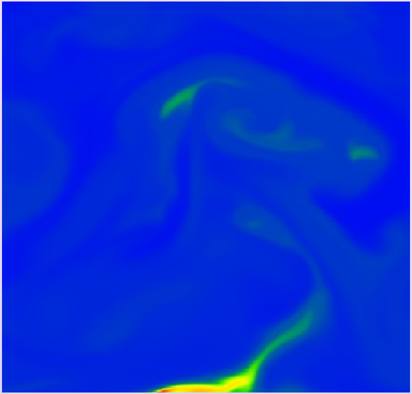

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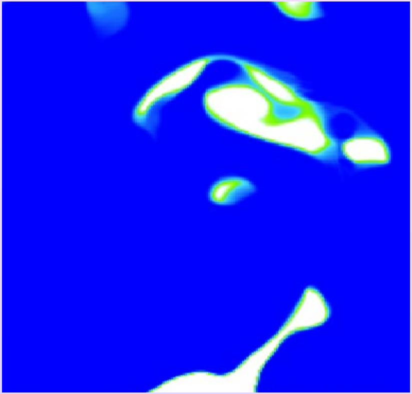

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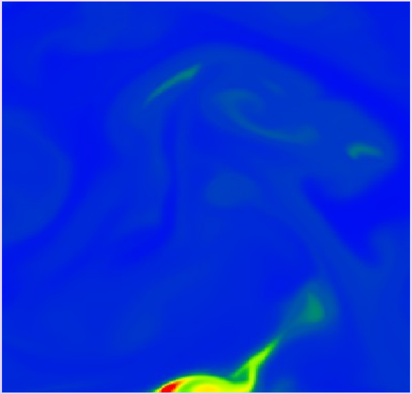

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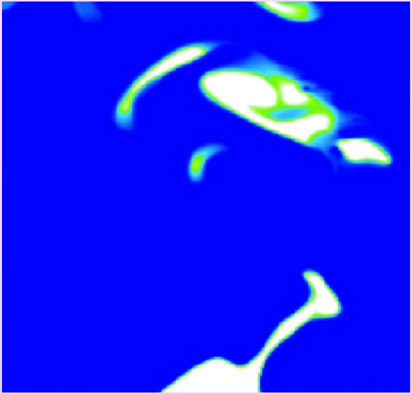

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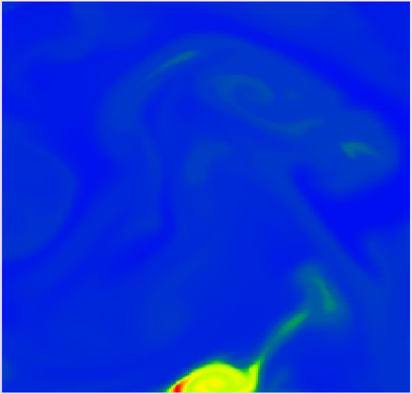

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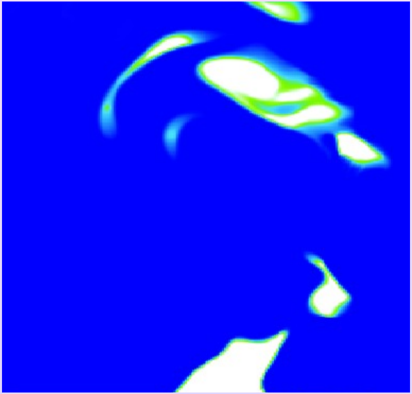

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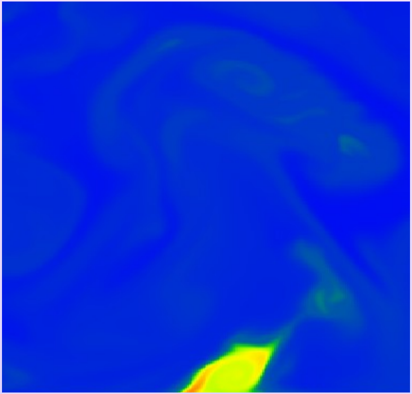

Geometry Play Skip

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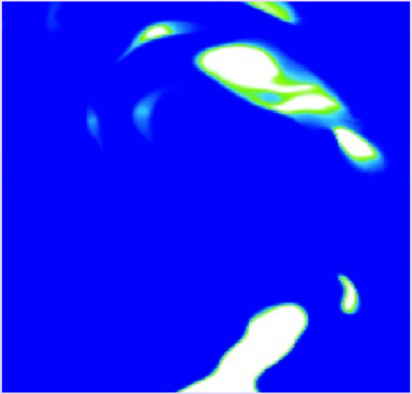
Mass Fraction y



