Metodo degli elementi finiti per l’approssimazione numerica del problema delle correnti parassite

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Panoramica

- Correnti parassite

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- Correnti parassite
- Formulazione variazionale in $\mathbf{H}$

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- Potenziale magnetico scalare

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- Iterazione per sottodomini
- Implementazione e prove numeriche

Correnti parassite

Equazioni di Maxwell in regime armonico

$$\begin{cases} \textbf{rot}\,\mathbf{E} = -i\omega\mu\mathbf{H} \\ \textbf{rot}\,\mathbf{H} = \ i\omega\varepsilon\mathbf{E}\ + \sigma\mathbf{E} + \mathbf{J}_e \end{cases}$$

Trascurando le correnti di spostamento
si ottiene il modello per le correnti parassite:

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- $\overline{\Omega}_C \subset \Omega$ conduttore
- $\Omega_I := \Omega \setminus \overline{\Omega}_C$ isolante perfetto
- $\Gamma := \partial\Omega_I \cap \partial\Omega_C$ interfaccia $\Rightarrow$

$$\begin{cases} \partial\Omega_C = \Gamma \\ \partial\Omega_I = \partial\Omega \cup \Gamma \end{cases}$$

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  - 0 in Ω_I
- $\mathbf{H} \times \mathbf{n} = \mathbf{0} \quad$ su $\partial\Omega$

Vincoli per l'esistenza

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[Poiché $\sigma_{|\Omega_I} = 0$]

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[Poiché $\mathbf{H} \times \mathbf{n} = \mathbf{0}$ su $\partial\Omega$ il dato $\mathbf{J}_e$ deve soddisfare inoltre]

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[Moltiplicando per una funzione test $\mathbf{v} \in V \subset H_0(\textbf{rot};\Omega)$ ed integrando su tutto il dominio Ω...]

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[Utilizzando la formula di Green...]

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$$\Big[Se \quad V := \{\mathbf{v}\in H_0(\textbf{rot};\Omega)\,|\,\textbf{rot}\,\mathbf{v}_I = \mathbf{0}\ \text{ in } \Omega_I\}\Big]$$

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$$\left[\mathbf{rot}\,\mathbf{H} = \sigma\mathbf{E} + \mathbf{J}_e \Rightarrow \mathbf{E}_C = \sigma^{-1}(\mathbf{rot}\,\mathbf{H}_C - \mathbf{J}_{e,C})\right]$$

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$$= \int_{\Omega_C} \sigma^{-1} \mathbf{J}_{e,C} \cdot \mathbf{rot}\, \overline{\mathbf{v}}_C$$

dove

$$\mathbf{H} \in V^{\mathbf{J}_{e,I}} := \{\mathbf{v} \in H_0(\mathbf{rot};\Omega) \mid \mathbf{rot}\, \mathbf{v}_I = \mathbf{J}_{e,I} \ \text{ in } \Omega_I\}$$

$$\mathbf{v} \ \in V = \{\mathbf{v} \in H_0(\mathbf{rot};\Omega) \mid \mathbf{rot}\, \mathbf{v}_I = \mathbf{0} \ \text{ in } \Omega_I\}$$

Esistenza ed unicità

$\mathbf{H}_e \in H_0(\textbf{rot};\Omega)$ tale che

$$\begin{cases} \textbf{rot}\,\mathbf{H}_{e,I} = \mathbf{J}_{e,I} & \text{in } \Omega_I \\ \mathbf{H}_{e,I} \times \mathbf{n} = \mathbf{0} & \text{su } \partial\Omega_I \setminus \Gamma \end{cases}$$

Indichiamo con $\mathbf{Z} := \mathbf{H} - \mathbf{H}_e$, allora $\mathbf{Z} \in V$ ed il problema diviene...

Esistenza ed unicità

Cercare $\mathbf{Z} \in V$ tale che $\forall\ \mathbf{v} \in V$

$$\mathcal{A}(\mathbf{Z}, \mathbf{v}) = L(\mathbf{v})$$

dove

$$\mathcal{A}(\mathbf{w}, \mathbf{v}) := \int_{\Omega_C} \sigma^{-1}\, \mathbf{rot}\, \mathbf{w}_C \cdot \mathbf{rot}\, \overline{\mathbf{v}}_C + i\omega \int_{\Omega} \mu \mathbf{w} \cdot \overline{\mathbf{v}}$$

$$L(\mathbf{v}) := -\mathcal{A}(\mathbf{H}_e, \mathbf{v}) + \int_{\Omega_C} \sigma^{-1} \mathbf{J}_{e,C} \cdot \mathbf{rot}\, \overline{\mathbf{v}}_C$$

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$\mathbf{v}_{|\Omega_I} = \nabla\psi_I \in H(\textbf{rot};\Omega_I)$

dove $\psi_I \in H^1_{0,\partial\Omega}(\Omega_I)$ (infatti $\mathbf{v}_I \times \mathbf{n} = \mathbf{0}$ su $\partial\Omega$ permette di scegliere $\psi_I = 0$ su $\partial\Omega$)

