Modelling and Simulation of Liquid-Vapor Phase Transition

A Boiling Crisis Study Contribution

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Boiling Crisis

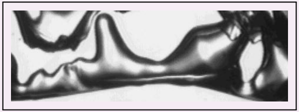
Phenomenon

Liquid phase heated by a wall at a fixed temperature T^{wall} (pool boiling).
When T^{wall} increases, we switch from a nucleate boiling to a film boiling.

Nucleate Boiling

→

Film Boiling



source: http://www.spaceflight.esa.int/users/fluids/TT_boiling.htm

Outline

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Euler System

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0, \\ \partial_t(\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u} + P\, \mathbb{I}) = \mathfrak{V}_{\text{vf}} - \mathfrak{S}_{\text{sf}}, \\ \partial_t \left(\rho \left(\frac{|\mathbf{u}|^2}{2} + \varepsilon \right) \right) + \operatorname{div} \left(\rho \left(\frac{|\mathbf{u}|^2}{2} + \varepsilon \right) \mathbf{u} + P\, \mathbf{u} \right) = (\mathfrak{V}_{\text{vf}} - \mathfrak{S}_{\text{sf}}) \cdot \mathbf{u} - \operatorname{div}(q). \end{cases}$$

- $(\mathbf{x}, t) \mapsto \rho$ specific density,
- $(\mathbf{x}, t) \mapsto \varepsilon$ specific internal energy,
- $(\mathbf{x}, t) \mapsto \mathbf{u}$ velocity;
- $(\rho, \varepsilon) \mapsto \mathfrak{V}_{\text{vf}}$ volumic forces,
- $(\rho, \varepsilon) \mapsto \mathfrak{S}_{\text{sf}}$ surface forces,
- $(\rho, \varepsilon) \mapsto \operatorname{div}(q)$ heat transfert.

$(\rho, \varepsilon) \mapsto P$ pressure law.

Euler System

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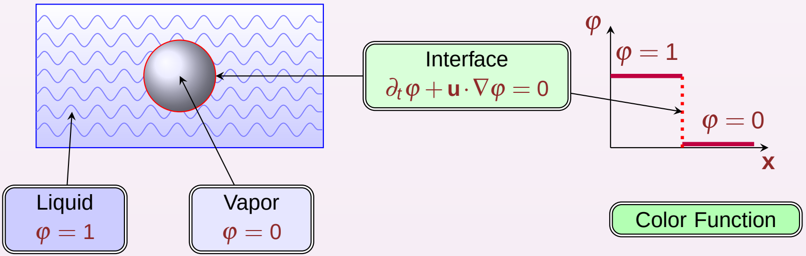
Liquid-Vapor Interface



$$(\rho, \varepsilon) \mapsto P = \begin{cases} P^{\text{liq}} & \text{if } \varphi = 1; \\ P^{\text{vap}} & \text{if } \varphi = 0. \end{cases}$$

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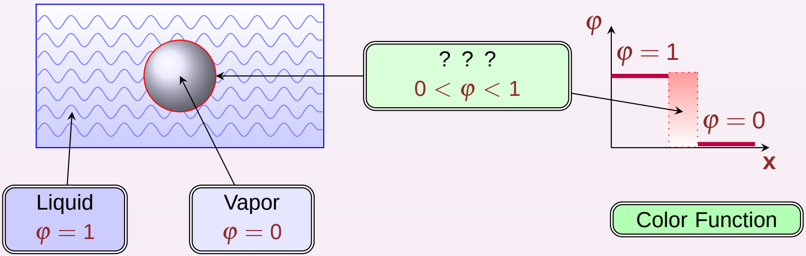

$$(\rho, \varepsilon) \mapsto P = \begin{cases} P^{\text{liq}} & \text{if } \varphi = 1; \\ ??? & \text{if } 0 < \varphi < 1; \\ P^{\text{vap}} & \text{if } \varphi = 0. \end{cases}$$

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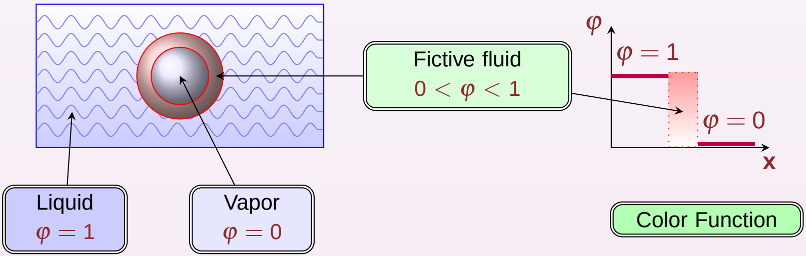

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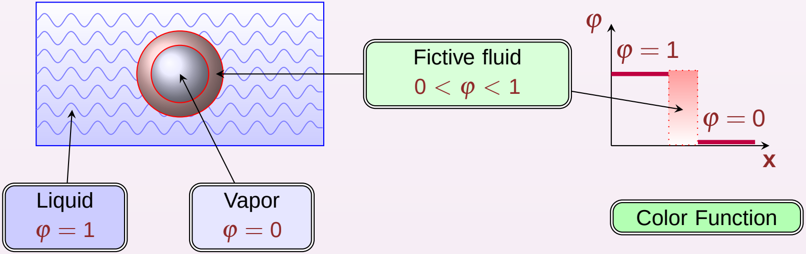

Goal: define a global pressure law such that

- $(\rho, \varepsilon, \mathbf{u}, P)$ are continuous (3 zones)
- the interface position and the phase change are implicit ($\rightsquigarrow$ ~~φ~~)
- coherence with classical thermodynamics [H. Callen]