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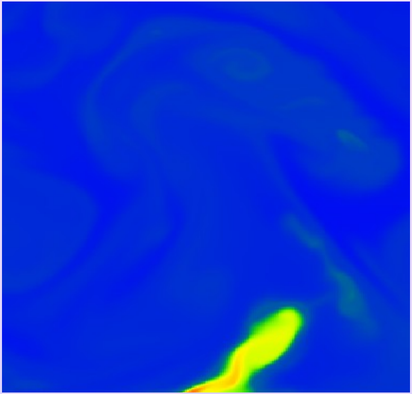

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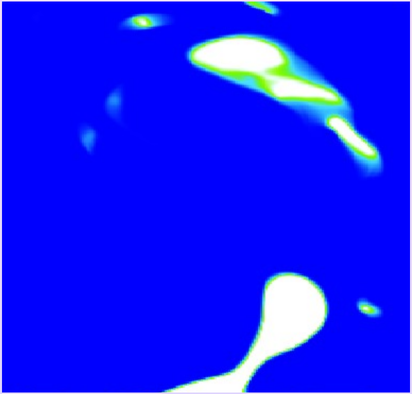

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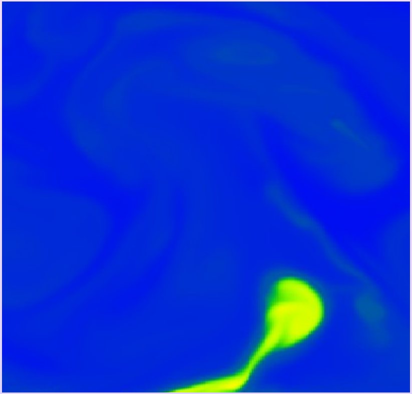

Geometry Play Skip

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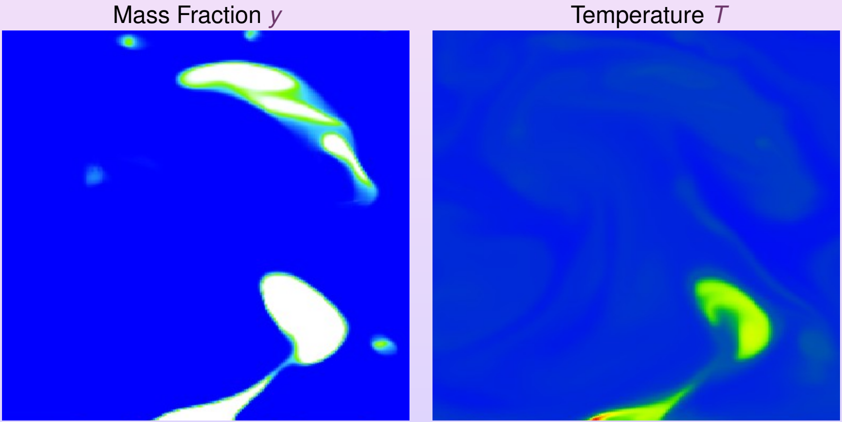
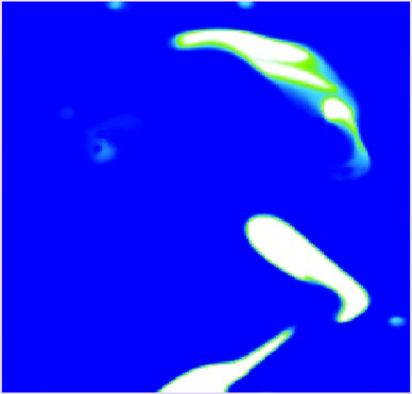


◂ Geometry ▸ Play ▸▸ Skip

Film

Mass Fraction y



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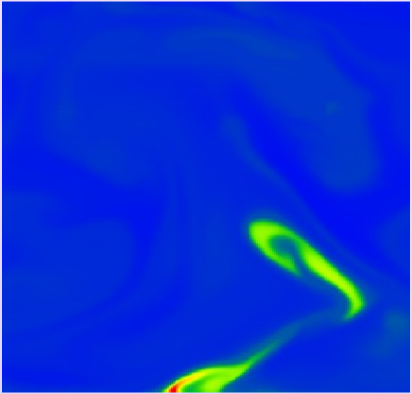