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Introduciamo allora lo spazio

$$W := \{(\mathbf{v}_C, \varphi_I) \in H(\textbf{rot}; \Omega_C) \times H^1_{0,\partial\Omega}(\Omega_I) \mid \mathbf{v}_C \times \mathbf{n}_C + \nabla\varphi_I \times \mathbf{n}_I = \mathbf{0} \text{ su } \Gamma\}$$

Formulazione bidominio

Cercare $(\mathbf{Z}_C, \psi_I) \in W$ tale che $\forall\, (\mathbf{v}_C, \varphi_I) \ \in W$

$$\int_{\Omega_C} [i\omega\mu_C \mathbf{Z}_C \cdot \overline{\mathbf{v}}_C + \sigma^{-1}\, \mathbf{rot}\, \mathbf{Z}_C \cdot \mathbf{rot}\, \overline{\mathbf{v}}_C] +$$

$$+ \int_{\Omega_I} [i\omega\mu_I \nabla\psi_I \cdot \nabla\overline{\varphi}_I] = F(\mathbf{v}_C, \varphi_I)$$

dove

$$F(\mathbf{v}_C, \varphi_I) := -\int_{\Omega_I} [i\omega\mu_I \mathbf{H}_{e,I} \cdot \nabla\overline{\varphi}_I] +$$

$$+ \int_{\Omega_C} \left[\sigma^{-1}(\mathbf{J}_{e,C} - \mathbf{rot}\, \mathbf{H}_{e,C}) \cdot \mathbf{rot}\, \overline{\mathbf{v}}_C - i\omega\mu_C \mathbf{H}_{e,C} \cdot \overline{\mathbf{v}}_C\right]$$

Approssimazione di Galerkin

$$W := \{(\mathbf{v}_C, \varphi_I) \in \; H(\textbf{rot}; \Omega_C) \; \times \; H^1_{0,\partial\Omega}(\Omega_I) \; | \\ \mathbf{v}_C \times \mathbf{n}_C + \nabla\varphi_I \times \mathbf{n}_I = \mathbf{0} \text{ su } \Gamma\}$$

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EF lineari Nédélec

EF lineari Lagrange

EF di Lagrange $\rightsquigarrow H^1_{0,\partial\Omega}(\Omega_I)$

- $V_{I,h} := \left\{ \psi_h \in C^0(\overline{\Omega}_I) \mid \psi_{h|K} \in \mathbb{Q}_1 \ \forall\, K \in T_h \right\}$

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- base $\rightsquigarrow \varphi_{i,h}(\mathbf{a}_j) = \delta_{ij} \qquad \forall i, j = 0 \ldots n_{h,L}$

EF di Nédélec $\rightsquigarrow H(\textbf{rot}, \Omega_C)$

- $N_{C,h} := \{\mathbf{v}_h \in H(\textbf{rot}; \Omega_C) \mid \mathbf{v}_{h|K} \in \mathbb{Q}_{0,1,1} \times \mathbb{Q}_{1,0,1} \times \mathbb{Q}_{1,1,0} \; \forall\, K \in T_h\}$

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$$\mathbf{v}_C \times \mathbf{n}_C + \nabla\varphi_I \times \mathbf{n}_I = \mathbf{0} \text{ su } \Gamma\}$$

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Formulazione forte

1. $\textbf{rot}(\sigma^{-1}\,\textbf{rot}\,\mathbf{H}_C) + i\omega\mu_C\mathbf{H}_C = \textbf{rot}(\sigma^{-1}\mathbf{J}_{e,C})$ in Ω_C

2. $\text{div}(\mu_I\nabla\psi_I) = -\,\text{div}(\mu_I\mathbf{H}_{e,I})$ in Ω_I
3. $\psi_I = 0$ su $\partial\Omega_I \setminus \Gamma$

4. $\mathbf{H}_C \times \mathbf{n}_C = -\nabla\psi_I \times \mathbf{n}_I - \mathbf{H}_{e,I} \times \mathbf{n}_I$ su Γ
5. $\mu_I\nabla\psi_I \cdot \mathbf{n}_I = -\mu_C\mathbf{H}_C \cdot \mathbf{n}_C - \mu_I\mathbf{H}_{e,I} \cdot \mathbf{n}_I$ su Γ

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Procedura Dirichlet-Neumann

Dato $\boldsymbol{\lambda}_h^0 \in N_{\Gamma,h}$ (spazio discreto della traccia di $N_{C,h}$ su Γ),
per ogni $k \geq 0$ calcolare $\mathbf{H}_{C,h}^k \in N_{C,h}$ tale che per ogni $\mathbf{v}_{C,h} \in N_{C,h}^0 := N_{C,h} \cap H_0(\textbf{rot}; \Omega_C)$

$$\begin{cases} \displaystyle\int_{\Omega_C} \left[\sigma^{-1} \textbf{rot}\, \mathbf{H}_{C,h}^k \cdot \textbf{rot}\, \overline{\mathbf{v}}_{C,h} + i\omega\mu_C \mathbf{H}_{C,h}^k \cdot \overline{\mathbf{v}}_{C,h}\right] = \\ \displaystyle = \int_{\Omega_C} \sigma^{-1} \mathbf{J}_{e,C} \cdot \textbf{rot}\, \overline{\mathbf{v}}_{C,h} \\ \mathbf{H}_{C,h}^k \times \mathbf{n}_C = -\boldsymbol{\lambda}_h^k - \mathbf{H}_{e,I,h} \times \mathbf{n}_I \end{cases}$$