EOS of each Phase $\alpha = 1, 2$

τ_α specific volume
ε_α specific internal energy $\Bigg\} \Rightarrow \quad \mathbf{w}_\alpha \overset{\text{def}}{=} (\tau_\alpha, \varepsilon_\alpha);$

$\mathbf{w}_\alpha \mapsto s_\alpha$ specific entropy (Hessian matrix neg. def.);

$$\begin{cases} T_\alpha \overset{\text{def}}{=} \left(\left.\dfrac{\partial s_\alpha}{\partial \varepsilon_\alpha}\right|_{\tau_\alpha} \right)^{-1} > 0 & \text{temperature,} \\ P_\alpha \overset{\text{def}}{=} T_\alpha \left.\dfrac{\partial s_\alpha}{\partial \tau_\alpha}\right|_{\varepsilon_\alpha} > 0 & \text{pressure,} \\ g_\alpha \overset{\text{def}}{=} \varepsilon_\alpha + P_\alpha \tau_\alpha - T_\alpha s_\alpha & \text{free enthalpy (Gibbs potential).} \end{cases}$$

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EOS without Phase Change

- $\mathbf{w} \stackrel{\text{def}}{=} y\mathbf{w}_1 + (1-y)\mathbf{w}_2$;
- y mass fraction;
- z volume fraction s.t. $y\tau_1 = z\tau$;
- ψ energy fraction s.t. $y\varepsilon_1 = \psi\varepsilon$.

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ENTROPY WITHOUT PHASE CHANGE

$$\sigma \stackrel{\text{def}}{=} ys_1(\mathbf{w}_1) + (1-y)s_2(\mathbf{w}_2) = ys_1\left(\frac{z}{y}\tau, \frac{\psi}{y}\varepsilon\right) + (1-y)s_2\left(\frac{1-z}{1-y}\tau, \frac{1-\psi}{1-y}\varepsilon\right)$$

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Entropy without phase change

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EOS WITH PHASE CHANGE

ENTROPY WITHOUT PH.CH.

$(\mathbf{w}, z, y, \psi) \mapsto \sigma$

→

ENTROPY AT EQUILIBRIUM

$\mathbf{w} \mapsto s^{\text{eq}}$

DEFINITION [H. CALLEN, PH. HELLUY . . .]

Optimization Problem:

$$s^{\text{eq}}(\mathbf{w}) \stackrel{\text{def}}{=} \max_{z,y,\psi \in [0,1]^3} \sigma(\mathbf{w}, z, y, \psi)$$

Optimality Condition:

$$\begin{cases} T_1(z,y,\psi) = T_2(z,y,\psi) \\ P_1(z,y,\psi) = P_2(z,y,\psi) \\ g_1(z,y,\psi) = g_2(z,y,\psi) \\ z,y,\psi \in]0,1[^3 \end{cases}$$

Solution: (z^*, y^*, ψ^*)

EOS with Phase Change

Entropy without Ph.Ch.

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Entropy at equilibrium

$\mathbf{w} \mapsto s^{\text{eq}}$

Definition [H. Callen, Ph. Helluy . . .]

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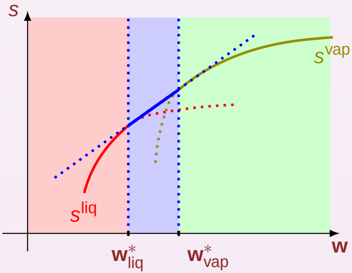

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$\mathbf{w} \mapsto s^{eq}$

Definition [H. Callen, Ph. Helluy ...]

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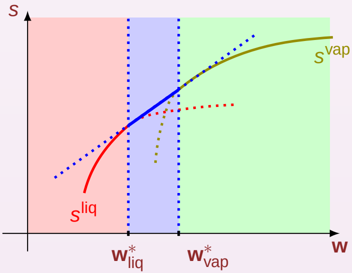

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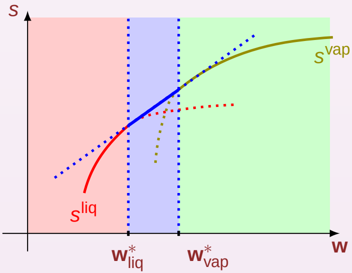

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Dynamic Liquid-Vapor Phase Change

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Properties [G. Allaire, G. Faccanoni, S. Kokh]

If $\tau_1^* \neq \tau_2^*$ and $\varepsilon_1^* \neq \varepsilon_2^*$ (first order phase transition) then

❶ $c(\mathbf{w}) > 0$, ❷ $s_{\tau\varepsilon}^{\mathrm{eq}}(\mathbf{w}) > 0$

❶ Euler system: strict hyperbolicity ($\neq$ p-system),

❷ Riemann problem: multitude of entropic (Lax) solutions [R. Menikoff, B. J. Plohr], uniqueness of Liu solution.

DYNAMIC LIQUID-VAPOR PHASE CHANGE

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Dynamic Liquid-Vapor Phase Change

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Outline

How to simulate the Liu solution

- Exact Riemann Solver [A. Voß]
- Viscuous Solver (the Liu solution is the only solution that has a viscuous profile) [S. Jaouen]
- Solver(s) based on Relaxation Approach [F. Coquel, B. Perthame], [Th. Barberon, Ph. Helluy], [Ph. Helluy, N. Seguin], [F. Coquel, F. Caro, D. Jamet, S. Kokh], ...