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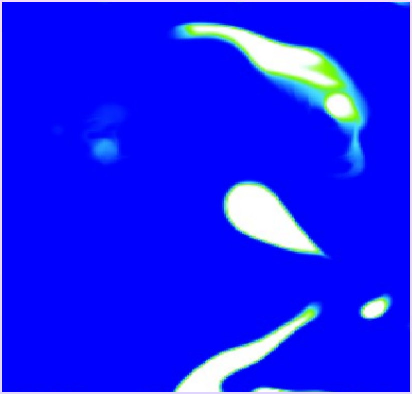

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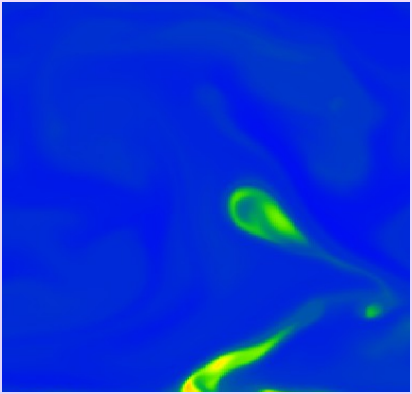

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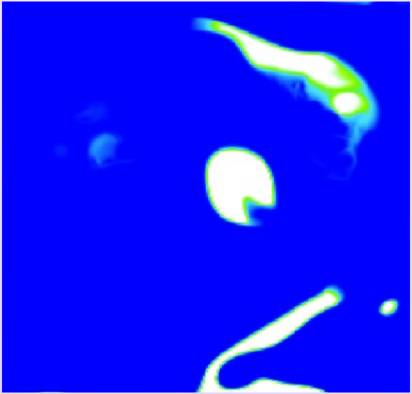

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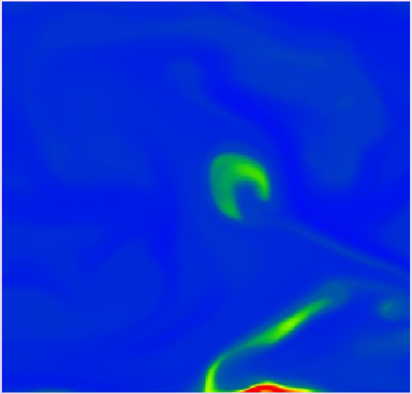

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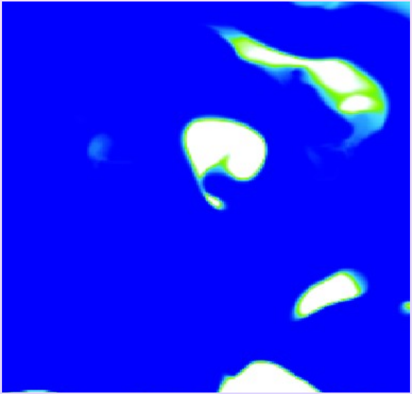

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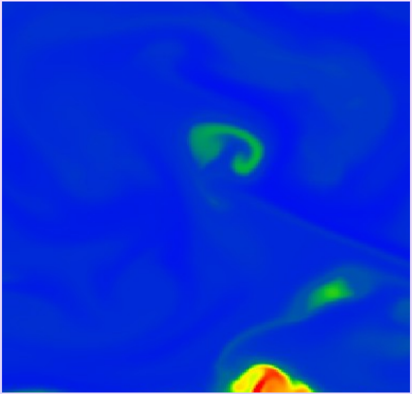

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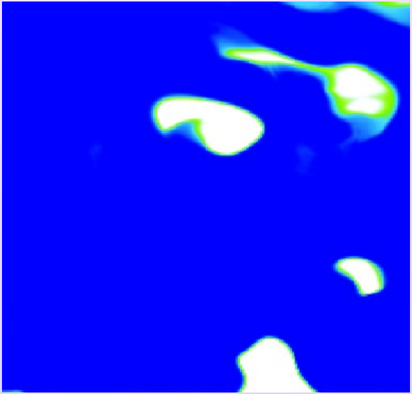

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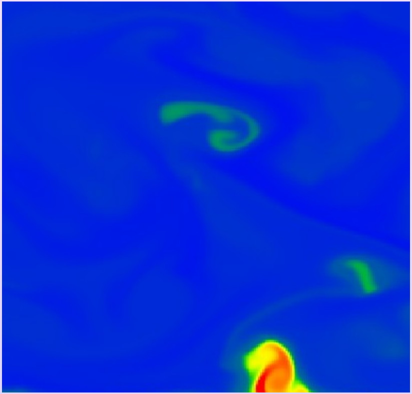