Procedura Dirichlet-Neumann

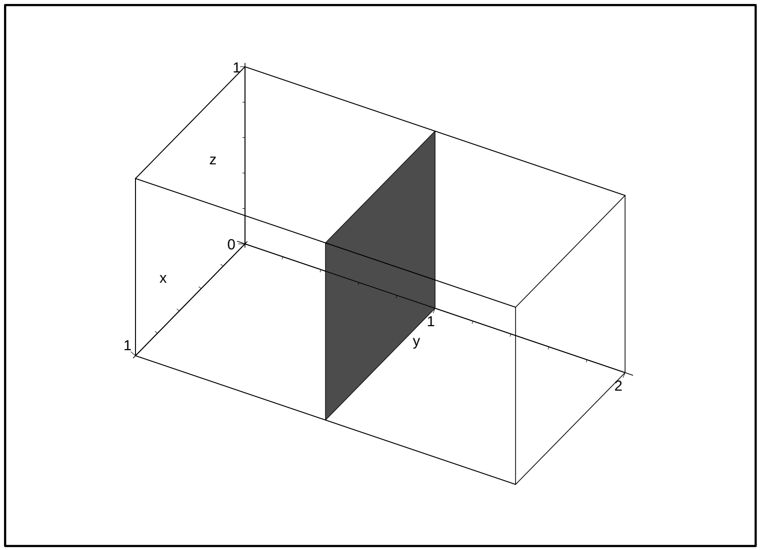
quindi calcolare $\psi^k_{I,h} \in V_{I,h}$ tale che per ogni $\varphi_{I,h} \in V^0_{I,h} := V_{I,h} \cap H^1_{0,\partial\Omega}(\Omega_I)$

$$\int_{\Omega_I} \mu_I \nabla \psi^k_{I,h} \cdot \nabla \overline{\varphi}_{I,h} =$$
$$= - \int_{\Omega_I} \mu_I \mathbf{H}_{e,I,h} \cdot \nabla \overline{\varphi}_{I,h} - \int_\Gamma \mu_C \mathbf{H}^k_{C,h} \cdot \mathbf{n}_C \, \overline{\varphi}_{I,h}$$

e aggiornare il dato

$$\boldsymbol{\lambda}^{k+1}_h = (1-\vartheta)\boldsymbol{\lambda}^k_h + \vartheta \, (\nabla \psi^k_{I,h} \times \mathbf{n}_I) \quad \text{su } \Gamma$$

Dominio Computazionale



- $\Omega = (0, 1) \times (0, 2) \times (0, 1)$
- $\Omega_C = (0, 1) \times (0, y_\Gamma) \times (0, 1)$
- $\Omega_I = (0, 1) \times (y_\Gamma, 2) \times (0, 1)$
- $\Gamma = \{(x, y_\Gamma, z) \,|\, x, z \in (0, 1)\}$

- $\mathbf{H}_C \times \mathbf{n}_C = \mathbf{0}$ su $\partial\Omega_C \cap \partial\Omega$

Convergenza

Errore all'iterazione k-esima:

$$\boldsymbol{\Xi}_C^k = \mathbf{H}_C - \mathbf{H}_C^k$$
$$\xi_I^k = \psi_I - \psi_I^k$$

$$\begin{cases} \textbf{rot}(\sigma^{-1}\,\textbf{rot}\,\boldsymbol{\Xi}_C^k) + i\omega\mu_C\boldsymbol{\Xi}_C^k = \mathbf{0} & \text{in } \Omega_C \\ \boldsymbol{\Xi}_C^k \times \mathbf{n}_C = \mathbf{0} & \text{su } \partial\Omega_C \\ \boldsymbol{\Xi}_C^k \times \mathbf{n}_C = -\boldsymbol{\chi}^k & \text{su } \Gamma \end{cases}$$

Convergenza

$$\begin{cases} -\,\mathrm{div}(\mu_I \nabla \xi_I^k) = 0 & \text{in } \Omega_I \\ \mu_I \nabla \xi_I^k \cdot \mathbf{n}_I = -\mu_C \boldsymbol{\Xi}_C^k \cdot \mathbf{n}_C & \text{su } \Gamma \\ \xi_I^k = 0 & \text{su } \partial\Omega_I \setminus \Gamma \end{cases}$$

$$\boldsymbol{\chi}^{k+1} = (1-\vartheta)\boldsymbol{\chi}^k + \vartheta(\nabla \xi_I^k \times \mathbf{n}_I) \quad \text{su } \Gamma$$