Relaxation Approach

$$\partial_t \mathbf{U} + \operatorname{div} \mathbf{F}(\mathbf{U}) = \mathbf{0}$$

Relaxation Approach

$$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \frac{1}{\mu}\mathbf{R}(\mathbf{V}) \quad \xrightarrow[\mu\to 0]{\text{Formally}} \quad \partial_t \mathbf{U} + \operatorname{div} \mathbf{F}(\mathbf{U}) = \mathbf{0}$$

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$$P^{\text{eq}}(\rho,\varepsilon) = \frac{s_\tau^{\text{eq}}}{s_\varepsilon^{\text{eq}}}, \quad e \stackrel{\text{def}}{=} \frac{|\mathbf{u}|^2}{2} + \varepsilon$$

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In the interface

$$\begin{cases} \partial_t z + \mathbf{u} \cdot \mathbf{grad}\, z = \\ \partial_t y + \mathbf{u} \cdot \mathbf{grad}\, y = \\ \partial_t \psi + \mathbf{u} \cdot \mathbf{grad}\, \psi = \end{cases}$$

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In the interface

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$$\xrightarrow[\mu_j\to 0]{\text{Formally}}$$

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$$\xrightarrow[\mu_j\to 0]{\text{Formally}}$$

Equilibrium System

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0 \\ \partial_t(\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u}\otimes \mathbf{u} + P^{\mathrm{eq}}\,\mathbb{I}) = 0 \\ \partial_t(\rho e) + \operatorname{div}((\rho e + P^{\mathrm{eq}})\mathbf{u}) = 0 \end{cases}$$

$$P^{\mathrm{eq}}(\rho,\varepsilon) = \frac{s_\tau^{\mathrm{eq}}}{s_\varepsilon^{\mathrm{eq}}}, \qquad e \stackrel{\mathrm{def}}{=} \frac{|\mathbf{u}|^2}{2} + \varepsilon$$

Remark: $\partial_t \psi + \mathbf{u}\cdot \mathbf{grad}\,\psi = 0 \rightsquigarrow T_1 = T_2$.

NUMERICAL SCHEME

$$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{S}(\mathbf{V}) + \frac{1}{\mu} \mathbf{R}(\mathbf{V})$$

$\mathbf{V}_i^n \longrightarrow \mathbf{V}_i^{n+1}$

❶ $\mu_j = +\infty$

$\Downarrow$

$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{S}(\mathbf{V})$

Aug. System: 5-eq. iso-T

Num. Scheme: op. splitting

Conv.: [G. Allaire and all.]

Surf. Tens.: [J. U. Brackbill and all.]

Heat: 2D implicit

❷ $\mu_j = 0$

$\Downarrow$

$\mathbf{R}(\mathbf{V}) = \mathbf{0}$

update fractions (y, z, ψ) by projecting $\mathbf{V}_i^{n+1/2}$ onto the P, T, g equilibrium

Numerical Scheme

$$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{S}(\mathbf{V}) + \frac{1}{\mu} \mathbf{R}(\mathbf{V})$$

$\mathbf{V}_i^n$ → $\mathbf{V}_i^{n+1}$

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$\mathbf{V}_i^n$ → $\mathbf{V}_i^{n+1/2}$ → $\mathbf{V}_i^{n+1}$

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$\Downarrow$
$\partial_t \mathbf{V} + \operatorname{div} \mathbf{G}(\mathbf{V}) = \mathbf{S}(\mathbf{V})$

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Num. Scheme: op. splitting
Conv.: [G. Allaire and all.]
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ANALYTICAL EOS

Water Example

(τ,ε) fixed

$(\tau_1,\varepsilon_1,\tau_2,\varepsilon_2,y)$ SOLUTION OF

$$\begin{cases} g_1(\tau_1,\varepsilon_1)=g_2(\tau_2,\varepsilon_2)\\ P_1(\tau_1,\varepsilon_1)=P_2(\tau_2,\varepsilon_2)\\ T_1(\tau_1,\varepsilon_1)=T_2(\tau_2,\varepsilon_2)\\ \tau=y\tau_1+(1-y)\tau_2\\ \varepsilon=y\varepsilon_1+(1-y)\varepsilon_2 \end{cases}$$

(P,T) SOLUTION OF

$$\begin{cases} g_1(P,T)=g_2(P,T)\\ \dfrac{\tau-\tau_2(P,T)}{\tau_1(P,T)-\tau_2(P,T)}=\dfrac{\varepsilon-\varepsilon_2(P,T)}{\varepsilon_1(P,T)-\varepsilon_2(P,T)} \end{cases}$$

$$T\mapsto P=P^{\text{sat}}(T)\approx P^{\text{sat}}(T)$$

T SOLUTION OF

$$\frac{\tau-\tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T)-\tau_2^{\text{sat}}(T)}=\frac{\varepsilon-\varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T)-\varepsilon_2^{\text{sat}}(T)} \quad \text{where} \quad \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T)\stackrel{\text{def}}{=}\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha(P^{\text{sat}}(T),T)$$

Analytical EOS

Water Example

(τ,ε) fixed

$(\tau_1,\varepsilon_1,\tau_2,\varepsilon_2,y)$ solution of

$$\begin{cases} g_1(\tau_1,\varepsilon_1)=g_2(\tau_2,\varepsilon_2)\\ P_1(\tau_1,\varepsilon_1)=P_2(\tau_2,\varepsilon_2)\\ T_1(\tau_1,\varepsilon_1)=T_2(\tau_2,\varepsilon_2)\\ \tau=y\tau_1+(1-y)\tau_2\\ \varepsilon=y\varepsilon_1+(1-y)\varepsilon_2 \end{cases}$$