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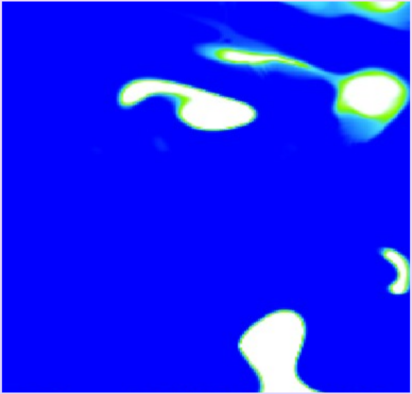

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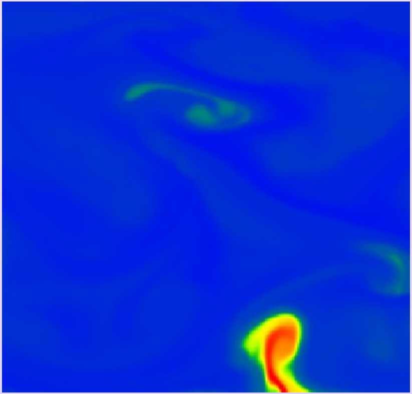

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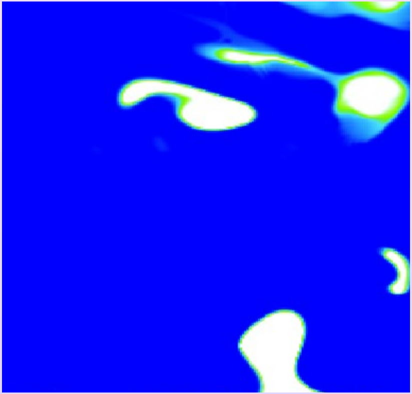

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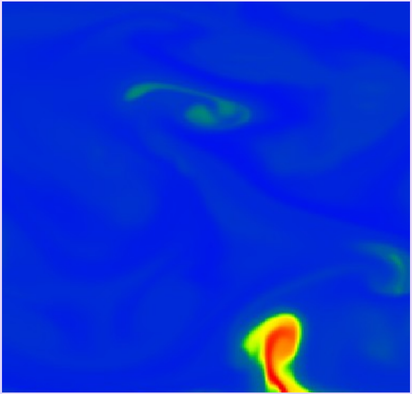

Geometry Play Skip

Outline

Summary & Perspectives

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  - ✓ general construction of the Equilibrium EOS (also for tabulated data),
  - ✓ strict hyperbolicity of the Euler system with the Equilibrium EOS,
- Numerical Method based on the relaxation approach: augmented systems with relaxation terms
  - ✓ operator splitting based on the 5-eqs iso-T with a Roe like solver [G. Allaire, S. Clerc, S. Kokh],
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  - ✗ **3D simulations (parallelization)**.

Appendix

- Stiffened Gas for Water
- Tabulated EOS for Water
- Speed of sound
- Isentropic curves
- Surface Tension
- Metastability
- Critical Point

Stiffened Gas for Water

Phase	c_v [J/(kg·K)]	γ	π [Pa]	q [J/kg]	m [J/(kg·K)]
Water	1816.2	2.35	10^9	-1167.056×10^3	-32765.55596
Steam	1040.14	1.43	0	2030.255×10^3	-33265.65947

Table: Parameters proposed by [O. LE METAYER] for water.

$$(\tau_\alpha, \varepsilon_\alpha) \mapsto s_\alpha = c_{v_\alpha} \ln(\varepsilon_\alpha - q_\alpha - \pi_\alpha \tau_\alpha) + c_{v_\alpha}(\gamma_\alpha - 1) \ln \tau_\alpha + m_\alpha$$

$$(P,T) \mapsto \varepsilon_\alpha = c_{v_\alpha} T \frac{P + \pi_\alpha \gamma_\alpha}{P + \pi_\alpha} + q_\alpha, \qquad (P,T) \mapsto \tau_\alpha = c_{v_\alpha}(\gamma_\alpha - 1) \frac{T}{P + \pi_\alpha}.$$

$$\left.\begin{array}{l} T^i = 278\text{K} \ldots 610\text{K}, \\ g_1(P,T^i) = g_2(P,T^i) \Rightarrow P^{\text{sat}}(T^i) \end{array}\right\} \Rightarrow \mathfrak{A} = \left\{ \left(T^i, P^{\text{sat}}(T^i)\right) \right\}_{i=0}^{83}$$

$\widehat{P}^{\text{sat}}$ defined by using a least square approximation of $\mathfrak{A}$:

$$T \mapsto P^{\text{sat}}(T) \approx \widehat{P}^{\text{sat}}(T) \stackrel{\text{def}}{=} \exp\left(\sum_{k=-8}^{k=8} a_k T^k\right)$$