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- indipendenza da h

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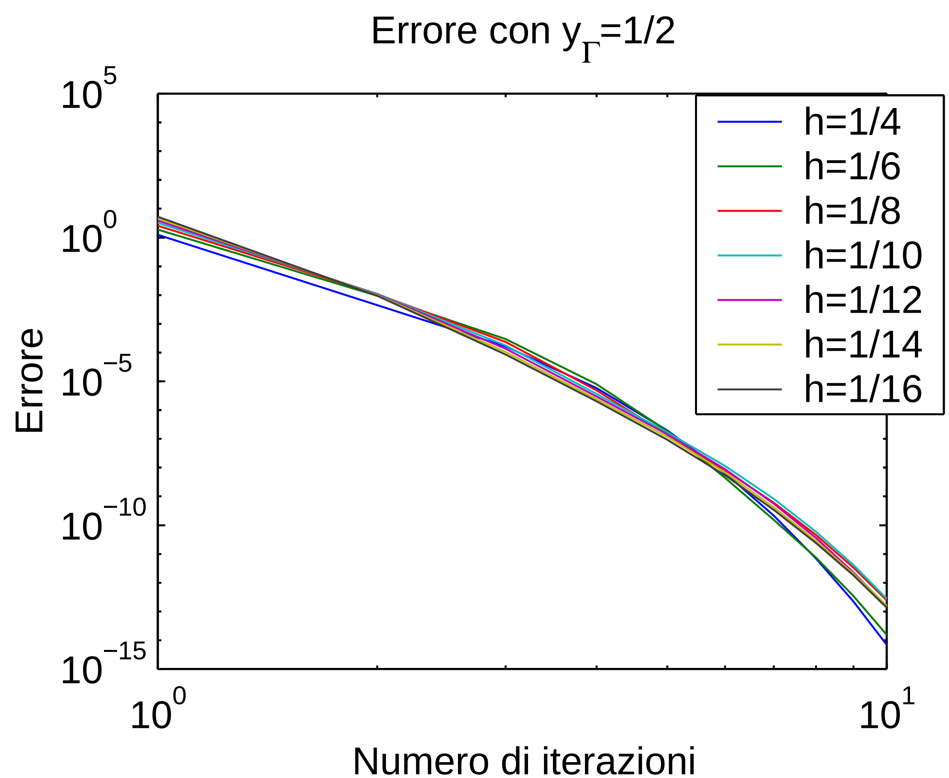
Indipendenza da h, $\boldsymbol{\lambda} : \Gamma \longrightarrow \mathbb{R}^3$



$$\text{Errore} := ||\boldsymbol{\Xi}_C^k||^2_{H(\textbf{rot};\Omega_C)} + ||\nabla \xi_I^k||^2_{L^2(\Omega_I)}$$

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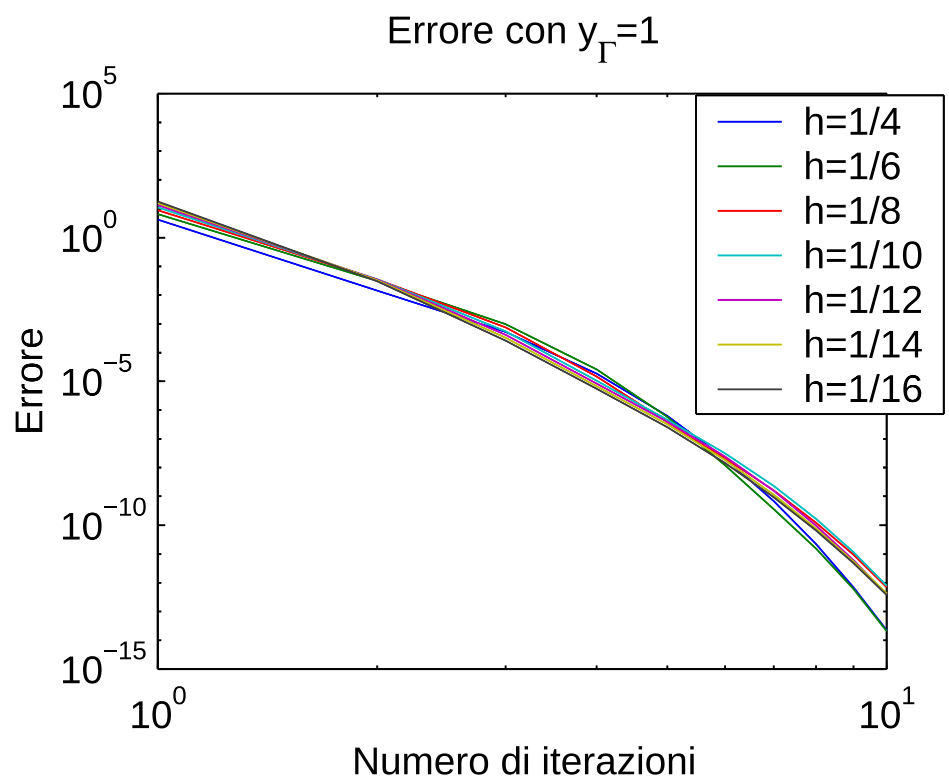