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$$\frac{\tau-\tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T)-\tau_2^{\text{sat}}(T)}=\frac{\varepsilon-\varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T)-\varepsilon_2^{\text{sat}}(T)} \quad \text{where} \quad \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T)\stackrel{\text{def}}{=}\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha(P^{\text{sat}}(T),T)$$

Analytical EOS

Water Example

(τ, ε) fixed

$(\tau_1, \varepsilon_1, \tau_2, \varepsilon_2, y)$ SOLUTION OF

$$\begin{cases} g_1(\tau_1, \varepsilon_1) = g_2(\tau_2, \varepsilon_2) \\ P_1(\tau_1, \varepsilon_1) = P_2(\tau_2, \varepsilon_2) \\ T_1(\tau_1, \varepsilon_1) = T_2(\tau_2, \varepsilon_2) \\ \tau = y\tau_1 + (1-y)\tau_2 \\ \varepsilon = y\varepsilon_1 + (1-y)\varepsilon_2 \end{cases}$$

(P, T) SOLUTION OF

$$\begin{cases} g_1(P,T) = g_2(P,T) \\ \dfrac{\tau - \tau_2(P,T)}{\tau_1(P,T) - \tau_2(P,T)} = \dfrac{\varepsilon - \varepsilon_2(P,T)}{\varepsilon_1(P,T) - \varepsilon_2(P,T)} \end{cases}$$

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$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{where} \quad \begin{pmatrix}\tau \\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \stackrel{\text{def}}{=} \begin{pmatrix}\tau \\ \varepsilon\end{pmatrix}_\alpha (P^{\text{sat}}(T), T)$$

Analytical EOS

Water Example

(τ,ε) fixed

$(\tau_1,\varepsilon_1,\tau_2,\varepsilon_2,y)$ solution of

$$\begin{cases} g_1(\tau_1,\varepsilon_1)=g_2(\tau_2,\varepsilon_2)\\ P_1(\tau_1,\varepsilon_1)=P_2(\tau_2,\varepsilon_2)\\ T_1(\tau_1,\varepsilon_1)=T_2(\tau_2,\varepsilon_2)\\ \tau=y\tau_1+(1-y)\tau_2\\ \varepsilon=y\varepsilon_1+(1-y)\varepsilon_2 \end{cases}$$

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Analytical EOS

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$$\begin{cases} g_1(\tau_1,\varepsilon_1)=g_2(\tau_2,\varepsilon_2)\\ P_1(\tau_1,\varepsilon_1)=P_2(\tau_2,\varepsilon_2)\\ T_1(\tau_1,\varepsilon_1)=T_2(\tau_2,\varepsilon_2)\\ \tau=y\tau_1+(1-y)\tau_2\\ \varepsilon=y\varepsilon_1+(1-y)\varepsilon_2 \end{cases}$$

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least square approximation

$$T\mapsto P=\widehat{P}^{\text{sat}}(T)\approx P^{\text{sat}}(T)$$

T solution of

$$\frac{\tau-\tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T)-\tau_2^{\text{sat}}(T)}=\frac{\varepsilon-\varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T)-\varepsilon_2^{\text{sat}}(T)}\quad \text{where } \begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T)\stackrel{\text{def}}{=}\begin{pmatrix}\tau\\ \varepsilon\end{pmatrix}_\alpha(\widehat{P}^{\text{sat}}(T),T)$$

Tabulated EOS

Water Examples

(τ, ε) fixed

T solution of

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix} \tau \\ \varepsilon \end{pmatrix}_\alpha^{\text{sat}}(T) \quad \text{tabulated}$$

$$\wr\wr$$

$$\frac{\tau - \widehat{\tau}_2^{\text{sat}}(T)}{\widehat{\tau}_1^{\text{sat}}(T) - \widehat{\tau}_2^{\text{sat}}(T)} = \frac{\varepsilon - \widehat{\varepsilon}_2^{\text{sat}}(T)}{\widehat{\varepsilon}_1^{\text{sat}}(T) - \widehat{\varepsilon}_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix} \widehat{\tau} \\ \widehat{\varepsilon} \end{pmatrix}_\alpha^{\text{sat}}(T)$$

Tabulated EOS

Water Examples

(τ, ε) fixed

T solution of

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix}\tau \\ \varepsilon\end{pmatrix}_\alpha^{\text{sat}}(T) \quad \text{tabulated}$$

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$$\frac{\tau - \widehat{\tau}_2^{\text{sat}}(T)}{\widehat{\tau}_1^{\text{sat}}(T) - \widehat{\tau}_2^{\text{sat}}(T)} = \frac{\varepsilon - \widehat{\varepsilon}_2^{\text{sat}}(T)}{\widehat{\varepsilon}_1^{\text{sat}}(T) - \widehat{\varepsilon}_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix}\widehat{\tau} \\ \widehat{\varepsilon}\end{pmatrix}_\alpha^{\text{sat}}(T)$$

least square approximations

TABULATED EOS

Water Examples

(τ, ε) fixed

T SOLUTION OF

$$\frac{\tau - \tau_2^{\text{sat}}(T)}{\tau_1^{\text{sat}}(T) - \tau_2^{\text{sat}}(T)} = \frac{\varepsilon - \varepsilon_2^{\text{sat}}(T)}{\varepsilon_1^{\text{sat}}(T) - \varepsilon_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix} \tau \\ \varepsilon \end{pmatrix}_\alpha^{\text{sat}}(T) \quad \text{tabulated}$$