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Phase	c_v [J/(kg·K)]	γ	π [Pa]	q [J/kg]	m [J/(kg·K)]
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Steam	1040.14	1.43	0	2030.255×10^3	-33265.65947

Table: Parameters proposed by [O. LE METAYER] for water.

$$(\tau_\alpha, \varepsilon_\alpha) \mapsto s_\alpha = c_{v_\alpha} \ln(\varepsilon_\alpha - q_\alpha - \pi_\alpha \tau_\alpha) + c_{v_\alpha}(\gamma_\alpha - 1) \ln \tau_\alpha + m_\alpha$$

$$(P,T) \mapsto \varepsilon_\alpha = c_{v_\alpha} T \frac{P + \pi_\alpha \gamma_\alpha}{P + \pi_\alpha} + q_\alpha, \qquad (P,T) \mapsto \tau_\alpha = c_{v_\alpha}(\gamma_\alpha - 1)\frac{T}{P + \pi_\alpha}.$$

$$\left.\begin{array}{l} T^i = 278\text{K} \dots 610\text{K}, \\ g_1(P,T^i) = g_2(P,T^i) \Rightarrow P^{\text{sat}}(T^i) \end{array}\right\} \Rightarrow \mathfrak{A} = \left\{ \left(T^i, P^{\text{sat}}(T^i)\right) \right\}_{i=0}^{83}$$

$\widehat{P}^{\text{sat}}$ defined by using a least square approximation of $\mathfrak{A}$:

$$T \mapsto P^{\text{sat}}(T) \approx \widehat{P}^{\text{sat}}(T) \stackrel{\text{def}}{=} \exp\left(\sum_{k=-8}^{k=8} a_k T^k\right)$$

Stiffened Gas for Water

Phase	c_v [J/(kg·K)]	γ	π [Pa]	q [J/kg]	m [J/(kg·K)]
Water	1816.2	2.35	10^9	-1167.056×10^3	-32765.55596
Steam	1040.14	1.43	0	2030.255×10^3	-33265.65947

Table: Parameters proposed by [O. Le Metayer] for water.

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Water Tabulated EOS

$$\left.\begin{array}{l} T^i = 278\text{K}\ldots 610\text{K}, \\ \varepsilon_\alpha^{\text{sat}}(T^i),\ \tau_\alpha^{\text{sat}}(T^i) \text{ found in the tables} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \mathfrak{A} = \left\{\left(T_i, \frac{1}{\varepsilon_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \\ \mathfrak{B} = \left\{\left(T_i, \frac{\varepsilon_{\text{liq}}^{\text{sat}}(T_i)}{\varepsilon_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \\ \mathfrak{C} = \left\{\left(T_i, \frac{1}{\tau_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \\ \mathfrak{D} = \left\{\left(T_i, \frac{\tau_{\text{liq}}^{\text{sat}}(T_i)}{\tau_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \end{array}\right.$$

$\widehat{\varepsilon}_\alpha^{\text{sat}}$ and $\widehat{\tau}_\alpha^{\text{sat}}$ defined by using a least square approximation of $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ and $\mathfrak{D}$:

$$T \mapsto \varepsilon_{\text{vap}}^{\text{sat}} \approx \widehat{\varepsilon}_{\text{vap}}^{\text{sat}} \stackrel{\text{def}}{=} \frac{1}{\sum_{k=0}^{6} a_k T^k} \qquad T \mapsto \varepsilon_{\text{liq}}^{\text{sat}} \approx \widehat{\varepsilon}_{\text{liq}}^{\text{sat}} \stackrel{\text{def}}{=} \widehat{\varepsilon}_{\text{vap}}^{\text{sat}}(T) \sum_{k=0}^{6} b_k T^k$$

$$T \mapsto \tau_{\text{vap}}^{\text{sat}} \approx \widehat{\tau}_{\text{vap}}^{\text{sat}} \stackrel{\text{def}}{=} \frac{1}{\sum_{k=0}^{8} c_k T^k} \qquad T \mapsto \tau_{\text{liq}}^{\text{sat}} \approx \widehat{\tau}_{\text{liq}}^{\text{sat}} \stackrel{\text{def}}{=} \widehat{\tau}_{\text{vap}}^{\text{sat}}(T) \sum_{k=0}^{9} d_k T^k$$