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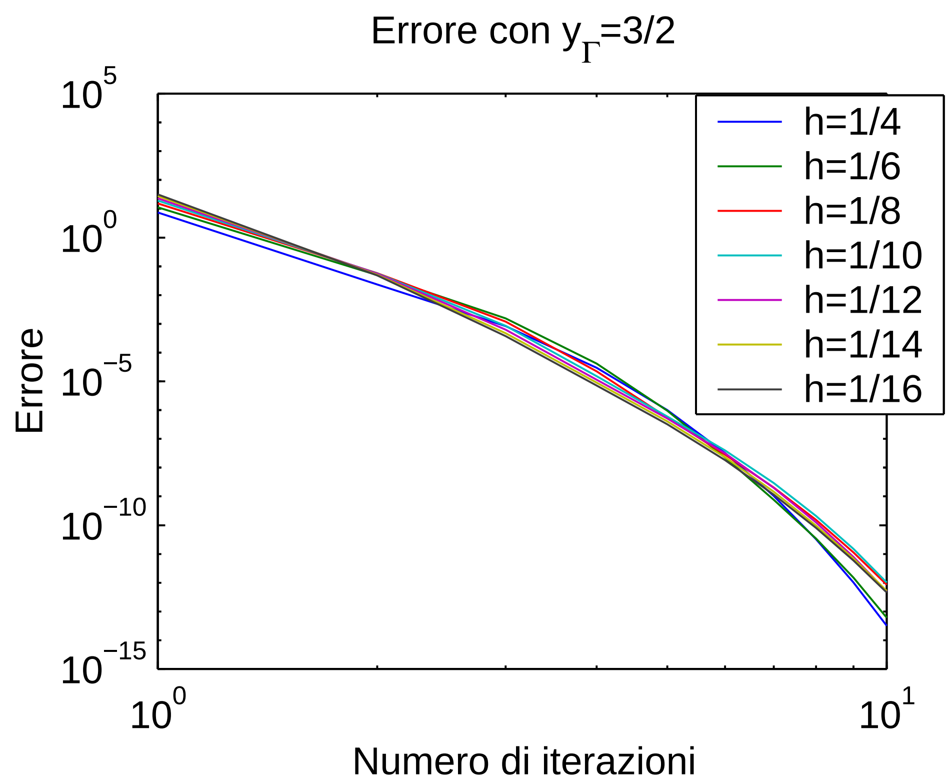

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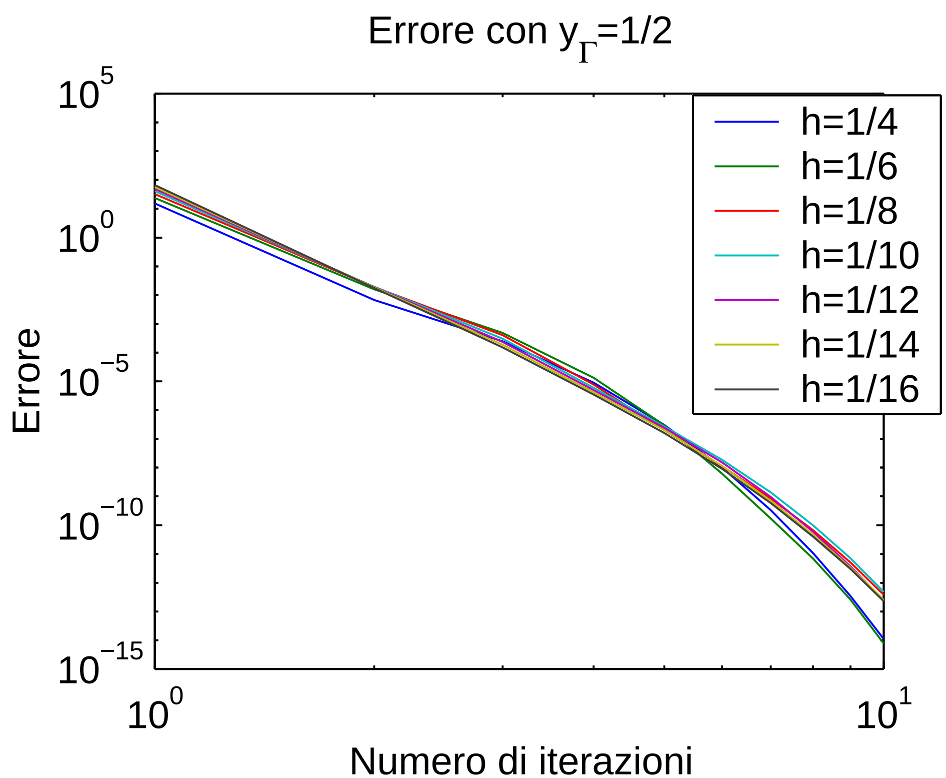