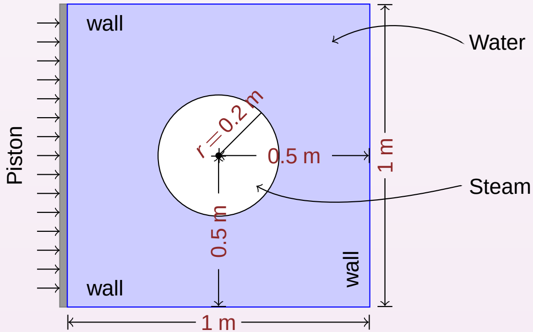
$$\wr\wr$$

$$\frac{\tau - \widehat{\tau}_2^{\text{sat}}(T)}{\widehat{\tau}_1^{\text{sat}}(T) - \widehat{\tau}_2^{\text{sat}}(T)} = \frac{\varepsilon - \widehat{\varepsilon}_2^{\text{sat}}(T)}{\widehat{\varepsilon}_1^{\text{sat}}(T) - \widehat{\varepsilon}_2^{\text{sat}}(T)} \quad \text{with} \quad \begin{pmatrix} \widehat{\tau} \\ \widehat{\varepsilon} \end{pmatrix}_\alpha^{\text{sat}}(T)$$

least square approximations

Outline

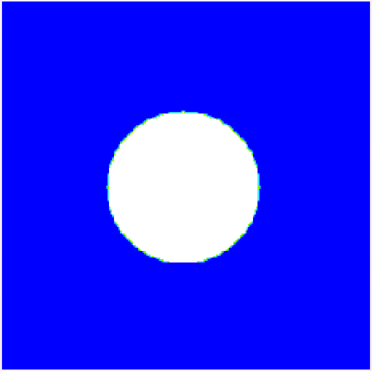
Compression of a Vapor Bubble



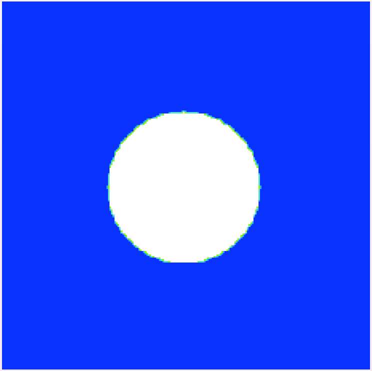
Compression of a 2D Vapor Bubble involving two Stiffened Gases for water and steam.
The piston moves towards right at constant speed $u_p = 30$ m/s.

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

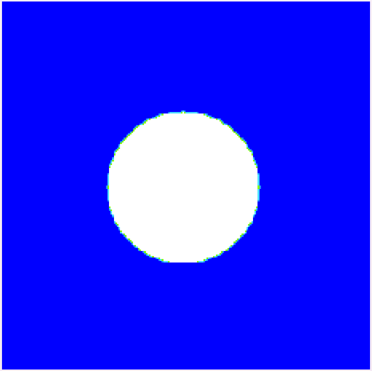


$t = 0.00$ ms

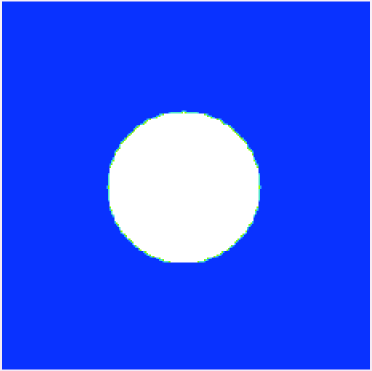
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

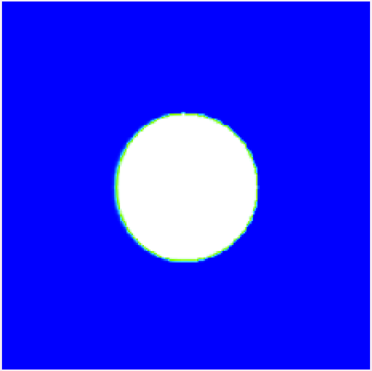


$t = 0.00$ ms

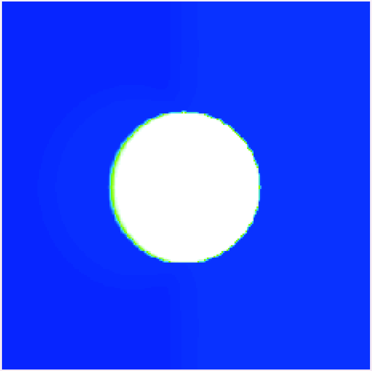
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

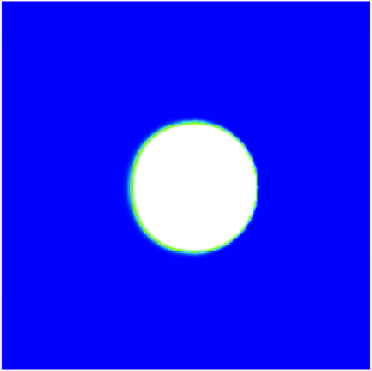


$t = 0.89$ ms

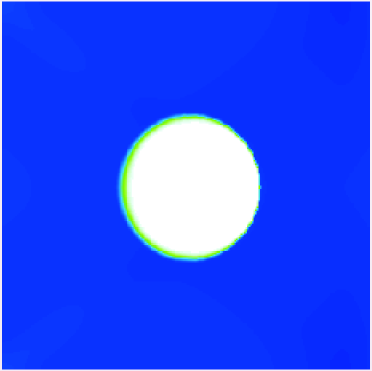
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