Speed of sound

$$c^2 \stackrel{\text{def}}{=} \tau^2 \left(P^{\text{eq}} \frac{\partial P^{\text{eq}}}{\partial \varepsilon}\bigg|_{\tau} - \frac{\partial P^{\text{eq}}}{\partial \tau}\bigg|_{\varepsilon} \right) = \underbrace{-\tau^2 T^{\text{eq}}}_{<0} \underbrace{\begin{bmatrix} P^{\text{eq}}, & -1 \end{bmatrix} \begin{bmatrix} s^{\text{eq}}_{\varepsilon\varepsilon} & s^{\text{eq}}_{\tau\varepsilon} \\ s^{\text{eq}}_{\tau\varepsilon} & s^{\text{eq}}_{\tau\tau} \end{bmatrix} \begin{bmatrix} P^{\text{eq}} \\ -1 \end{bmatrix}}_{\leq 0}$$

Hessian matrix of $\mathbf{w} \mapsto s^{\text{eq}}$

- for all $\mathbf{w}$ pure phase state

$$\mathbf{v}^T \, \mathrm{d}^2 s^{\text{eq}}(\mathbf{w}) \, \mathbf{v} < 0 \quad \forall \, \mathbf{v} \neq 0,$$

- for all $\mathbf{w}$ equilibrium mixture state

$$\exists \, \mathbf{v}(\mathbf{w}) \neq 0 \;\text{ s.t. } (\mathbf{v}(\mathbf{w}))^T \, \mathrm{d}^2 s^{\text{eq}}(\mathbf{w}) \, \mathbf{v}(\mathbf{w}) = 0.$$

Speed of sound

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Speed of sound

$$c^2 \stackrel{\text{def}}{=} \tau^2 \left(P^{eq} \frac{\partial P^{eq}}{\partial \varepsilon}\Big|_\tau - \frac{\partial P^{eq}}{\partial \tau}\Big|_\varepsilon \right) = \underbrace{-\tau^2 T^{eq}}_{<0} \underbrace{\begin{bmatrix} P^{eq}, & -1 \end{bmatrix} \begin{bmatrix} s^{eq}_{\varepsilon\varepsilon} & s^{eq}_{\tau\varepsilon} \\ s^{eq}_{\tau\varepsilon} & s^{eq}_{\tau\tau} \end{bmatrix} \begin{bmatrix} P^{eq} \\ -1 \end{bmatrix}}_{\leq 0}$$

HESSIAN MATRIX OF $\mathbf{w} \mapsto s^{eq}$

- for all $\mathbf{w}$ pure phase state

$$\mathbf{v}^T \, \mathrm{d}^2 s^{eq}(\mathbf{w}) \, \mathbf{v} < 0 \quad \forall \, \mathbf{v} \neq 0,$$

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Speed of sound

$$c^2 \stackrel{\text{def}}{=} \tau^2 \left(P^{\text{eq}} \left.\frac{\partial P^{\text{eq}}}{\partial \varepsilon}\right|_{\tau} - \left.\frac{\partial P^{\text{eq}}}{\partial \tau}\right|_{\varepsilon} \right) = \underbrace{\boxed{-\tau^2 T^{\text{eq}}}}_{\leq 0} \boxed{\underbrace{\begin{bmatrix} P^{\text{eq}}, & -1 \end{bmatrix} \begin{bmatrix} s^{\text{eq}}_{\varepsilon\varepsilon} & s^{\text{eq}}_{\tau\varepsilon} \\ s^{\text{eq}}_{\tau\varepsilon} & s^{\text{eq}}_{\tau\tau} \end{bmatrix} \begin{bmatrix} P^{\text{eq}} \\ -1 \end{bmatrix}}_{\leq 0}}$$

HESSIAN MATRIX OF $\mathbf{w} \mapsto s^{\text{eq}}$

- for all $\mathbf{w}$ pure phase state

$$\mathbf{v}^T \, \mathrm{d}^2 s^{\text{eq}}(\mathbf{w}) \, \mathbf{v} < 0 \quad \forall \, \mathbf{v} \neq 0,$$

- for all $\mathbf{w}$ equilibrium mixture state

$$\exists \, \mathbf{v}(\mathbf{w}) \neq 0 \;\; \text{s.t.} \; (\mathbf{v}(\mathbf{w}))^T \, \mathrm{d}^2 s^{\text{eq}}(\mathbf{w}) \, \mathbf{v}(\mathbf{w}) = 0.$$

$$\boxed{\forall \mathbf{w} \text{ equilibrium mixture state, } \mathbf{v}(\mathbf{w}) \stackrel{?}{\equiv} [P^{\text{eq}}(\mathbf{w}), -1]}$$