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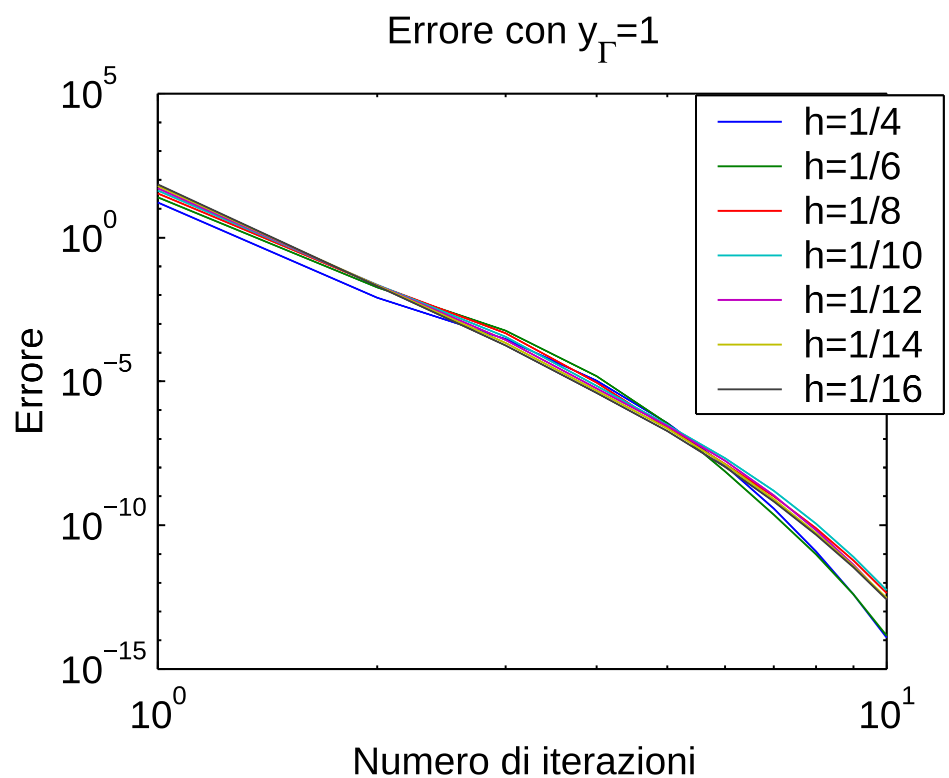

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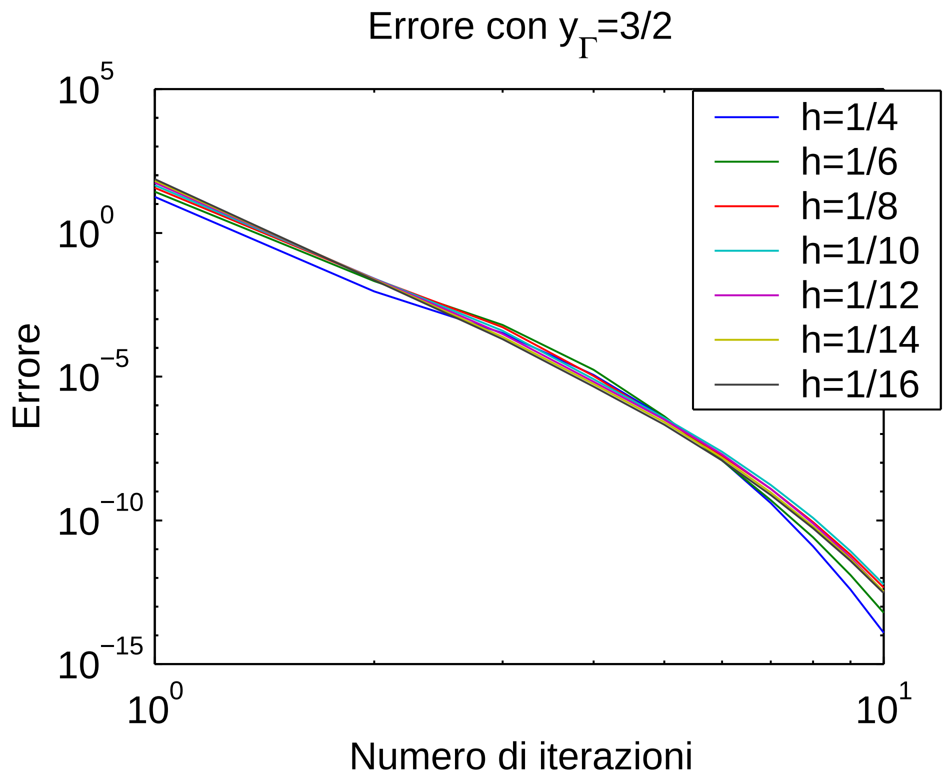