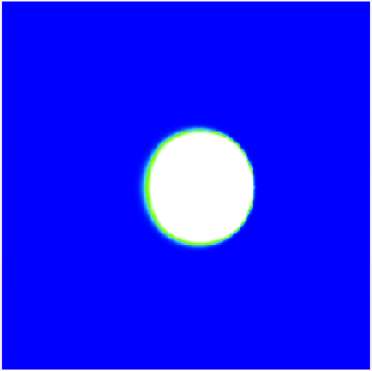


$t = 1.49$ ms

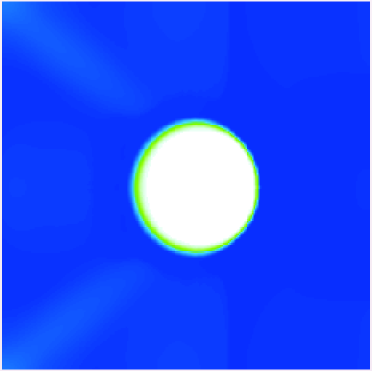
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

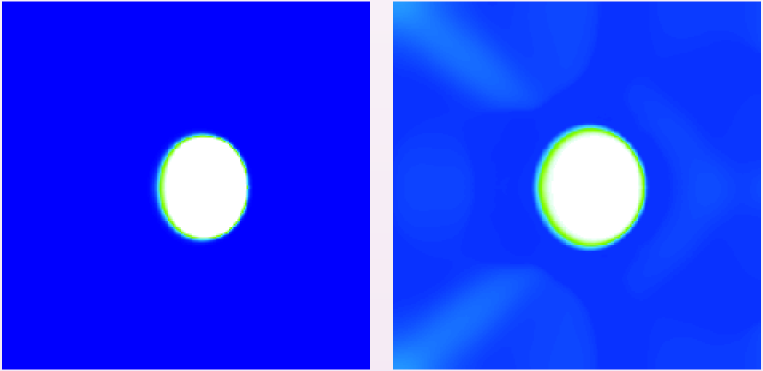


$t = 2.09$ ms

Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y Density ρ

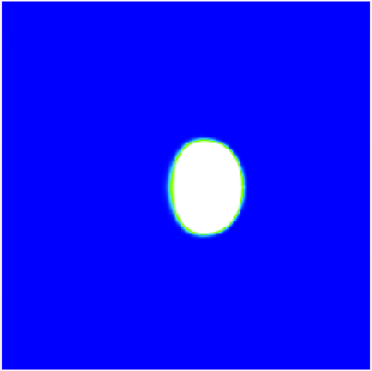


$t = 2.69$ ms

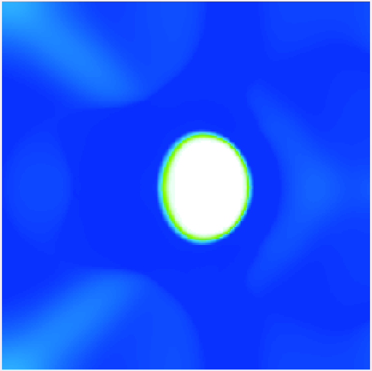
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

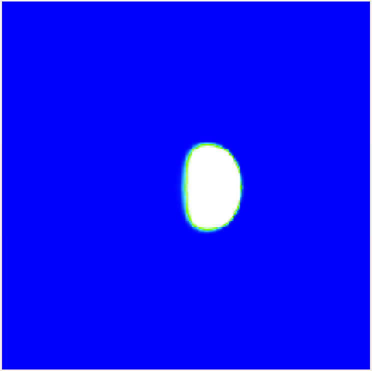


$t = 3.30$ ms

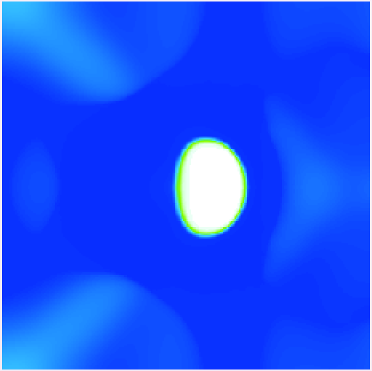
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

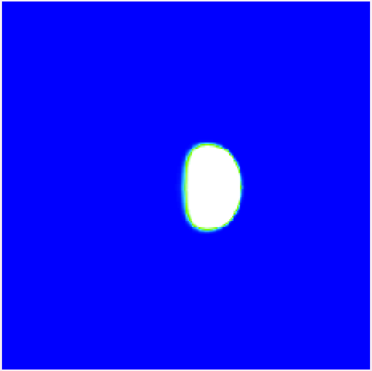


$t = 3.60$ ms

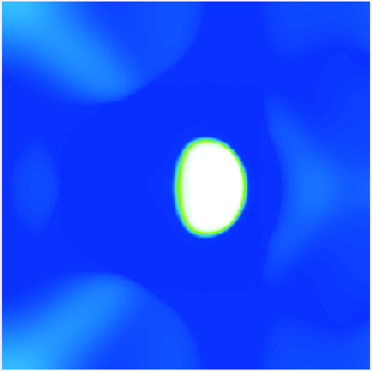
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