Speed of sound

$$c^2 \stackrel{\text{def}}{=} \tau^2 \left(P^{\text{eq}} \left.\frac{\partial P^{\text{eq}}}{\partial \varepsilon}\right|_\tau - \left.\frac{\partial P^{\text{eq}}}{\partial \tau}\right|_\varepsilon \right) = \underbrace{-\tau^2 T^{\text{eq}}}_{<0} \underbrace{\begin{bmatrix} P^{\text{eq}}, & -1 \end{bmatrix} \begin{bmatrix} s^{\text{eq}}_{\varepsilon\varepsilon} & s^{\text{eq}}_{\tau\varepsilon} \\ s^{\text{eq}}_{\tau\varepsilon} & s^{\text{eq}}_{\tau\tau} \end{bmatrix} \begin{bmatrix} P^{\text{eq}} \\ -1 \end{bmatrix}}_{\leq 0}$$

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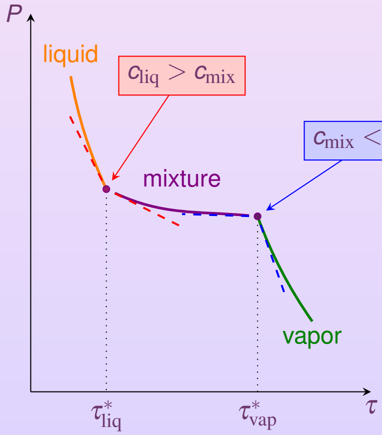
$\forall \mathbf{w}$ equilibrium mixture state, $\mathbf{v}(\mathbf{w}) \overset{?}{\not\asymp} [P^{\text{eq}}(\mathbf{w}), -1]$

Isentropic curves



$$\gamma \stackrel{\text{def}}{=} -\frac{\tau}{P}\left.\frac{\partial P}{\partial \tau}\right|_s$$

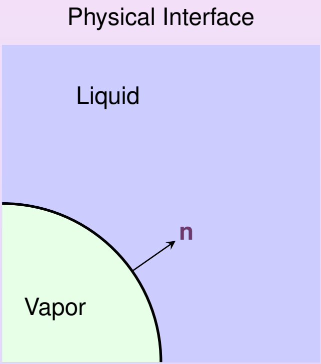
$$\Gamma \stackrel{\text{def}}{=} \tau \left.\frac{\partial P}{\partial \varepsilon}\right|_\tau$$

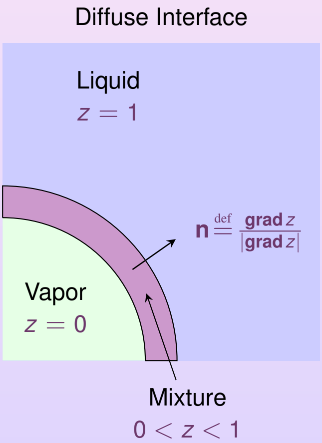
$$\mathfrak{G} \stackrel{\text{def}}{=} \frac{\tau^2}{2\gamma P}\left.\frac{\partial^2 P}{\partial \tau^2}\right|_s$$

- Pure Phases
  - (H) $\gamma > 0$
  - (H) $\Gamma > 0$
  - (H) $\mathfrak{G} > 0$
- Mixture
  - (P) $\gamma > 0$
  - (P) $\Gamma > 0$
  - (H) $\mathfrak{G} > 0$

- Regularity: [J. Correia, P.G. LeFloch, M.D. Thanh]
- Loss of convexity: [A. Voss]

Continuum Surface Force (CSF) Approach

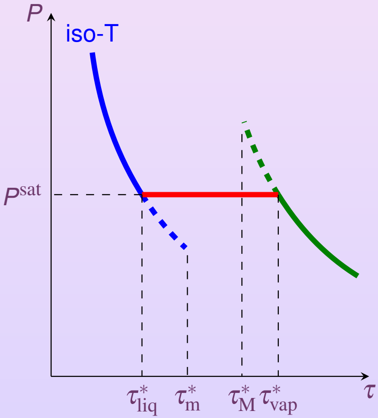




$$\Pi_{\text{tension}} = -\sigma \, \text{div}(\mathbf{n})\mathbf{n}$$

[J.U. BRACKBILL, D.B. KOTHE, C. ZEMACH]