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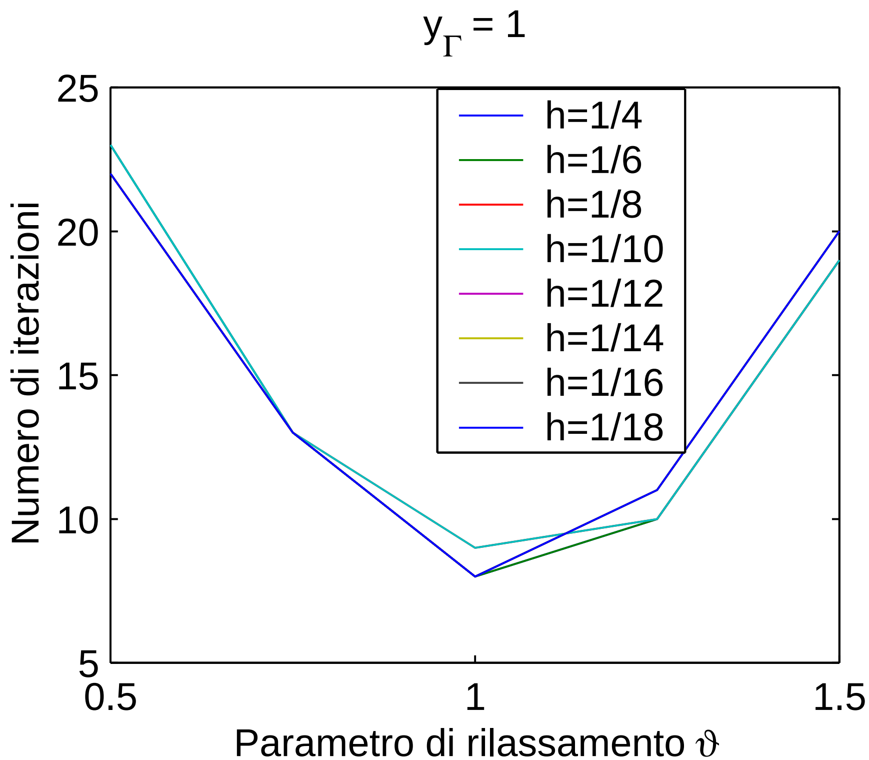

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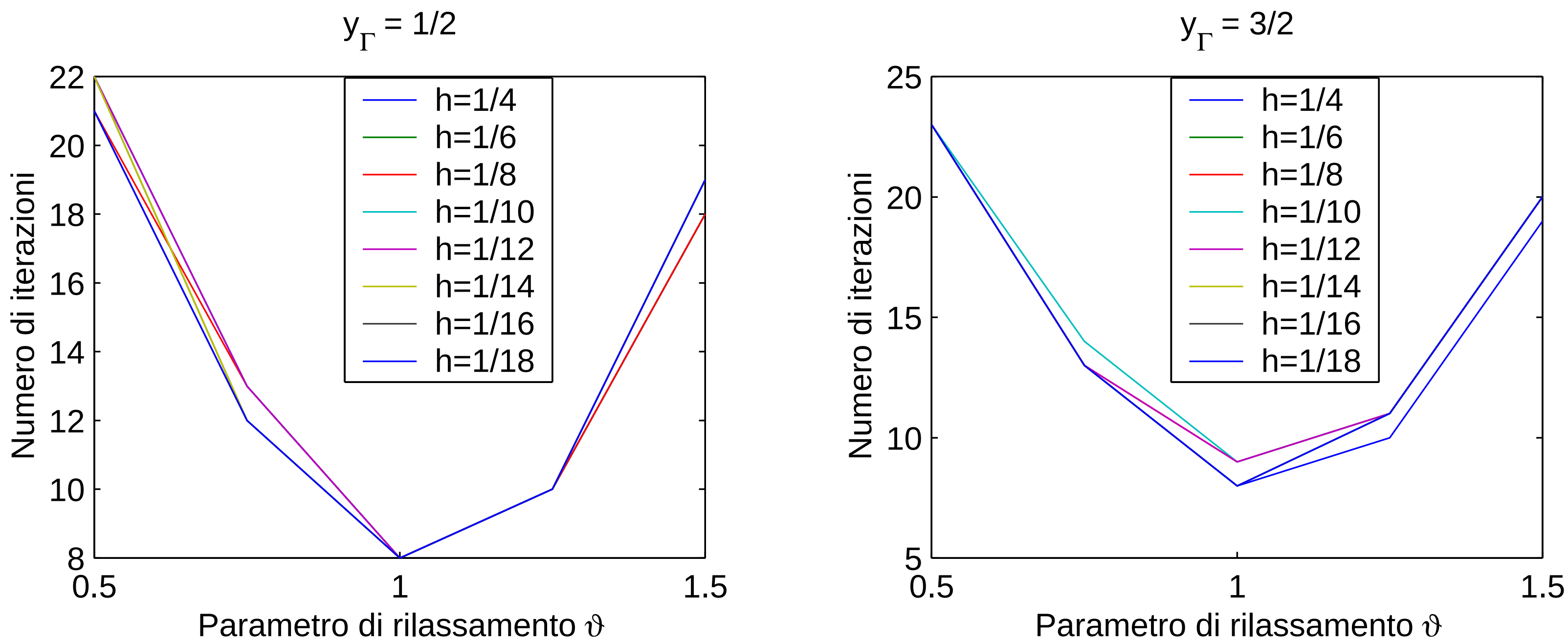