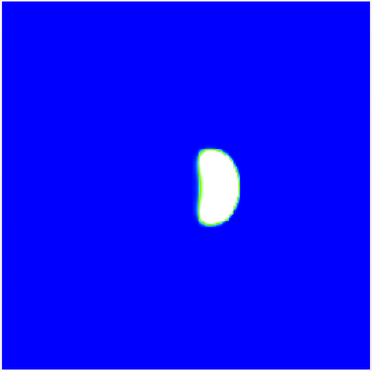


$t = 3.60$ ms

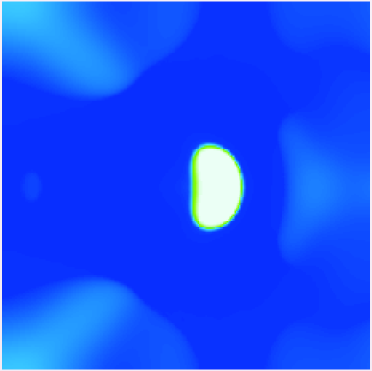
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

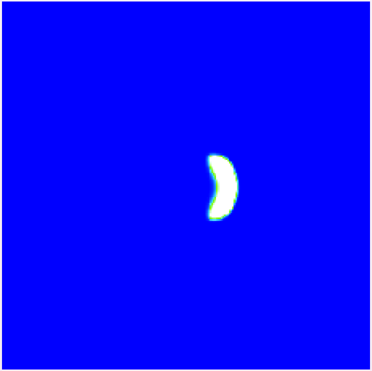


$t = ?$ ms

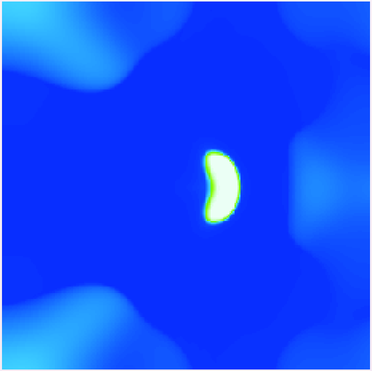
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

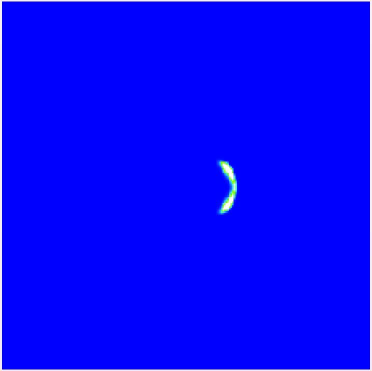


$t = ?$ ms

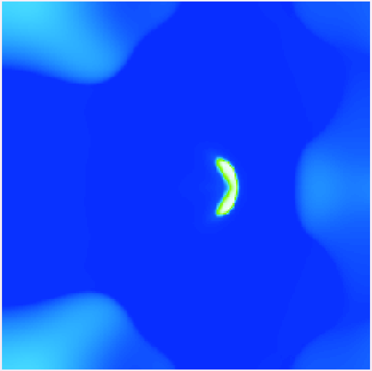
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

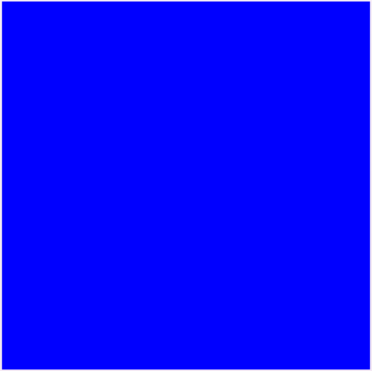


$t = ?$ ms

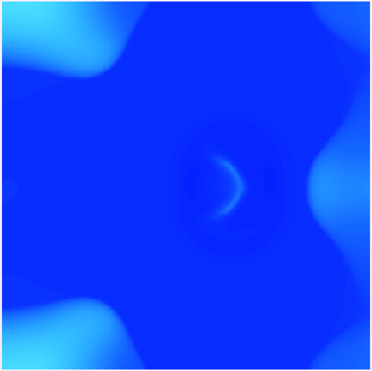
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

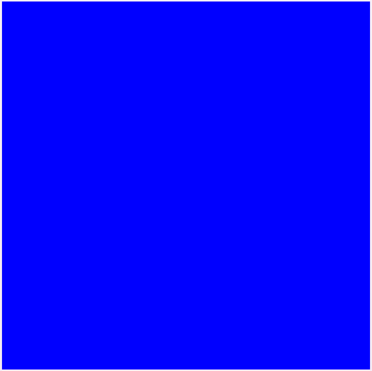


$t = ?$ ms

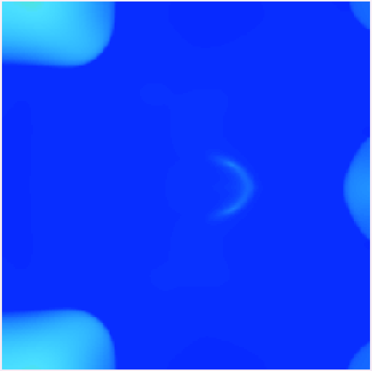
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ