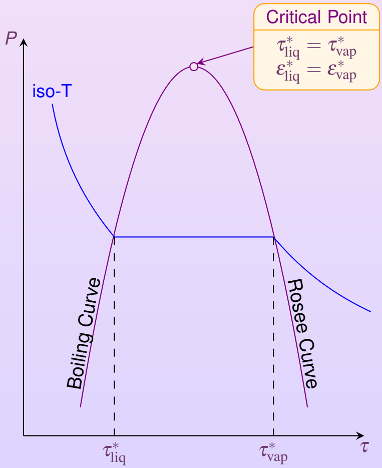
Metastability



$$P^{\text{eq}} = \begin{cases} P_{\text{liq}}, & \text{if } \tau < \tau^*_{\text{liq}}, \\ P^{\text{sat}}, & \text{if } \tau^*_{\text{liq}} < \tau < \tau^*_{\text{vap}}, \\ P_{\text{vap}}, & \text{if } \tau^*_{\text{vap}} < \tau. \end{cases}$$

$$P^{\text{met}} = \begin{cases} P_{\text{liq}}, & \text{if } \tau < \tau^*_{\text{liq}}, \\ [P^{\text{sat}} \text{ or } P_{\text{liq}}], & \text{if } \tau^*_{\text{liq}} < \tau < \tau^*_{\text{m}}, \\ P^{\text{sat}}, & \text{if } \tau^*_{\text{m}} < \tau < \tau^*_{\text{M}}, \\ [P^{\text{sat}} \text{ or } P_{\text{vap}}], & \text{if } \tau^*_{\text{M}} < \tau < \tau^*_{\text{vap}}, \\ P_{\text{vap}}, & \text{if } \tau^*_{\text{vap}} < \tau, \end{cases}$$

Critical Point



Physic

- 2 Pure Phases EOS $(\tau, \varepsilon) \mapsto P_\alpha$
- 1 Saturation EOS $\tau \mapsto P^{\text{sat}}$

Eq

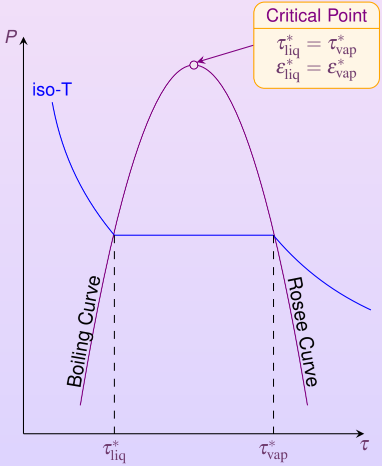
EOS

PG $\varepsilon^*_{\text{liq}} = \varepsilon^*_{\text{vap}} \Leftrightarrow c_{v_{\text{liq}}} = c_{v_{\text{vap}}}$ (indip. of T)

SG $\{\tau_i, P_i^{\text{sat,e}}\}_i \rightsquigarrow (\tau, \varepsilon) \mapsto P_\alpha \rightsquigarrow \tau \mapsto P^{\text{sat}}$
$\tau^*_{\text{liq}} = \tau^*_{\text{vap}}$ but $\varepsilon^*_{\text{liq}} \neq \varepsilon^*_{\text{vap}}$

TAB $\{\tau_i, P_i^{\text{sat,e}}\}_i \rightsquigarrow \tau \mapsto P^{\text{sat}}$
$\{(\tau_i, \varepsilon_i), (P^{\text{e}}_\alpha)_i\}_i \rightsquigarrow (\tau, \varepsilon) \mapsto P_\alpha$

Critical Point



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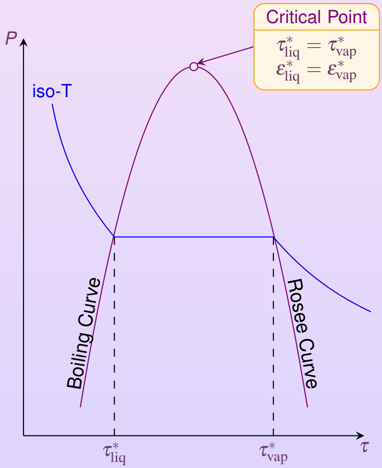

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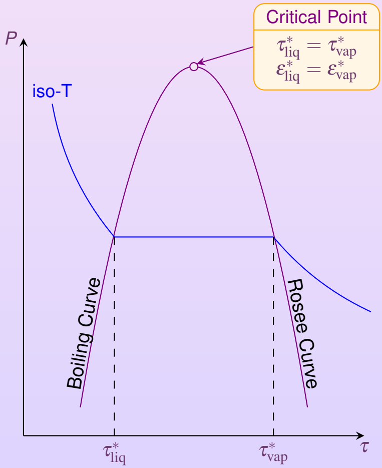

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