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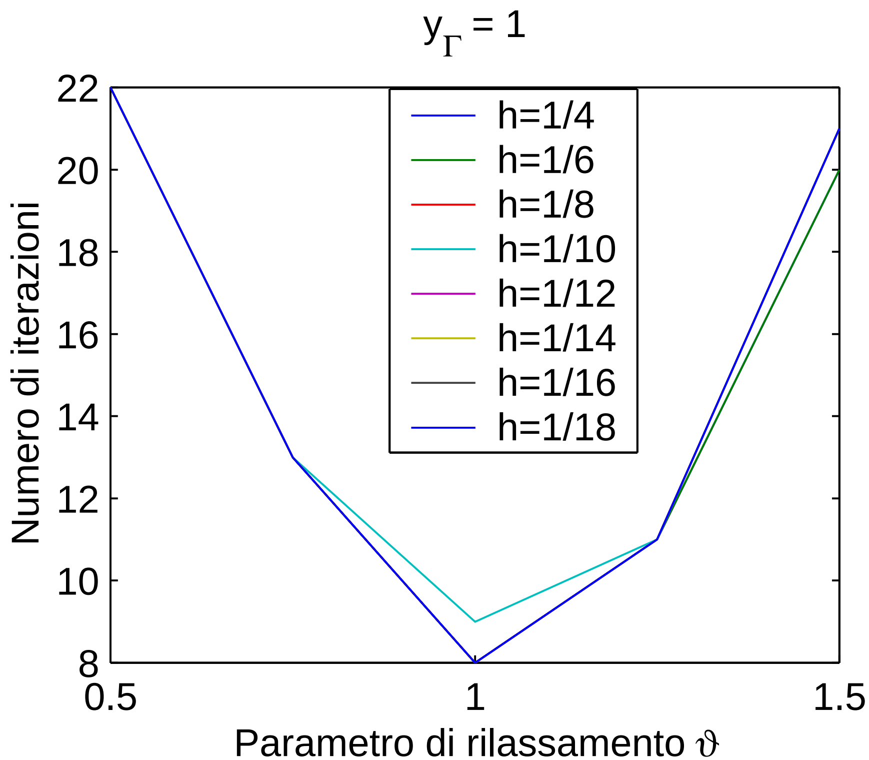

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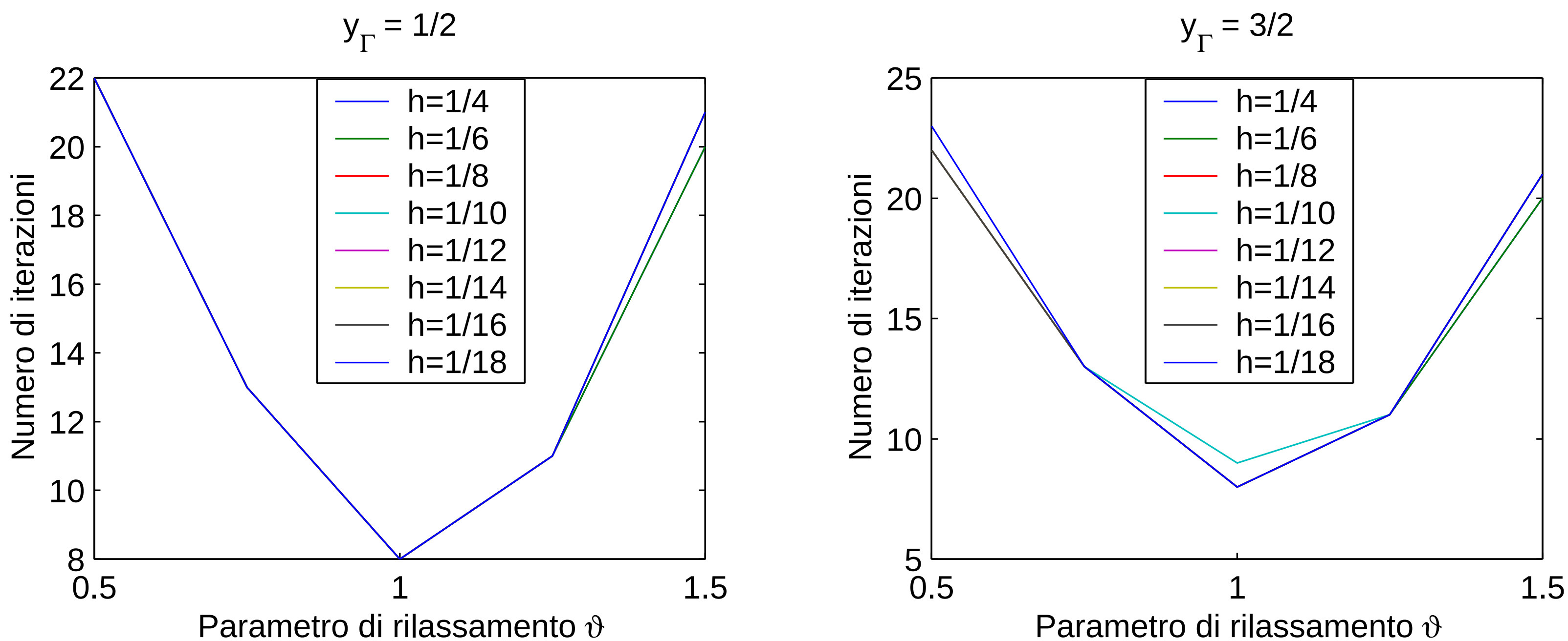

Robustezza rispetto a $k := \omega\sigma\mu$, $\boldsymbol{\lambda} : \Gamma \longrightarrow \mathbb{R}^3$

h \ k	10^{-1}	1	10	10^2
1/6	11	8	5	4
1/8	12	9	5	4
1/10	14	9	5	4
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