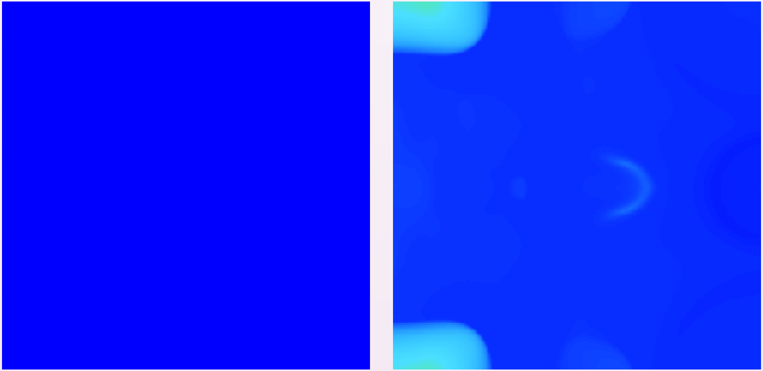


$t = ?$ ms

Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y Density ρ

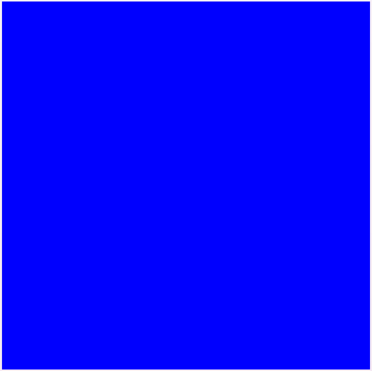


$t = ?$ ms

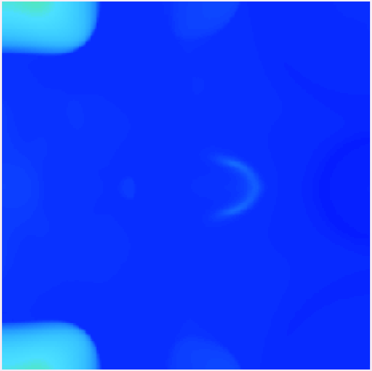
Geometry Play Skip

Compression of a Vapor Bubble

Mass Fraction y



Density ρ



$t = ?$ ms

Geometry Play Skip

Outline

LIQUID-VAPOR PHASE TRANSITION

- Diffuse Interface Model
  - global EOS always at equilibrium (entropy maximization),
  - strict hyperbolicity of the Euler system,
  - uniqueness of Liu solution for the Riemann problem;
- Relaxation Approach
  - 6 (or 5) equation system with relaxation terms;
- Numerical Method
  - operator splitting,
  - general approximate construction of global EOS (and resolution of projection step).

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Return

Stiffened Gas for Water

$$(\tau_\alpha,\varepsilon_\alpha)\mapsto s_\alpha = c_{v_\alpha}\ln(\varepsilon_\alpha - q_\alpha - \pi_\alpha\tau_\alpha) + c_{v_\alpha}(\gamma_\alpha - 1)\ln\tau_\alpha + m_\alpha$$

Phase	c_v [J/(kg·K)]	γ	π [Pa]	q [J/kg]	m [J/(kg·K)]
Water	1816.2	2.35	10^9	-1167.056×10^3	-32765.55596
Steam	1040.14	1.43	0	2030.255×10^3	-33265.65947

TABLE: Parameters proposed by [Le Metayer] for water.

$$(P,T)\mapsto\varepsilon_\alpha = c_{v_\alpha}T\frac{P+\pi_\alpha\gamma_\alpha}{P+\pi_\alpha}+q_\alpha, \qquad (P,T)\mapsto\tau_\alpha = c_{v_\alpha}(\gamma_\alpha-1)\frac{T}{P+\pi_\alpha}.$$

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$\widehat{P}^{\text{sat}}$ defined by using a least square approximation of $\mathfrak{A}$:

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STIFFENED GAS FOR WATER

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Water Tabulated EOS

$$\left.\begin{array}{l} T^i = 278\text{K}\ldots 610\text{K}, \\ \varepsilon_\alpha^{\text{sat}}(T^i),\ \tau_\alpha^{\text{sat}}(T^i) \text{ found in the tables} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \mathfrak{A} = \left\{\left(T_i, \frac{1}{\varepsilon_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \\ \mathfrak{B} = \left\{\left(T_i, \frac{\varepsilon_{\text{liq}}^{\text{sat}}(T_i)}{\varepsilon_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \\ \mathfrak{C} = \left\{\left(T_i, \frac{1}{\tau_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \\ \mathfrak{D} = \left\{\left(T_i, \frac{\tau_{\text{liq}}^{\text{sat}}(T_i)}{\tau_{\text{vap}}^{\text{sat}}(T_i)}\right)\right\}_i \end{array}\right.$$

$\widehat{\varepsilon}_\alpha^{\text{sat}}$ and $\widehat{\tau}_\alpha^{\text{sat}}$ defined by using a least square approximation of $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ and $\mathfrak{D}$:

$$T \mapsto \varepsilon_{\text{vap}}^{\text{sat}} \approx \widehat{\varepsilon}_{\text{vap}}^{\text{sat}} \stackrel{\text{def}}{=} \frac{1}{\sum_{k=0}^{6} a_k T^k} \qquad T \mapsto \varepsilon_{\text{liq}}^{\text{sat}} \approx \widehat{\varepsilon}_{\text{liq}}^{\text{sat}} \stackrel{\text{def}}{=} \widehat{\varepsilon}_{\text{vap}}^{\text{sat}}(T) \sum_{k=0}^{6} b_k T^k$$

$$T \mapsto \tau_{\text{vap}}^{\text{sat}} \approx \widehat{\tau}_{\text{vap}}^{\text{sat}} \stackrel{\text{def}}{=} \frac{1}{\sum_{k=0}^{8} c_k T^k} \qquad T \mapsto \tau_{\text{liq}}^{\text{sat}} \approx \widehat{\tau}_{\text{liq}}^{\text{sat}} \stackrel{\text{def}}{=} \widehat{\tau}_{\text{vap}}^{\text{sat}}(T) \sum_{k=0}^{9} d_k T^k$$